9.3 HW
1/15

$\boxed{3}$ $(x^2+1)\frac{dy}{dx} = xy$

$\Rightarrow \int \frac{dy}{y} = \int \frac{x}{x^2+1}\,dx$

$\Rightarrow \ln|y| = \frac{1}{2}\ln|x^2+1| + c$

$\Rightarrow \ln|y| = \ln\sqrt{x^2+1} + c$

$\Rightarrow |y| = e^{\ln\sqrt{x^2+1} + c}$

$\Rightarrow |y| = A\sqrt{x^2+1}$

$\Rightarrow y = \pm A\sqrt{x^2+1}$

$\boxed{4}$ $\frac{dy}{dx} = \frac{\sqrt{x}}{e^y}$

$\Rightarrow \int e^y\,dy = \int \sqrt{x}\,dx$

$\Rightarrow e^y = \frac{2}{3}x^{3/2} + c$

$\Rightarrow y = \ln\left(\frac{2}{3}x^{3/2} + c\right)$

#2 (unassigned)

$\boxed{4}$ $\frac{dy}{dx} = y^2\sin x$

$\Rightarrow \int y^{-2}\,dy = \int \sin x\,dx$

$\Rightarrow +\frac{1}{y} = +\cos x + C$

$\Rightarrow y = \frac{1}{\cos x + C}$ ($y=0$ is also a solution).

