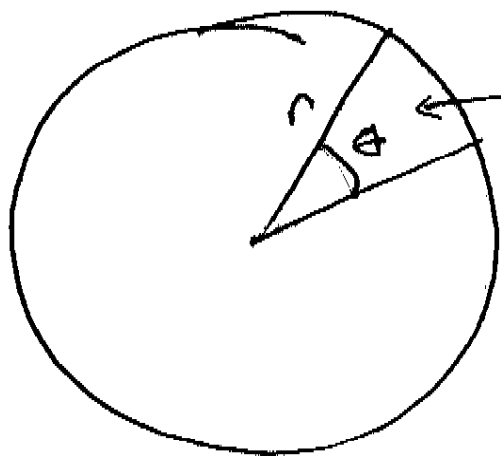


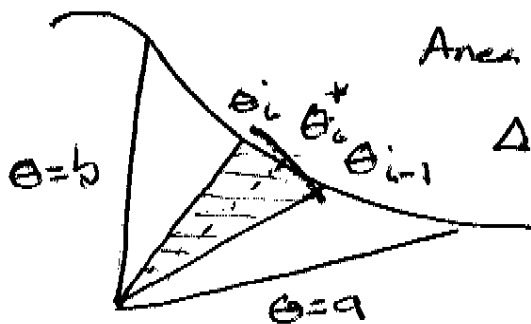
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## 10.4: Areas and Lengths in Polar Coordinates



Area of the wedge

$$A = \frac{1}{2} r^2 \theta.$$



Area of wedge

$$\Delta A_i = \frac{1}{2} r(\theta_i^*)^2 \Delta \theta$$

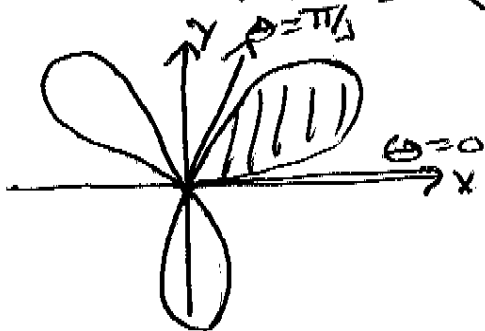
$$A \approx \sum_{i=1}^n \frac{1}{2} [r(\theta_i^*)]^2 \Delta \theta$$

AND

$$A = \int_a^b \frac{1}{2} r^2 d\theta.$$

Ex 1: Find the area of the petals of

$$r(\theta) = \sin(3\theta)$$

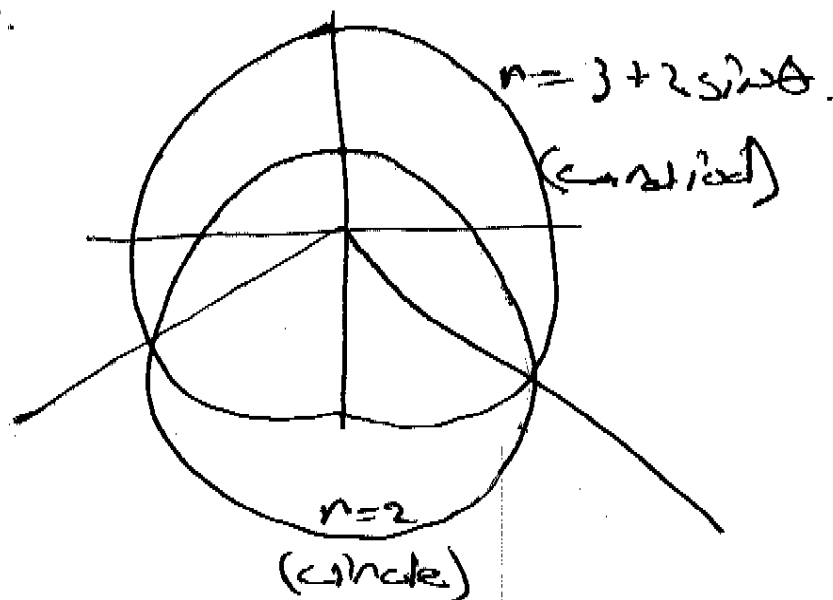


NOTE: Petal shaded on  $[0, \pi/3]$

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Ex 2: Find the area inside  $r = 4 \sin \theta$  and outside  $r = 2$ .

Ex 3: Find the area inside  $r = 2$  and  $r = 3 + 2 \sin \theta$ .



Ex 4: Find all pts of intersection of  $r = \cos \theta$  and  $r = 1 - \cos \theta$ .

Recall from 10.2, that the arc length of  $(x(t), y(t))$  on  $a \leq t \leq b$  is

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

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In polar, this is :

$$L = \int_{\theta=a}^{\theta=b} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta$$

$$\frac{dx}{d\theta} = r' \cos \theta - r \sin \theta$$

$$\left(\frac{dx}{d\theta}\right)^2 = \left(\frac{dr}{d\theta}\right)^2 \cos^2 \theta - 2 \frac{dr}{d\theta} r \sin \theta \cos \theta + r^2 \sin^2 \theta$$

$$\left(\frac{dy}{d\theta}\right)^2 =$$

$$\Rightarrow \left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = \left(\frac{dr}{d\theta}\right)^2 + r^2$$

$$\text{so } L = \int_a^b \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2} d\theta$$

Ex 5: Find the exact length of  $r = e^{2\theta}$   
on  $\theta \in [0, 2\pi]$ .