Notetaker's name Emma Miller Class CALC 125 Date 6/2/09

8.3 - Physics Apps.

3 Topics:

(1) Hydrostatic Pressure

(2) Centroid (center of mass)

(3) Theorem of Pappus

1) Hydrostatic Pressure

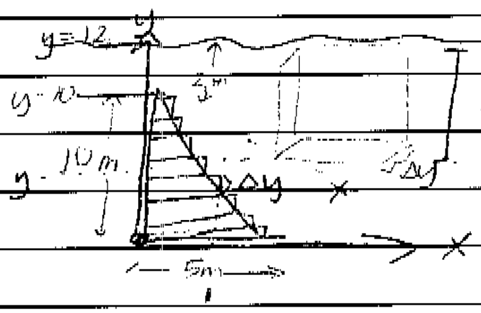
→ water → non-moving → area → Rectangles

force → gravity

mass → density, volume

Fact: At any pt in a static liquid, the pressure is the same in all directions.

Example 1: Find P



x → need x in terms of y

$\Delta P = 9.8(1000)(12-y)(x)\Delta y$ $\quad y = -2x+10$

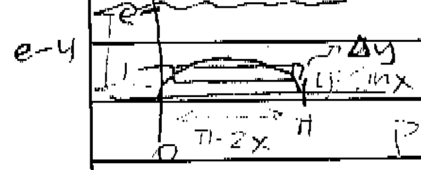
gravity, density, volume $\quad x = \frac{y-10}{-2}$

so

$\Delta P = 9.8(1000)(12-y)\left(\frac{y-10}{-2}\right)\Delta y$

$P = \int_0^{10} 9.8(1000)(12-y)\left(\frac{y-10}{-2}\right)dy$

Example 2: set up integral for pressure on gate



$\Delta P = 9.8(1000)(e-y)(\pi-2x)\Delta y$

$\Delta P = 9.8(1000)(e-y)(\pi-2\arcsin y)\Delta y$

$P = \int_0^1 9.8(1000)(e-y)(\pi-2\arcsin y)\,dy$

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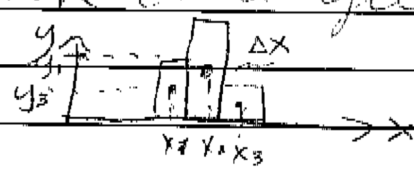
8.3 Apps to Physics – Centroid (center of mass)

1st: Weighted Averages

A) Fix distance B) Fixed mass (area)

$(90(10) + 70(60))/70$

2nd: Center of a geometric region — moment about the y-axis



$M_y = x_1 \cdot y_1 \cdot \Delta x + x_2 y_2 \Delta x + x_3 y_3 \Delta x$

$= (x_1 y_1 + x_2 y_2 + x_3 y_3)\Delta x$

$M_y = \sum_{i=1}^{3} x_i y_i \Delta x$ } sum of weights

$\frac{M_y}{A \text{(area)}} = \bar{x}$ = avg. distance

$A = y_1 \Delta x + y_2 \Delta x + y_3 \Delta x = \sum_{i=1}^{3} y_i \Delta x$

so $M_y = \int_a^b x \cdot f(x)\,dx$

so $\bar{x} = \frac{M_y}{A}$ and $A = \int_a^b f(x)\,dx$

so, $\bar{x} = \frac{\int_a^b x f(x)\,dx}{\int_a^b f(x)\,dx}$

Example 1: Find the centroid of region under $y = \sin x$ on $[0, \pi]$.

Need: $\bar{x} = M_y / A$

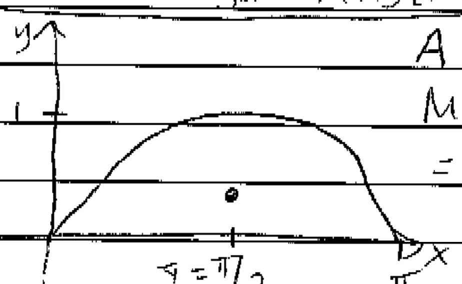
$A = \int_0^\pi \sin x\,dx = 2$

$M_y = \int_0^\pi x \sin x\,dx$ $u = x$ $dv = \sin x\,dx$

$= [-x\cos x + \int \cos x\,dx]_0^\pi$ $du = dx$ $v = -\cos x$

$= [-x\cos x + \sin x]_0^\pi$ so $\bar{x} = \frac{M_y}{A} = \frac{\pi}{2}$

$= -\pi(-1) = \pi$



$M_x = \left(\frac{y_1}{2}\right) \underbrace{y_1 \Delta x}_{\text{area}} + \left(\frac{y_2}{2}\right) y_2 \Delta x \ldots = \sum_{i=1}^{3} \frac{1}{2}(y_i)^2 \Delta x = \int_a^b \frac{1}{2} f(x)^2\,dx$

$\bar{y} = \frac{M_x}{A} = \frac{\int_a^b \frac{1}{2} f(x)^2\,dx}{\int_a^b f(x)\,dx} = \frac{\int_0^\pi \frac{1}{2}\sin^2 x\,dx}{2} = \frac{\pi/4}{2}$

$\frac{1}{2}\int_0^\pi \sin^2 x\,dx = \frac{1}{4}\int_0^\pi 1 - \cos 2x\,dx$ so, $\bar{y} = \pi/8$

$= \frac{1}{4}\left[x - \frac{1}{2}\sin 2x\right]_0^\pi$ ≈ 0.39

$= \pi/4$