9.3 HW
2/15

$\boxed{5}$ $(1+\tan y)\dfrac{dy}{dx} = x^2+1$

$\Rightarrow \int (1+\tan y)\,dy = \int (x^2+1)\,dx$

$\Rightarrow y + \ln|\cos y| = \dfrac{x^3}{3} + x + c$

$\boxed{9}$ $\dfrac{du}{dt} = 2 + 2u + t + tu$

$= 2(1+u) + t(1+u)$

$= (2+t)(1+u)$

$\Rightarrow \int \dfrac{du}{1+u} = \int (2+t)\,dt$

$\Rightarrow \ln|1+u| = 2t + \dfrac{t^2}{2} + c$

$\Rightarrow |1+u| = k e^{2t+\frac{1}{2}t^2}$

$\Rightarrow u = -1 \pm k e^{2t+\frac{1}{2}t^2}$

9.3 HW
3/15

[12] $\frac{dy}{dx} = \frac{y\cos x}{1+y^2}$; $y(0)=1$

$\Rightarrow \int \frac{1+y^2}{y}\,dy = \int \cos x\,dx$

$\Rightarrow \int (\frac{1}{y} + y)\,dy = \sin x + C$

$\Rightarrow \ln|y| + \frac{1}{2}y^2 = \sin x + C$. AND $y(0)=1$

solve $\ln|1| + \frac{1}{2} = \sin(0) + C$

$\Rightarrow C = \frac{1}{2}$

[15] $\frac{du}{dt} = \frac{2t + \sec^2 t}{2u}$; $u(0) = -5$

$\Rightarrow \int 2u\,du = \int (2t + \sec^2 t)\,dt$

$\Rightarrow u^2 = t^2 + \tan t + C$

$\Rightarrow u = \pm\sqrt{t^2 + \tan t + C}$

$\Rightarrow -5 = - \sqrt{0 + 0 + C}$

so $C = 25$

9.3 HW
4/15

$\boxed{16}$ $xy' + y = y^2$; $y(1) = -1$

$\Rightarrow x\dfrac{dy}{dx} = y^2 - y$

$\dfrac{1}{y^2 - y} = \dfrac{A}{y} + \dfrac{B}{y-1}$

$\Rightarrow \dfrac{dy}{y^2 - y} = \dfrac{dx}{x}$

$1 = Ay - A + By$

$A = -1$ & $B = 1$

$\Rightarrow \int \dfrac{-1}{y} + \dfrac{1}{y-1}\, dy = \ln|x| + c$

$\Rightarrow -\ln|y| + \ln|y-1| = \ln|x| + c$

$\Rightarrow \ln\left|\dfrac{y-1}{y}\right| = \ln|x| + c$

$\Rightarrow \left|\dfrac{y-1}{y}\right| = ke^{\ln|x|}$

$\Rightarrow \left|\dfrac{y-1}{y}\right| = k|x|$

solve $\left|\dfrac{-1-1}{-1}\right| = k|1|$

$\Rightarrow 2 = k$

8.3 HW
5/15

$f'(x) = f(x)(1 - f(x))$; $f(0) = \frac{1}{2}$

$\Rightarrow \frac{dy}{dx} = y(1-y)$

$\Rightarrow \frac{dy}{y(1-y)} = dx$

$\Rightarrow \int \frac{+1}{y} + \frac{+1}{1-y}\, dy = x + c$

~~$\Rightarrow \ln|y(1-y)| = x + c$~~

~~$\Rightarrow$ solve $\ln\left|\frac{1}{2}(1-\frac{1}{2})\right| = 0 + c$~~

~~$\Rightarrow \ln\left|\frac{1}{4}\right| = c$~~

~~$\Rightarrow \ln\left(\frac{1}{4}\right) = c$~~

$\ln|y| - \ln|1-y| = x + c$.

$\Rightarrow -\ln|y| + \ln|1-y| = -x + c$

$\Rightarrow \ln\left|\frac{1-y}{y}\right| = -x + c$

$\Rightarrow \left|\frac{1-y}{y}\right| = e^{-x+c}$

$\Rightarrow \frac{1}{y} - 1 = c e^{-x}$

$\Rightarrow \frac{1}{y} = 1 + c e^{-x}$

$\Rightarrow y = \frac{1}{1 + c e^{-x}}$

AND $y(0) = \frac{1}{2}$

$\frac{1}{2} = \frac{1}{1 + c}$

$\Rightarrow c = 1$

So $y = \frac{1}{1 + e^{-x}}$

9.3 Hw
6/15

$\boxed{21}$ $y' = x + y$ Let $u = x + y$

$\frac{du}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow \frac{du}{dx} - 1 = u$

$\Rightarrow \frac{du}{u+1} = dx$

$\Rightarrow \ln|u+1| = x + c$

$\Rightarrow |u+1| = ke^{x}$

$\Rightarrow u+1 = \pm ke^{x}$

$\Rightarrow y = -1 - x + ke^{x}$

check $k=0$

$u = -1$

$y = -1 - x$

so $k=0$ is ok.

$\boxed{23}$ (a) $y' = 2x\sqrt{1-y^2}$

$\Rightarrow \int \frac{dy}{\sqrt{1-y^2}} = \int 2x\,dx$

$\Rightarrow \arcsin(y) = x^2 + c$

$\Rightarrow y = \sin(x^2 + c)$.

(b) $y(0) = 0$ .... $c = k\pi$, $k \in \mathbb{Z}$.

(c) $y(0) = 2$ has no solution (outside the range of sine.

9.3 Hw 7/15

$\boxed{25}$ $y' = \dfrac{\sin x}{\sin y}$; $y(0) = \dfrac{\pi}{2}$

$\Rightarrow \int \sin y \, dy = \int \sin x \, dx$

$\Rightarrow + \cos y = + \cos \cancel{y}x + c$

$\Rightarrow y = \arccos(\cos x + c)$

Solve $\dfrac{\pi}{2} = \arccos(1 + c)$

$\Rightarrow c = -1.$

$\boxed{30}$ $y^2 = kx^3$ (find orthogonal trajectories).

$k = \dfrac{y^2}{x^3}$

$\Rightarrow \dfrac{dy}{dx} \cdot 2y = 3kx^2$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{3kx^2}{2y}$

$= \dfrac{3y^2x^2}{2yx^3}$

$= \dfrac{3y}{2x}$ (slope of solutions).

slope of orthogonals $-\dfrac{2x}{3y}$ ... so $\dfrac{dy}{dx} = -\dfrac{2x}{3y}$

$\Rightarrow \int 3y \, dy = \int -2x \, dx$

$\Rightarrow \dfrac{3y^2}{2} = -x^2 + c \Longrightarrow$ or $x^2 + \frac{3}{2}y^2 = c$

$\Rightarrow y = \pm\sqrt{-\frac{2}{3}x^2 + c}$ which are ellipses.

