

11.3
 1/4

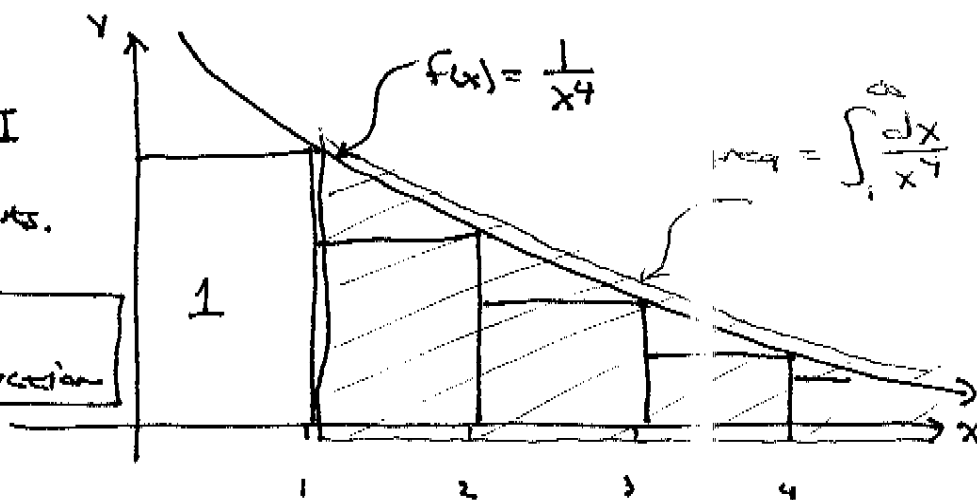
11.3: Integral Test for Estimating Sums

Ex1: $\sum_{n=1}^{\infty} \frac{1}{n^4} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \dots$

Reminds us of the convergent $\int_1^{\infty} \frac{dx}{x^4} = \lim_{b \rightarrow \infty} \left. -\frac{1}{3x^3} \right|_1^b = \frac{1}{3}$

Since I believe it will converge, I use right endpoints.

The series is below the function



AND we have $0 < \sum_{n=1}^{\infty} \frac{1}{n^4} < 1 + \frac{1}{3}$. Hence it converges. Using Mathematica, we know that it converges to $\pi^4/90$. But, most importantly it converges.

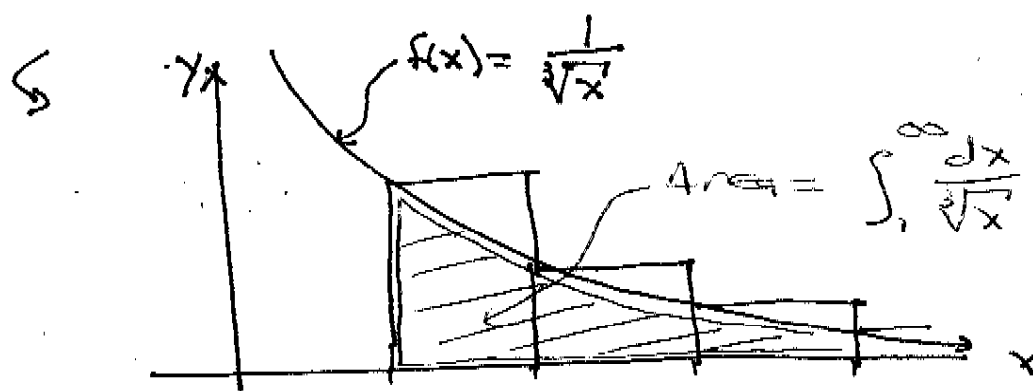
Ex2: $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}} = 1 + \frac{1}{\sqrt[3]{2}} + \frac{1}{\sqrt[3]{3}} + \dots$

This reminds us of the divergent $\int_1^{\infty} \frac{dx}{\sqrt{x}}$.

Since I believe the series will diverge, I use left endpoints.

7

11.3
2/4



sequence
The series
is above the
function

and $\int_1^{\infty} \frac{dx}{\sqrt{x}} < \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$. So the series diverges.

The Integral Test Suppose f is a cont., pos., decreasing fct on $1 \leq x < \infty$ and let $a_n = f(n)$. Then $\sum_{n=1}^{\infty} a_n$ is convergent iff $\int_1^{\infty} f(x) dx$ is convergent.

Ex 3: Does $\sum_{n=1}^{\infty} n e^{-n}$ converge?

Ex 4: For what values of p does $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge?

