$\boxed{\text{Review}}$

7.7 : Approx $\int_1^4 \frac{dx}{\sqrt{x}}$ w/ T_6

Find ~~Ep~~ an error bound...

Find E_T.

7.8 : Comparison Test

Improper Integral $\int_0^2 z^2 \ln z \, dz$ ... requires #

Comparison Test $\int_0^\infty \frac{\arctan x}{2+e^x} \, dx$

8.1 Estimate the length of $x = y + \sqrt{y}$ on $1 \le y \le 2$ using S_{10} ...

Find the error bounds

8.2 : Rotate $y = x^{1/3}$ about the y-axis on $1 \le y \le 2$.

$$\int_1^8 2\pi x \sqrt{1 + \frac{1}{9x^{4/3}}} \, dx = \int_1^8 \frac{2}{3}\pi x^{1/3} \sqrt{9x^{4/3} + 1} \, dx$$

or

$$\int_1^2 2\pi y^3 \sqrt{1 + 9y^4} \, dy.$$

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Review for TEST 4 - 8.1, 8.2, 7.7, 7.8

improper integral

Example 1: $\int_0^2 z^2 \ln z \, dz = \lim_{t\to 0^+} \int_t^2 z^2 \ln z \, dz$

$\int_t^2 z^2 \ln z \, dz$ let $u = \ln z$ $dv = z^2$

$du = \frac{1}{z} dz$ $v = z^3/3$

$\lim_{t\to 0^+}\left[\frac{z^3}{3}\ln z - \int \frac{z^2}{3} dz\right] = \lim_{t\to 0^+}\left[\frac{z^3}{3}\ln z - \frac{z^3}{9}\right]$

$= \left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) - \lim_{t\to 0^+}\left(\frac{t^3}{3}\ln t - \frac{\cancel{t^3}}{\cancel{9}}\right)$ 0

$= \frac{8}{3}\ln 2 - \frac{8}{9} - \frac{1}{3}\underbrace{\lim_{t\to 0^+} t^3 \ln t}$

use L'Hospital's Rule $\lim_{t\to 0^+} t^3 \ln t = \lim_{t\to 0^+} \frac{\ln t}{t^{-3}}$

$= \lim_{t\to 0^+} \frac{1/t}{-3t^{-4}} =$

$= \lim_{t\to 0^+} -\frac{1}{3}t^3 = 0$

$\frac{8}{3}\ln 2 - \frac{8}{9} - 0$

$= \frac{8}{3}\ln 2 - \frac{8}{9}$

Example 2: Find the error bound for T_6 when it's used to approximate $\int_1^4 \frac{dx}{\sqrt{x}}$.

$|E_T| \le \frac{K(b-a)^3}{12N^2}$ where $K \ge |f''(x)|$ on $[a,b]$

max of $y = \frac{3}{4}x^{-5/2}$ on $[1,4]$ @ $x=1$, $K = \frac{3}{4}$

$f(x) = x^{-1/2}$ $|E_T| \le \frac{3/4(3)^3}{12\cdot 6^2} = .046875$

$f' = -\frac{1}{2}x^{-3/2}$ $\hookrightarrow$ max error

$f'' = \frac{3}{4}x^{-5/2}$ = worst case in T_6

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Example 2 (cont'd): What N must we choose so that T_N is w/in .0001 of the exact answer

$|E_T| \le \frac{k(b-a)^3}{12N^2} \le .0001$ $k = 3/4$ $b-a = 3$

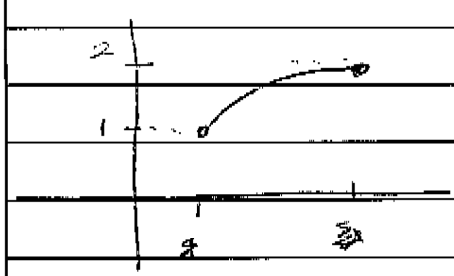
$\frac{\frac{3}{4}(3)^3}{12N^2} \le .0001$; $N^2 \le \frac{\frac{3}{4}(27)}{12(.0001)}$

$N \le \sqrt{\frac{3/4(27)}{12(.0001)}}$

so, choose 130 for N

for Simpson's Rule, you must round up to next <u>EVEN N</u> #

Example 3: Rotate $y = \sqrt[3]{x}$ on $1 \le y \le 2$ about y-axis + Find SA



(1) $\int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

(2) $\int_a^b 2\pi \cancel{x} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

(1) $x = y^3$ $dx/dy = 3y^2$

$SA = \int_1^2 2\pi y^3 \sqrt{1 + 9y^4}\, dy$ let $u = 1 + 9y^4$

$\frac{du}{36} = y^3 dy$

$\frac{2\pi}{36}\int_{10}^{145} \sqrt{u}\, du$

$\frac{2\pi}{36}\left[\frac{2}{3}u^{3/2}\right]_{10}^{145}$

$= \pi/27\left(145^{3/2} - 10^{3/2}\right) = 63.5\pi$

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(2) $\int_1^8 2\pi x\sqrt{1+\left(\frac{1}{3x^{2/3}}\right)^2}\,dx$ $y=\sqrt[3]{x}$ $dy/dx=$

$= 2\pi\int_1^8 x\sqrt{\frac{9x^{4/3}+1}{9x^{4/3}}}\,dx$

$\frac{2\pi}{3}\int_1^8 x^{1/3}\sqrt{9x^{4/3}+1}\,dx$ let $u=9x^{4/3}+1$

$\frac{du}{12}=x^{1/3}\,dx$

$\frac{2\pi}{3\cdot 12}\int_{10}^{145}\sqrt{u}\,du$

$\frac{2\pi}{36}\int_{10}^{145}\sqrt{u}\,du \rightarrow$ same as #(1).

Example 4: $\int_0^\infty \frac{\arctan x}{2+e^x}\,dx \rightarrow$ converge or diverge?

$\int_0^\infty \frac{\arctan x}{2+e^x}\,dx \leq \int_0^\infty \frac{\arctan x}{e^x}\,dx \leq \int_0^\infty \frac{\pi/2}{e^x}\,dx$

so $\frac{\pi}{2}\int_0^\infty e^{-x}\,dx = \frac{\pi}{2}\lim_{t\to\infty}\int_0^t e^{-x}\,dx$

$= \frac{\pi}{2}\lim_{t\to\infty}\left[-e^{-x}\right]_0^t$

$= \frac{\pi}{2}\lim_{t\to\infty}\left(\cancel{-e^{-t}}^{\,0}+1\right) = \frac{\pi}{2} \rightarrow$ converges

So, by comparison $\int_0^\infty \frac{\arctan x}{2+e^x}\,dx$ converges