8.3 HW
8/15

[31] $\dfrac{dy}{dx} = -\dfrac{k}{x^2}$

$= -\dfrac{xy}{x^2}$

$= -\dfrac{y}{x}$ (slope of solutions.)

so the slope of the ortho trajectories.

is $\dfrac{dy}{dx} = \dfrac{x}{y}$

$\Rightarrow \int y\,dy = \int x\,dx$

$\Rightarrow \dfrac{y^2}{2} = \dfrac{x^2}{2} + c$

$\Rightarrow y^2 - x^2 = c$ (hyperbolas).

[35] $\dfrac{dP}{dt} = k(M-P)$

$\Rightarrow \int \dfrac{dP}{k(M-P)} = \int dt$

$\Rightarrow -\dfrac{1}{k}\ln|k(M-P)| = t + c$

$\Rightarrow \ln|k(M-P)| = -kt + c_1$

$\Rightarrow |k(M-P)| = e^{c_1}e^{-kt}$

$\Rightarrow k(M-P) = c_2 e^{-kt}$

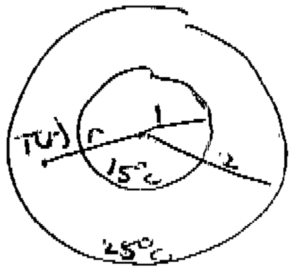
$\Rightarrow M - P = c_3 e^{-kt}$

$\Rightarrow P = M - c_3 e^{-kt}$

As $t \mapsto \infty$, $P(t) \mapsto M$.

$\boxed{38}$

$\boxed{\begin{array}{c} 9.3\ \text{HW} \\ 9/15 \end{array}}$



$$\frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = 0$$

Let $s = \dfrac{dT}{dr}$

then $\dfrac{ds}{dr} = \dfrac{d^2T}{dr^2}$

AND $\dfrac{ds}{dR} + \dfrac{2}{r}s = 0$

$$\Rightarrow \frac{ds}{dR} = -\frac{2s}{r}$$

$$\Rightarrow \int \frac{ds}{2s} = -\int \frac{dr}{r}$$

$$\Rightarrow \frac{1}{2}\ln|2s| = -\ln|r| + c$$

$$\Rightarrow |2s| = k\left|\frac{1}{r^2}\right|$$

$$\Rightarrow s = \pm k\left|\frac{1}{r^2}\right|$$

$T(1\ \ 15$
$T(2) = 25$
$15 = \dfrac{k}{1} + c$
$25 = \dfrac{k}{2} + c$
$-10 = \dfrac{1}{2}k$
$k = -20$ AND
$c = 35$

So, subsituite back

$$\frac{dT}{dr} = \pm k\left|\frac{1}{r^2}\right| \Rightarrow dT = \frac{\pm k}{r^2}\,dr$$

$$\cancel{\Rightarrow dT = \pm k|r|\,dr}$$

$$\cancel{\Rightarrow T = \pm \frac{k r^2}{2} + c \Rightarrow T = \pm k \ln|r| + c}$$

$$\Rightarrow T = \mp\frac{k}{r} + c$$

$$T(r) = \frac{-20}{r} + 35$$

9.3 HW
10/15

39 $\frac{dc}{dt} = r - kc$

$\Rightarrow \frac{dc}{r-kc} = dt$

$\Rightarrow -\frac{1}{k} \ln|r-kc| = t + A_1$

$\Rightarrow \ln|r-kC| = -kt + A_2$

$\Rightarrow |r-kc| = A_3 e^{-kt}$

$\Rightarrow r-kc = A_4 e^{-kt}$

$\Rightarrow kC = r - A_4 e^{-kt}$

$\Rightarrow C(t) = \frac{r - A_4 e^{-kt}}{k}$

(b) If $c_0 < \frac{r}{k}$

$\lim_{t\to\infty} C(t) = \frac{r}{k}$

c_0 impacts the rate @ which c decreases to $\frac{r}{k}$.

