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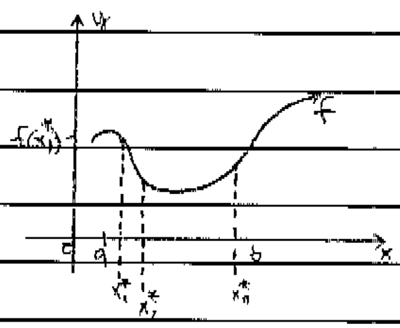
Section §6.5

These notes were taken in class by a helpful student:-)

Average value of a function.

$\{f_1, f_2, f_3 \ldots f_n\}$

the average is $f_{ave} \approx \dfrac{f_1 + f_2 + f_3 \ldots f_n}{n}$



average of f on $a \le x \le b$.

y-values $f(x_1^*), f(x_2^*), \ldots f(x_n^*)$.

weight y-values to get differential elements

$\dfrac{f(x_1^*)}{n}, \dfrac{f(x_2^*)}{n}, \ldots \dfrac{f(x_n^*)}{n}$.

Average

$f_{ave} \approx \dfrac{f(x_1^*) + f(x_2^*) + \ldots + f(x_n^*)}{n}$

$f_{ave} \approx \dfrac{(f(x_1^*) + f(x_2^*) + \ldots f(x_n^*))\,\Delta x}{b-a}$

Recall: $\dfrac{b-a}{n} = \Delta x$

$\Rightarrow \dfrac{b-a}{\Delta x} = n$.

$\approx \dfrac{f(x_1^*)\Delta x + f(x_2^*)\Delta x + \ldots + f(x_n^*)\Delta x}{b-a}$

$\approx \dfrac{\sum_{i=1}^{n} f(x_i^*)\Delta x}{b-a} \Rightarrow f_{ave} = \dfrac{\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x}{b-a}$

$$\boxed{f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

total Area / the width

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Ex: 1

Find the average value of the function

$y = (x-3)^2$ on $[2,5]$

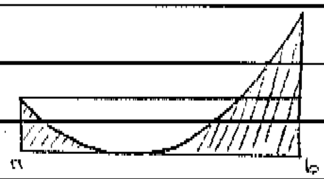
$$f_{ave} = \frac{1}{5-2}\int_2^5 (x-3)^2\,dx \qquad \text{let } u = x-3$$

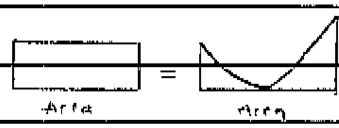
$$= \frac{1}{3}\int_{-1}^{2} u^2\,du \qquad du = dx$$

$$= \frac{1}{3}\left[\frac{u^3}{3}\right]_{-1}^{2} \qquad u(2) = -1$$

$$= \frac{1}{9}(8-(-1)) = \frac{1}{9}(9) \qquad u(5) = 2$$

$$= 1$$





Picture to go w/ the Mean Value theorem for definite integrals.

Mean Value Theorem

If f is continuous on $[a,b]$ then there exists $c \in [a,b]$ such that

$$f(c) = \underbrace{\frac{1}{b-a}\int_a^b f(x)\,dx}$$

$$\text{or} \quad f(c) = f_{ave}.$$

Ex 1: Find c such that (5.4).

$f(c) = f_{ave}.$

Solve: $(c-3)^2 = 1$

$\Rightarrow c-3 = \pm 1$

$\Rightarrow c = 3 \pm 1$

$c = 2$ or $c = 4$.

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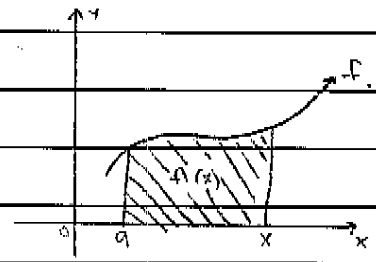
Review:

This is a theoretical use of the MVT for definite integrals ... that is, it helped us when we derived our FToC.

Derivation of FToC (Step 1)

$$A(x) = \int_a^x f(x)\,dx$$

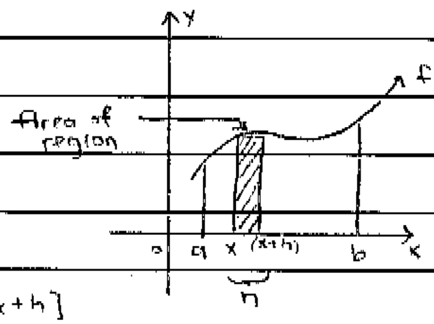


comulative area function

$$\int_x^{x+h} f(x)\,dx = A(x+h) - A(x)$$

$$= \underbrace{f(c)}_{f\text{ ave}} \cdot h$$

where $c \in [x, x+h]$



$$\Rightarrow f(c) = \frac{A(x+h) - A(x)}{h}$$

$$\lim_{h \to 0} f(c) = \lim_{h \to 0} \frac{A(x+h) - A(x)}{h}$$

$$\Rightarrow f(x) = A'(x)$$