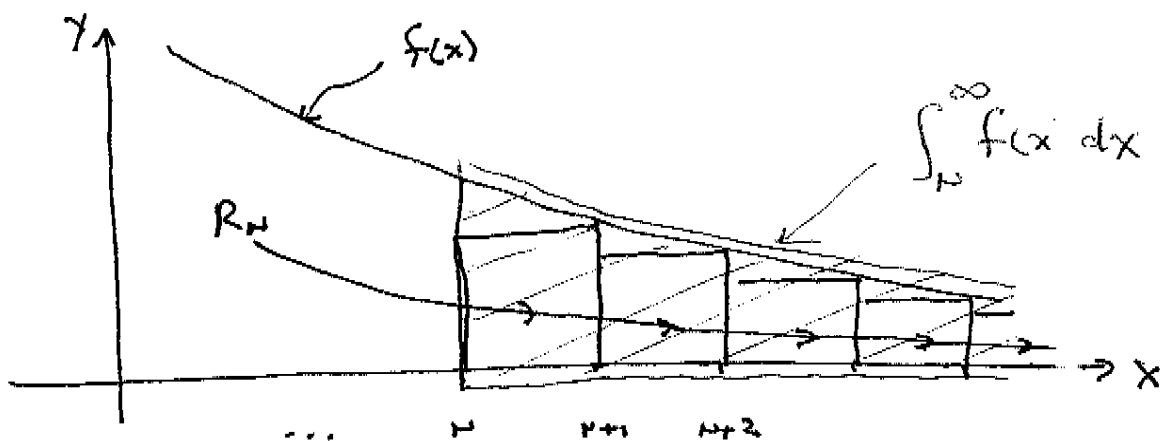
Estimating Sums

Suppose we know $\sum a_n$ converges but we cannot evaluate it directly. So, we want to estimate the series w/ the n th partial sum S_n . But, this leaves us w/ an important question.

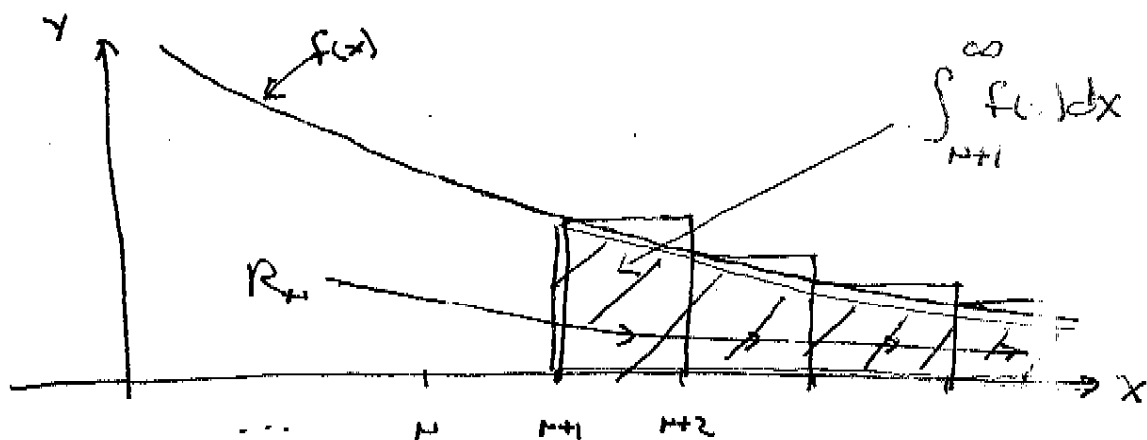
11.3
3/4

What is the error associated w/ each partial sum!

Remainder: $R_n = a_{n+1} + a_{n+2} + \dots$
in S_n .



and $R_n \leq \int_n^{\infty} f(x) dx$. similarly



and $\int_{n+1}^{\infty} f(x) dx \leq R_n$.

Remainder Estimate for the Integral Test

If $\sum a_n$ converges by the I.T. and $R_n = S - S_n$,

then $\int_{n+1}^{\infty} f(x) dx \leq R_n \leq \int_n^{\infty} f(x) dx$.

11.4
4/4

Ex 5: Suppose $\sum_{n=1}^{\infty} \frac{1}{n^5} = S$.

- find S_5
- bound R_5
- For what N 's is $R_N \leq 0.00005$?

$$a) \frac{1}{1^5} + \frac{1}{2^5} + \frac{1}{3^5} + \frac{1}{4^5} + \frac{1}{5^5} \approx 1.03662.$$

$$b) \int_6^{\infty} \frac{dx}{x^5} \leq R_N \leq \int_5^{\infty} \frac{dx}{x^5}$$

$$\Rightarrow \lim_{t \rightarrow \infty} \left[-\frac{1}{4x^4} \right]_6^t \leq R_N \leq \lim_{t \rightarrow \infty} \left[-\frac{1}{4x^4} \right]_5^t$$

$$\Rightarrow \frac{1}{4 \cdot 6^4} \leq R_N \leq \frac{1}{4 \cdot 5^4}$$

$$c) R_N \leq \int_N^{\infty} \frac{dx}{x^5} \leq 0.00005$$

AND $\lim_{t \rightarrow \infty} \left[-\frac{1}{4x^4} \right]_N^t$

$$\Rightarrow R_N \leq \frac{1}{4N^4} \leq 0.00005$$

so, solve $\frac{1}{4N^4} \leq 0.00005$

$$\Rightarrow \frac{1}{4(0.00005)} \leq N^4$$

$$8.41 \leq N$$

so, so long as $N \geq 9$, R_N will be less than 0.00005.