(a) If $C(0) = C_0$

$\Rightarrow C_0 = \frac{r - A_4}{k}$

$\Rightarrow C_0 k = r - A_4$

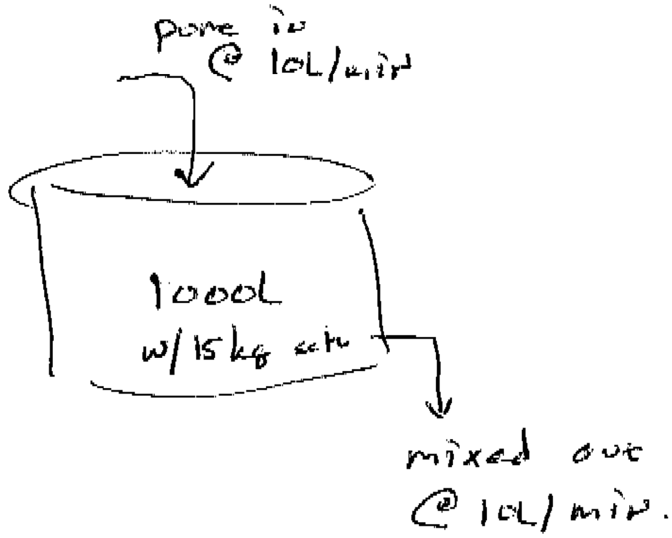
$\Rightarrow A_4 = r - C_0 k$

so $C(t) = \frac{r - (r - C_0 k)e^{-kt}}{k}$

$= \left(C_0 - \frac{r}{k}\right)e^{-kt} + \frac{r}{k}$

9.3 HW
11/15

41



$S(t)$ = kgs of salt in the tank after t min.

$$\frac{ds}{dt} = (\text{rate in}) - (\text{rate out})$$

$$= \frac{0\,kg}{min} - \frac{10 \cdot S}{1000}\frac{kg}{min}$$

(a) $\Rightarrow \dfrac{ds}{dt} = -\dfrac{S}{100}$

$\Rightarrow \dfrac{ds}{S} = -\dfrac{1}{100}\,dt$

$\Rightarrow \ln|S| = -\dfrac{1}{100}t + C$

$\Rightarrow S = k e^{-\frac{t}{100}}$

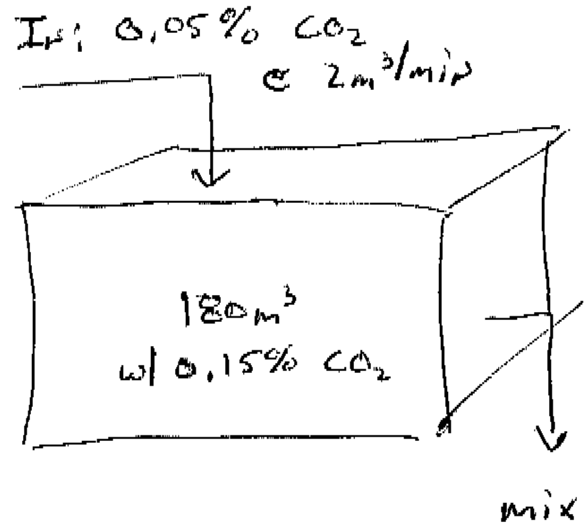
AND $S(0) = 15$ so $k = 15$.

$\Rightarrow S(t) = 15 e^{-t/100}$

(b) $S(20) = 12.28$ kg.

