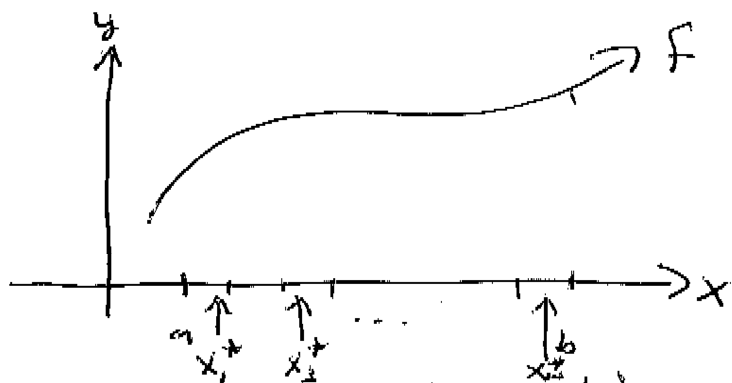
6.5 : Ave. value of a fct

To average n values... $y_1, \dots, y_n$

$$y_{ave} = \frac{y_1 + y_2 + \dots + y_n}{N} = \frac{y_1}{n} + \frac{y_2}{n} + \dots + \frac{y_n}{n}$$

If we want to find the average of a fct $f(x)$ on $[a,b]$



(1) subdivide (equal widths) $\Delta x = \frac{b-a}{n} \Rightarrow n = \frac{b-a}{\Delta x}$

(2) sample points.

(3) differential fct elements

$$f_{ave} \approx \frac{f(x_1^*)}{n} + \dots + \frac{f(x_n^*)}{n} = \frac{f(x_1^*) + \dots + f(x_n^*)}{n}$$

$$= \frac{f(x_1^*) + \dots + f(x_n^*)}{\frac{b-a}{\Delta x}}$$

$$= \frac{1}{b-a}\left(f(x_1^*)\Delta x + \dots + f(x_n^*)\Delta x \right)$$

$$= \frac{1}{b-a} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

Motivate the average value of a function discussion by talking about climate change and the average temp on earth ... how many thermometers in Seattle, the Sahara, and the Pacific Ocean? ... these are not equally weighted ... this leads nicely to the alternative definition of the definite integral where we look at lim(dx)->0.

6.5
2/3

(4) limit of the Riemann sums

$$f_{ave} = \frac{1}{b-a} \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

(5) write as a definite integral.

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx$$

Ex1: Find the average value of $f(x) = (x-3)^2$ on $[2, 5]$

$$f_{ave} = \frac{1}{5-2}\int_2^5 (x-3)^2 dx$$

$$= \frac{1}{5-2} \cdot \left[\frac{1}{3}(x-3)^3\right]_2^5$$

$$= \frac{1}{9}\left(8 - (-1)\right)$$

$$= \underline{1}.$$

MVT for Integrals If f is cont. on $[a,b]$, then $\exists$ $c \in [a,b]$ s.t.

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

or

$$\int_a^b f(x)dx = f(c)(b-a).$$

$\boxed{\begin{matrix} 6,5 \\ 3/3 \end{matrix}}$

Ex1 rev: Find c s.t. $f(c) = f_{ave}$.

solve $f(c) = f_{ave}$

$\Rightarrow (c-3)^2 = 1$

$\Rightarrow c - 3 = \pm 1$

$\Rightarrow c = 3 \pm 1$

$\Rightarrow c = 2$ or $c = 4$

Draw the picture.