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9.3 #2 Solve

$\frac{dy}{dx} = y(1-y)$ $\quad y(0) = \frac{1}{11}$

$\int \frac{dy}{y(1-y)} = \int dx$ $\quad \frac{1}{y(y-1)} = \frac{A}{y} + \frac{B}{-y+1}$

$\int \frac{1}{y} + \frac{1}{(1-y)}\, dy = x + C$

$1 = A(1-y) + By$

$\Rightarrow \ln|y| - \ln|1-y| = x + C$ $\quad 1 = A - Ay + By$

$\ln\left|\frac{y}{1-y}\right| = x + C$ $\quad A = 1$ $\quad A + B = 0$

$\left|\frac{y}{1-y}\right| = e^{x+C}$ $\quad \pm e^C = k$ $\quad B = 1.$

$\frac{y}{1-y} = Ke^x$

$\frac{1-y}{y} = K_2 e^{-x}$

$\frac{1}{y} - 1 = K_2 e^{-x}$

$\frac{1}{y} = 1 + K_2 e^{-x}$

$y = \frac{1}{1 + K_2 e^{-x}}$

Solve:

$y(0) = \frac{1}{11} = \frac{1}{1+K_2}$ $\quad C_2 = 10$

So $y = \frac{1}{1 + 10e^{-x}}$.

Section 7.4

Partial Fractions

Ex: 1 $\int \frac{x-1}{x^2+9x+20}\, dx = \int \frac{6}{x+5} - \frac{5}{x+4} = 6\ln|x+5| - 5\ln|x+4| + C$

rational expression $= \ln\left|\frac{(x+5)^6}{(x+4)^5}\right| + C.$

degree in denominator = 2

numerator = 1

$2 > 1$ use P.F.

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partial fractions scratch work.

$$\frac{x-1}{x^2+7x+10} = \frac{x-1}{(x+5)(x+4)} = \frac{A}{x+5} + \frac{B}{(x+4)}$$

$$\frac{x-1}{(x+5)(x+4)} = \frac{A}{(x+5)} + \frac{B}{(x+4)}$$

$$x-1 = A(x+4) + B(x+5)$$

$$x-1 = Ax + 4A + Bx + 5B$$

$$\begin{cases} 1x = Ax + Bx \\ -1 = 4A + 5B \end{cases}$$

$$\begin{cases} 1 = A + B \\ -1 = 4A + 5B \end{cases}$$

A B =

[1 1 1
-4 -5 -1]

row reduce:

so A = 6

B = -15

ind. matrix, edit. 2x3
ind. matrix. math. frac

Ex: 2 $$\int \frac{1-2x^2}{x^3+4x^2+4x}\,dx = \int \frac{\frac{1}{4}}{x} + \frac{-\frac{9}{4}}{x+2} + \frac{\frac{7}{2}}{(x+2)^2}\,dx = \frac{1}{4}\ln|x| - \frac{9}{4}\ln|x+2| - \frac{7}{2}\cdot\frac{1}{x+2} + C$$

P.F. scratch work:

$$\frac{1-2x^2}{x^3+4x^2+4x} = \frac{1-2x^2}{x(x+2)^2} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$ ← every single power up 1.

$$1-2x^2 = A(x+2)^2 + Bx(x+2) + Cx$$

$$-2x^2 = Ax^2 + Bx^2$$

$$0x = 4Ax + 2Bx + Cx$$

$$1 = 4A$$

[1 1 0 : -2
4 2 1 : 0
4 0 0 : 1]

3x4

$$A = .25 = \frac{1}{4}$$

$$B = -2.25 = -\frac{9}{4}$$

$$C = 3.5 = \frac{7}{2}$$

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Ex: 3 $\int \frac{x^2+2x}{x-3}\,dx =$

$= \int x+5+\frac{15}{x-3}\,dx$

$= \frac{x^2}{2}+5x+15\ln|x-3|+C.$

Scratch work:

$$x-3\ \overline{)\ x^2+2x+0}\ \ \ x+5$$

$$-(x^2-3x)$$

$$0+5x$$

$$-(5x-15)$$

$$0+15$$

$$\frac{x^2+2x}{x-3} = x+5+\frac{15}{x-3}$$

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Section 7.4

Partial Fraction.

Review P.F. decomposition.

① $\frac{2x+3}{(x+4)(3x-2)} = \frac{A}{(x+4)} + \frac{B}{(3x-2)}$

② $\frac{2x+3}{(x+4)^3(3x-2)} = \frac{A}{(x+4)} + \frac{B}{(x+4)^2} + \frac{C}{(x+4)^3} + \frac{D}{(3x-2)}$

③ $\frac{2x^2-4x+5}{(x+2)}$ ⇒ use polynomial long div.

④ $\frac{2x-3}{(x-4)(x^2+4)} = \frac{A}{(x-4)} + \frac{Bx+C}{x^2+4}$ (irreducible quadratic)

⑤ $\frac{3x^2-4x+5}{x^2+2}$ ⇒ use poly. long div.