9.3 #9
12/15

$\boxed{42}$



$C(t) =$ ~~Concentration (tenth of a %)~~ (hundredth) @ time t.
Amt. of CO_2 in the room

$$\frac{dC}{dt} = (\text{rate in}) - (\text{rate out})$$

$$\Rightarrow \frac{dC}{dt} = \frac{2(5)\,m^3}{min} - \frac{2C}{180}\frac{m^3}{min}$$

$$\Rightarrow \frac{dC}{dt} = 10 - \frac{C}{90}$$

$$\Rightarrow \frac{dC}{10 - \frac{C}{90}} = 1\,dt$$

$$\Rightarrow -90 \ln\left|10 - \frac{C}{90}\right| = t + c$$

$$\Rightarrow \ln\left|10 - \frac{C}{90}\right| = -\frac{t}{90} + c$$

$$\Rightarrow 10 - \frac{C}{90} = k e^{-t/90}$$

$$\Rightarrow C(t) = 900 - k e^{-t/90}$$

$$\Rightarrow C(t) = \frac{900 + 1800 e^{-t/90}}{10000}$$

$$\rightarrow C(t) = 0.09 + 0.18 e^{-t/90}$$

To find the expected % CO_2 ...

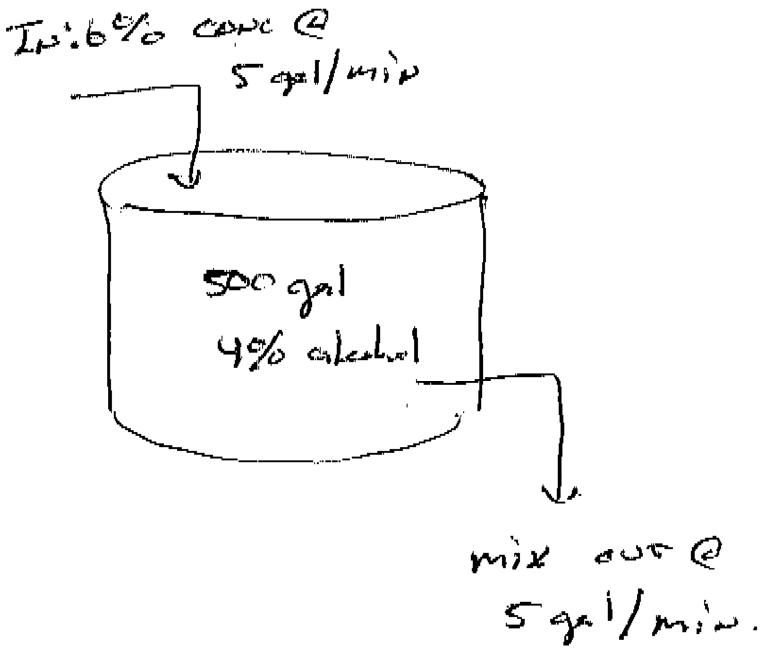
$$\lim_{t \to \infty} \frac{C(t)}{180} = \frac{0.09}{180}$$

$$= 0.0005$$

or 0.05%

9.3 HW
13/15

$\boxed{43}$



$c(t) =$ Amt. of alcohol in the vat @ time t.

