Notetaker's name ______ Class ______ Date Apr. 24.09

§7.1

Integration by Parts

$(u \cdot v)' = uv' + u'v$

$uv' = (uv)' - u'v$

$$\boxed{\int u\, dv = uv - \int v\, du}$$

Ex 1: $\int x\cos(x)\, dx = x\sin(x) - \int \sin(x)\, dx = x\sin(x) + \cos(x) + C.$

$u = x \qquad dv = \cos(x)\, dx$

$du = dx \qquad v = \sin(x)$

Check:

$y' = \sin(x) + x\cos(x) - \sin(x)$

$= x\cos(x).$

* Always put sin or cos on dv side.

Ex 2: $\int \ln(x)\, dx = x\ln(x) - \int x \cdot \frac{1}{x}\, dx = x\ln(x) - \int dx$

$= x\ln(x) - x + C.$

$u = \ln(x) \qquad dv = dx$

$du = \frac{1}{x}\, dx \qquad v = x$

Check:

$y' = \ln(x) + x \cdot \frac{1}{x} - 1$

$= \ln(x)$

Ex 3:

$\int x^2 \sin(\pi x)\, dx = -\frac{x^2\cos(\pi x)}{\pi} - \int 2x\left(-\frac{\cos(\pi x)}{\pi}\right)$

$u = x^2 \qquad dV = \sin(\pi x)\, dx$

$du = 2x\, dx \qquad V = -\cos(\pi x) \cdot \frac{1}{\pi}$

$= -\frac{x^2\cos(\pi x)}{\pi} + \frac{2}{\pi}\int x\cos(\pi x)\, dx$

$= -\frac{x^2\cos(\pi x)}{\pi} + \frac{2}{\pi}\left(\frac{x\cdot\sin(\pi x)}{\pi} - \int \frac{\sin(\pi x)}{\pi}\, dx\right)$

$u = x \qquad dv = \cos(\pi x)\, dx$

$du = dx \qquad v = \frac{\sin(\pi x)}{\pi}$

$= -\frac{x^2\cos(\pi x)}{\pi} + \frac{2}{\pi}\left(\frac{x\sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}\right) + C$

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7.1 Integration by Parts.

$$\boxed{\int u\,dv = uv - \int v\,du.}$$

Ex: 4 $\int \sqrt{t}\ln(t)\,dt$ $= \frac{2}{3}t^{3/2}\ln(t) - \int \frac{2}{3}t^{3/2}\frac{1}{t}\,dt$

$u = \ln(t)$ $dv = \sqrt{t}\,dt$ $= \frac{2}{3}t^{3/2}\ln(t) - \frac{4}{9}t^{3/2} + C$

$du = \frac{1}{t}\,dt$ $v = \frac{2}{3}t^{3/2}$

Ex: 4 rev:

$\int_1^4 \sqrt{t}\ln(t)\,dt$ $= \left[\frac{2}{3}t^{3/2}\ln(t) - \int \frac{2}{3}t^{1/2}\right]_1^4$

$u = \ln(t)$ $dv = \sqrt{t}\,dt$ $= \left[\frac{2}{3}t^{3/2}\ln(t) - \frac{4}{9}t^{3/2}\right]_1^4$

$du = \frac{1}{t}\,dt$ $v = \frac{2}{3}t^{3/2}$. $= \left(\frac{2}{3}\cdot 8\cdot\ln(4) - \frac{4}{9}\cdot 8\right) - \left(\frac{2}{3}\cdot 0 - \frac{4}{9}\right)$

$= \frac{16}{3}\ln(4) - \frac{28}{9}$

✓ Choose your u in the following order.

- Log
- Inverse trig
- Algebratic
- Trig
- Exponential.

Ex: 5 $\int_1^{\sqrt{3}} \tan^{-1}\left(\frac{1}{x}\right)\,dx$ $= \left[x\arctan\left(\frac{1}{x}\right) + \int x\cdot\frac{1}{x^2+1}\,dx\right]_1^{\sqrt{3}}$

$u = \tan^{-1}\left(\frac{1}{x}\right)$ $dv = dx$

$du = \frac{1}{1+\frac{1}{x^2}}\cdot -\frac{1}{x^2}\,dx$ $v = x$ $= \left[x\underbrace{\tan^{-1}\left(\frac{1}{x}\right)}_{\frac{\pi}{6}} + \frac{1}{2}\ln|x^2+1|\right]_1^{\sqrt{3}}$ let $u = x^2+1$

$= -\frac{1}{1+\frac{1}{x^2}}\cdot\frac{dx}{\frac{1}{x^2}}$ $du = 2x\,dx$

$= \left(\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \frac{1}{2}\ln(4)\right) -$ $\frac{du}{2} = x\,dx.$

$= -\frac{1}{x^2+1}\cdot\frac{dx}{x^2}$ $- \left(\underbrace{\tan^{-1}(1)}_{\frac{\pi}{4}} + \frac{1}{2}\ln(2)\right)$

$= \frac{-x^2\,x^2}{x^2+1}\cdot\frac{dx}{x^2}$

$= \pi\left(\frac{\sqrt{3}}{6} - \frac{1}{4}\right) + \frac{1}{2}\ln(2).$

$= \frac{-1}{x^2+1}$

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Ex: 6

$$\int e^{2\theta} \sin(3\theta)\,d\theta = \frac{1}{2}e^{2\theta}\cdot\sin(3\theta) - \frac{3}{2}\int e^{2\theta}\cdot\cos(3\theta)\,d\theta =$$

$u = \sin(3\theta)$ $\quad dv = e^{2\theta}\,d\theta$ | $u = \cos(3\theta)$ $\quad dv = e^{2\theta}\,d\theta$

$du = 3\cos(3\theta)\,d\theta$ $\quad v = \frac{1}{2}e^{2\theta}$ | $du = -3\sin(3\theta)\,d\theta$ $\quad v = \frac{1}{2}e^{2\theta}$

$$= \frac{1}{2}e^{2\theta}\sin(3\theta) - \frac{3}{4}e^{2\theta}\cos(3\theta) - \frac{9}{4}\int e^{2\theta}\sin(3\theta)\,d\theta.$$

back we're we start.

$$\Rightarrow 1\int e^{2\theta}\sin(3\theta)\,d\theta = \frac{1}{2}e^{2\theta}\sin(3\theta) - \frac{3}{4}e^{2\theta}\cos(3\theta) - \frac{9}{4}\int e^{2\theta}\sin(3\theta)\,d\theta.$$

/ 13/4 $\quad \Rightarrow \frac{13}{4}\int e^{2\theta}\sin(3\theta)\,d\theta = \frac{1}{2}e^{2\theta}\sin(3\theta) - \frac{3}{4}e^{2\theta}\cos(3\theta)$

$$\Rightarrow \int e^{2\theta}\sin(3\theta)\,d\theta = \frac{2}{13}e^{2\theta}\sin(3\theta) - \frac{3}{13}\cos(3\theta) + C.$$