5.3: Applications of Exponentials and Logarithms
Math 111

Objectives:

1. The importance of negotiation!
2. Comparing linear and exponential growth
3. Comparing exponential growth and exponential decay

1. The Importance of Negotiation!

According to a study by the National Association of Colleges and Employers, the average salary for a nursing graduate with a bachelor's is $52,129.
Source: http://www.cnn.com/2008/LIVING/worklife/04/28/cb.salaries.grads/index.html

Imagine that Olivia and Ahmed both graduated with a B.S. in nursing in 2008 and go to work at Highline Hospital. Olivia is offered $45,000 per year and takes it. Ahmed negotiates his salary and starts at $55,000. Both receive a 5% raise each year. After 10 years how much higher will Ahmed's salary be compared to Olivia? To answer this question, fill in the table below:

Year	Ahmed's Salary	Olivia's Salary
0	55,000	45,000
1	57750	
2	60638	
3		
4		
5	70195	57433
6		
7		
8		
9		
10	89589	73300

Can you figure out how much higher Ahmed's salary will be after 20 years compare to Olivia's?

Ahmed will make about $26K more/yr after 20 yrs.

2. *Comparing linear and exponential growth*

Jill starts a job in 2005 and her salary per year can be modeled by $S_J(t) = 53500 + 2000t$ where t is the number of years after 2005. Allan's salary can be modeled by $S_A(t) = 53500(1.02)^t$ under the same conditions.

a) Fill out the following tables.

t	$S_J(t)$
0	53.5k
1	55.5
2	57.5
3	59.5
4	61.5
5	63.5

Table J

t	$S_A(t)$
0	53.5
1	54.6
2	55.7
3	56.8
4	59.1
5	59.1 ~~58.2~~

Table A

b) Which table models a linear growth? How do you know? What is your slope?

Jill – increases 2000/yr. Slope = 2000

Jill gets a \$2000/yr raise.

c) Which table models the exponential growth? How do you know? What is your multiplier?

Allan... increases by a constant factor... of 1.02

Allan gets a 2% raise each yr.

d) Explain each model in everyday language.

\$2000 vs 2% raise.

e) Find $S_A(10)$ and $S_J(10)$ then explain what these numbers mean in everyday language.

After 10 yrs, Allan makes

\$65.2k/yr & Jill \$73.5k/yr.

f) Solve $S_J(t) = 90000$ and $S_A(t) = 90000$ then explain these numbers in everyday language.

what yr will they earn 90k?

S_J: solve $90000 = 53500 + 2000t$

$\Rightarrow 36500 = 2000t$

$\Rightarrow t = \frac{36500}{2000} = 18.25$

S_A: Solve $90000 = 53500(1.02^t)$

$\Rightarrow \frac{90000}{53500} = 1.02^t$

$\Rightarrow t = \log_{1.02}\left(\frac{900}{535}\right)$

$= \frac{\ln\left(\frac{900}{535}\right)}{\ln(1.02)} \simeq 26.26$

Jill earns 90k/y in 2023 but Allan must wait until 2031.

g) Graph the two functions in the window [0, 100] x [53500, 300000]. What is the moral of the story?

exponential growth always beats linear in the long run.

The short run varies.

3. *Comparing exponential growth and exponential decay*

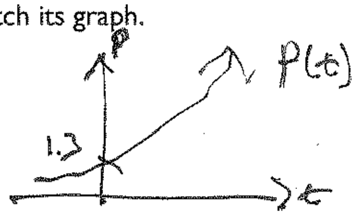
China's current population is 1.3 billion, with growing rate of 0.49% per year.
Source: https://www.cia.gov/library/publications/the-world-factbook/geos/ch.html

a) What is the mathematical model of this growth?

$$P(t) = P_0 (1+r)^t$$

$$P(t) = 1.3(1+0.0049)^t$$, P in billions.

b) Sketch its graph.



c) What is the population of China in 100 years?

$$P(100) = 2.12$$

in 2112, the pop will hit 2.12 billion

d) When does their population double?

$$2.6 = 1.3 \cdot 1.0049^t$$

$$\Rightarrow 2 = 1.0049^t$$

$$\Rightarrow \ln 2 = t \ln(1.0049)$$

$$\Rightarrow t = \frac{\ln 2}{\ln(1.0049)}$$

$$= 141.8$$

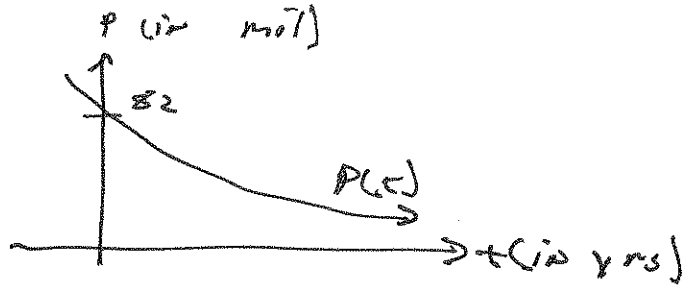
yrs until the pop. doubles.

Germany's current population is 82 million, with growing rate of -0.06% (decay of 0.06%) per year.
Source: https://www.cia.gov/library/publications/the-world-factbook/geos/gm.html

a) What is the mathematical model of this decay?

$P(t) = 82\,(1 - 0.0006)^t$

b) Sketch its graph.



c) What is the population of Germany in 100 years?

$P(100) = 77.223$

The pop will be 77.22 mil. in 100 yrs.

d) When does their population reduce to half of their current one?

$41 = 82\,(0.9994)^t$

$\Rightarrow \frac{1}{2} = 0.9994^t$

$\Rightarrow \ln \frac{1}{2} = t \ln 0.9994$

$\Rightarrow t = \dfrac{\ln 1/2}{\ln 0.9994}$

$\simeq 1154.9$

It will take about 1150 yrs for Germany's pop. to half.