$$\Rightarrow \frac{dc}{dt} = (\text{rate in}) - (\text{rate out})$$

$$= 5(0.06) - 5\left(\frac{c}{500}\right)$$

$$\Rightarrow \frac{dc}{dt} = 0.3 - \frac{c}{100}$$

$$\Rightarrow \frac{dc}{0.3 - \frac{c}{100}} = dt$$

$$\Rightarrow -100 \ln\left|0.3 - \frac{c}{100}\right| = t + C$$

$$\Rightarrow 0.3 - \frac{c}{100} = k e^{-t/100}$$

$$\Rightarrow c(t) = 30 - k e^{-t/100}$$

$c(0) = 0.04(500)$
$= 20$

$\Rightarrow k = 10$

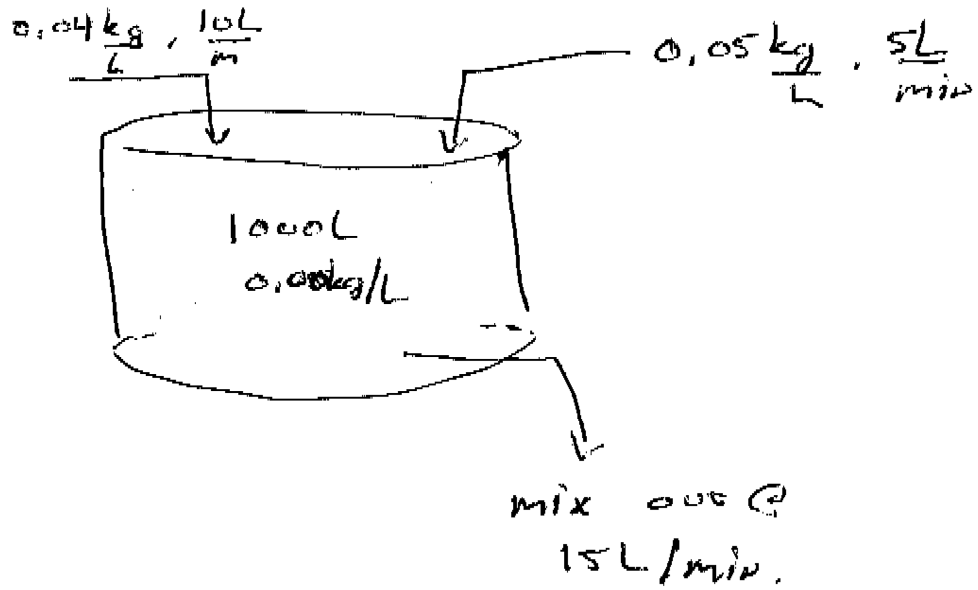
$c(60) = 24.5$ gal of alcohol in the vat.

percent $= \frac{24.5}{500}$

$= 4.9\%$.

9.3 HW
14/15

$\boxed{44}$



$c(t)$ = Amt of salt in the tank @ time t.

$$\Rightarrow \frac{dc}{dt} = (\text{rate in}) - (\text{rate out})$$

$$= \left(.25\frac{kg}{min} + .4\frac{kg}{min} \right) - \left(\frac{c}{1000} \cdot 15 \frac{kg}{min} \right)$$

$$\Rightarrow \frac{dc}{dt} = 0.65 - \frac{15}{1000}c$$

$$\Rightarrow \frac{dc}{650 - 15c} = \frac{dt}{1000}$$

$$\Rightarrow -\frac{1}{15}\ln|650-15c| = \frac{t}{1000} + k$$

$$\Rightarrow \ln|650-15c| = \frac{-15t}{1000} + k.$$

$$\Rightarrow 650 - 15c = ke^{-15t/1000}$$

$$\Rightarrow c(t) = \frac{650}{15} - ke^{-15t/1000}$$

$c(0) = 0$

$\Rightarrow k = -\frac{650}{15}$

After 1 hour.

$$c(60) = \frac{650}{15}\left(1 - e^{-15(60)/1000}\right)$$

$= 25.72$ kg ✓

9.3 HW
15/15

$\boxed{45}$ $(mv)' = gm$

$\Rightarrow m'v + mv' = gm$

we are given that $m' = km$

$\Rightarrow kmv + mv' = gm$

$\Rightarrow kv + v' = g$

$\Rightarrow \frac{dv}{dt} = g - kv$

$\Rightarrow \frac{dv}{g-kv} = dt$

$\Rightarrow -\frac{1}{k} \ln|g-kv| = t + c$

$\Rightarrow \ln|g-kv| = -kt + c$

$\Rightarrow g - kv = ce^{-kt}$

$\Rightarrow kv = g - ce^{-kt}$

$\Rightarrow v(t) = \frac{g}{k} - ce^{-kt}$

AND $v(0) = 0 \Rightarrow c = \frac{g}{k}$

$\Rightarrow v(t) = \frac{g}{k}(1 - e^{-kt})$

Terminal Velocity is $\lim_{t \to \infty} v(t) = \frac{g}{k}$.