11.5: Alternating Series

examples of alternating series — form

i) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$ $\quad (-1)^{n+1} b_n,\ b_n > 0$

ii) $-1 + 1 + -1 + +1 + -1 + 1 - 1 \dots$ $\quad (-1)^n b_n,\ b_n > 0.$

The alternating series test

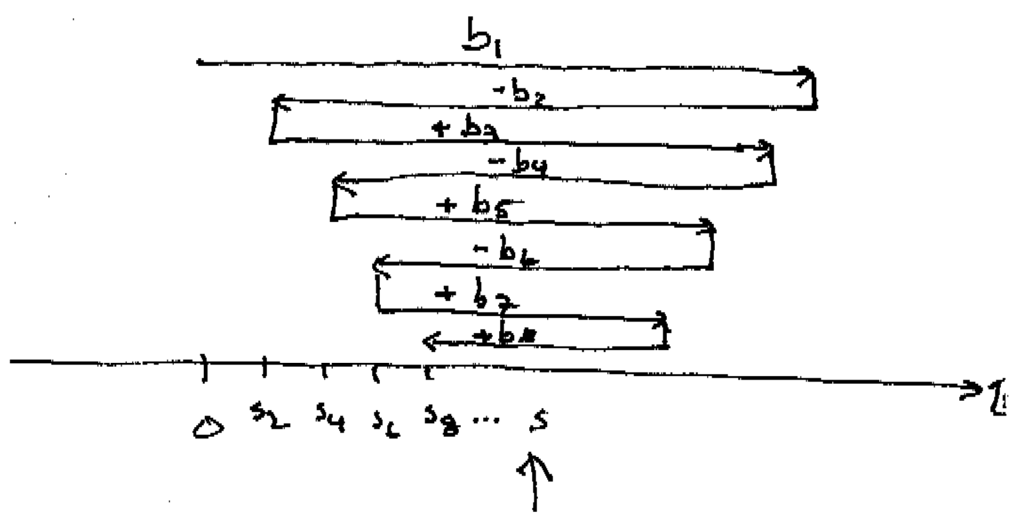
If $\sum_{n=1}^{\infty} (-1)^{n+1} b_n = b_1 - b_2 + b_3 - \dots \quad (b_n > 0)$

magnitude decreasing (monotonic)

satisfies the conditions $b_{n+1} \le b_n \ \forall n$ and $\lim_{n\to\infty} b_n = 0$, then the series converges.

decreases to zero

The picture



□ proof.

Recall from 11.1 #72 that if $\lim_{n\to\infty} a_{2n} = \lim_{n\to\infty} a_{2n+1} = L$ that $\lim_{n\to\infty} a_n = L$.

So, notice

$$s_2 = b_1 - b_2 \geq 0$$
$$s_4 = s_2 + (b_3 - b_4) \geq s_2$$
$$s_6 = s_4 + (b_5 - b_6) \geq s_4$$
$$\vdots$$
$$s_{2n} = s_{2n-2} + (\quad) \geq s_{2n-2}$$

So, $\{s_{2n}\}$ is increasing.

Now $s_{2n} = b_1 - (b_2 - b_3) - \dots - (b_{2n-2} - b_{2n-1}) - b_{2n} < b$

So $\{s_{2n}\}$ is increasing and bounded above. Hence it converges. and $\lim_{n\to\infty} s_{2n} = s$.

Now $\lim_{n\to\infty} s_{2n+1} = \lim_{n\to\infty} (s_{2n} + b_{2n+1})$

$$= \lim_{n\to\infty} s_{2n} + \lim_{n\to\infty} b_{2n+1}$$
$$= s + 0$$
$$= s$$

Since the even & odd partial sums converge to s, $\sum_{n=1}^{\infty} (-1)^{n+1} b_n = s$. ▪

11.5
3/3

Ex1: Test for convergence: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

Ex2: Test for convergence: $\sum_{n=1}^{\infty} (-1)^{n+1}$.

~~Ex3~~: What do I do if I can't easily see that $\{b_n\}$ is decreasing?

~~Ex~~1 What if the series begins by increasing?

Estimating Sums (Friendly)

From the picture and proof, the sum s always lives between s_n and s_{n+1} so $|s - s_n| \leq |s_{n+1} - s_n| = b_{n+1}$. So if $s = \sum_{n=1}^{\infty} (-1)^{n+1} b_n$ is the sum of an alternating series and $0 \leq b_{n+1} \leq b_n$ and $\lim_{n \to \infty} b_n = 0$, then

$$|R_n| = |s - s_n| \leq b_{n+1}.$$

Ex4: Approx $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$ to 4 places.