Ex1: Solve $2^x = 32$

$3^x = 27$

$7^x = 49$

Ex2: Solve w/o calculator

$2^x = 7$

$3^x = 10$

$7^x = 40.$

Now, if we wanted to solve $x^3 = 7$ we would use the rule $x^3 = y \iff x = \sqrt[3]{y}.$

Definition: If $a > 0$ and $a \neq 1$, then

$$x = a^y \iff y = \log_a(x) \quad (\text{w/domain } x > 0).$$

Ex3: write in log form.

$9 = 3^2$ $\qquad 16 = 2^4$ $\qquad \frac{1}{49} = 7^{-2}$

Ex4: write in exp form

$\log_3(81) = 4$ $\qquad \log_e(e^{-1}) = -1$ $\qquad \log_a(x) = y.$

Special Notation

$\log_{10}(x) = \log(x)$

$\log_e(x) = \ln(x)$

Ex5: P is a continually compounded investment that grows at 7% per year. The value after t years is given by $A = Pe^{.07t}$. How long until ~~#~~ P triples?

Ex6: Solve $m = 20\ln\left(\frac{40}{40-n}\right)$ for n.

Log Properties

Assume $b>0$ and $b \neq 1$

I: $\log_b(b^x) = x$, $x \in \mathbb{R}$

II: $b^{\log_b(x)} = x$, $x>0$

III: If $m, n > 0$, $\log_b(mn)$

IV: If $m, n > 0$, $\log_b\left(\frac{m}{n}\right)$

V: If $m>0$, $n \in \mathbb{R}$, $\log_b(m^n)$

Ex7: Let $u = \log_b(2)$ and $v = \log_b(7)$. Express the following in terms of u & v.

a) $\log_b(7/2)$

b) $\log_b(28)$

c) $\log_b\left(\frac{16}{49}\right)$

Ex1: Write as a sum/diff of logs. w/o exponents

$\boxed{5.2b \; 1/2}$

a) $\log\left(\frac{x-7}{x+6}\right)$

b) $\ln\left[(x+1)(4x+5)\right]$

c) $\log_7\left(x\sqrt[3]{x+4}\right)$

Ex2: Write as a single log.

a) $\ln(x) - \ln(y)$

b) $\log_3(x+1) + \log_3(x-1)$

c) $\log(2x+1) - \frac{1}{3}\log(x+1)$

Ex3: How much stronger is a 7.7 magnitude earthquake on the Richter scale than a 7.3 if $R = \log\left(\frac{I}{I_0}\right)$ where I_0 is a constant.

$$7.7 = \log\left(\frac{I_1}{I_0}\right) \quad \text{and} \quad 7.3 = \log\left(\frac{I_2}{I_0}\right)$$

Find $\frac{I_1}{I_2}$

Change of Base Formula

$$\log_b(a) = \frac{\ln(a)}{\ln(b)} = \frac{\log(a)}{\log(b)}$$

Ex4: Evaluate

a) $\log_5(17)$ b) $\log_\pi(e)$ c) $\log_2(10)$

Ex5: Graph on the calculator

a) $y = \log_a(x)$ b) $y = \log_2(-x)$.

5.2b
2/2

Claim: If $b>0$ and $b\neq 1$, $m>0$ and $r\in\mathbb{R}$, then
$$\log_b(m^r) = r\log_b(m).$$

□ proof.

Suppose $u = \log_b(m)$
$\Leftrightarrow b^u = m$.

$$\text{Now } \log_b(m^r) = \log_b\left((b^u)^r\right)$$
$$= \log_b(b^{ur})$$
$$= ur$$
$$= r\log_b(m).$$