⑥ $\frac{2x-7}{(x-4)(x^2+4)^2} = \frac{A}{(x-4)} + \frac{Bx+C}{(x^2+4)} + \frac{Dx+C}{(x^2+4)^2}$.

Ex: 4 $\int \frac{x-1}{x^2+3x}\,dx = \int \frac{-\frac{1}{3}}{x} + \frac{\frac{1}{3}x+1}{x^2+3}\,dx$

$\Rightarrow -\frac{1}{3}\ln|x| + \frac{1}{3}\int \frac{x}{x^2+3}\,dx + \int \frac{1}{x^2+3}\,dx =$ (u-sub) (arctan)

$\Rightarrow -\frac{1}{3}\ln|x| + \frac{1}{6}\ln|x^2+3| + \frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + C.$

P.F.

$\frac{x-1}{x^2+3x} = \frac{x-1}{x(x^2+3)}$ ⇒ (irreducible quadratic)

$\Rightarrow \frac{A}{x} + \frac{Bx+C}{x^2+3} = \frac{x-1}{x(x^2+3)}$

$\Rightarrow A(x^2+3) + (Bx+C)x = x-1$

$x^2: 0 = A + B + 0C$

$x: 1 = 0A + 0B + C$

$1: -1 = 3A + 0B + 0C$

$A = -\frac{1}{3}, \quad C = 1, \quad B = \frac{1}{3}.$

Ex: 5 $\int \frac{x^3-3x+1}{x^4+4x^2+4}\,dx = \int \left(\frac{1x}{x^2+2} + \frac{*2x+1}{(x^2+2)^2}\right)dx$

$\Rightarrow \int \frac{x}{x^2+2}\,dx - 2\int \frac{x}{(x^2+2)^2}\,dx + \int \frac{1}{(x^2+2)^2}\,dx.$ (u-sub) (u-sub) (trig sub)

$\Rightarrow \frac{1}{2}\ln(x^2+2) + \frac{2}{2(x^2+2)} + \frac{\sqrt{2}}{4}\arctan\left(\frac{x}{\sqrt{2}}\right) + \frac{1}{4(x^2+2)} + C.$

(B) Trig subst. $x = \sqrt{2}\tan\theta$, $dx = \sqrt{2}\sec^2\theta\,d\theta$

$\int \frac{dx}{(x^2+2)^2} = \int \frac{\sqrt{2}\sec^2\theta\,d\theta}{(2\tan^2\theta+2)^2} = \frac{\sqrt{2}}{4}\int \frac{\sec^2\theta\,d\theta}{(\tan^2\theta+1)^2}$

$= \frac{\sqrt{2}}{4}\int \frac{d\theta}{\sec^2\theta} = \frac{\sqrt{2}}{4}\int \cos^2\theta = \frac{\sqrt{2}}{4}\left(\theta + \frac{\sin(2\theta)}{2}\right) \Rightarrow$ Sub back.

(*) let $u = x^2+2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

(**) let $v = \frac{x}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\int \frac{dv}{v^2+1}$

$\sqrt{2}\,dv = dx$

$= \frac{1}{\sqrt{2}}\arctan(v)$

$= \frac{1}{\sqrt{2}}\arctan\left(\frac{x}{\sqrt{2}}\right)$

let $u = x^2+2$, $du = 2x$

P.F. $\frac{x^3-3x+1}{x^4+4x^2+4} = \frac{x^3-3x+1}{(x^2+2)^2} = \frac{Ax+B}{(x^2+2)} + \frac{Cx+D}{(x^2+2)^2}$

$x^3-3x+1 = (Ax+B)(x^2+2) + Cx + D.$

$x^3-3x+1 = Ax^3+2Ax+Bx^2+2B+Cx+D.$

$x^3: 1 = A + 0 \Rightarrow A = 1$

$x^2: 0 = 0A + B + 0C + 0D \Rightarrow B = 0$

$x: -3 = 2A + 0B + C \Rightarrow C = -5$

$1: 1 = 2B + D \Rightarrow D = 1$

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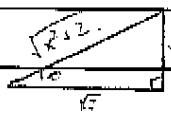

$= \frac{\sqrt{2}}{2} \arctan\left(\frac{x}{\sqrt{2}}\right) + \frac{\sqrt{2}}{2} \sin(2\theta)$

$= \frac{\sqrt{2}}{2} \arctan\left(\frac{x}{\sqrt{2}}\right) + \frac{\sqrt{2}}{2} \cdot 2\sin\theta\cdot\cos\theta$

$\frac{x}{\sqrt{x^2+2}} \quad \frac{\sqrt{2}}{\sqrt{x^2+2}}$

$= \frac{\sqrt{2}}{2} \arctan\left(\frac{x}{\sqrt{2}}\right) + \frac{1}{2} \frac{x}{x^2+2} + C.$