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Our question: Why does the abstract concept of the definite Integral so perfectly describe the physical world?

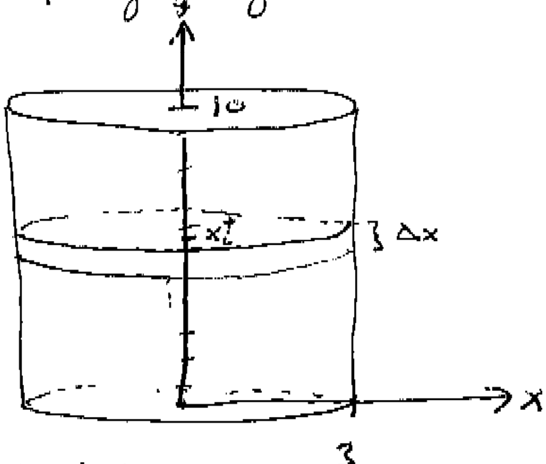
So far, we have used the definite integral to find areas & volumes ... both are physical quantities you can touch. Today, we will use it to determine "work" which can't be seen.

Work Definitions.

Dfn. $F = m \cdot a$ (Newton's 2^{nd} law of motion)

Dfn. $W = F \cdot d$ (work)

<u>Ex1</u>: Find the work required to pump all the water out of a completely filled water tank (see pic). The density of water is $1000\,kg/m^3$ and the acceleration of gravity $\approx 10\,m/s^2$.



(1) subdivide

(2) sample points

(3) differential <u>work</u> element.

Area	9π
Volume	$9\pi\,\Delta x$
mass	$9{,}000\pi\,\Delta x$
force	$90{,}000\pi\,\Delta x$
work	$90{,}000\pi(10-x_i^*)\,\Delta x$

(4) limit of Riemann sums

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} 90{,}000\pi(10-x_i^*)\,\Delta x$$

(5) Definite Integral

$$W = \int_0^{10} 90{,}000\pi(10-x)\,dx$$

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Work

Defn: $F = m \cdot a$ (Newton's 2nd law of motion).

Defn: $W = f \cdot d$. (work).

Ex 1:

a) How much work to lift an 10 lb book 2 feet above the table.

b) How much work to lift a 10 kg bag up 2 meters. ($g \approx 10 m/s^2$).

If an object moves along the x-axis in the positive direction from a to b and a force $f(x)$ acts on the object where f is a cont. fct, then the work done is

$$W = \int_a^b f(x)\,dx$$

Ex 2: A particle is moved along the x-axis by a force that measures 5x pounds at a point x-ft from the origin. Find the work done in moving the particle from the origin to a pt 4 ft from the origin.

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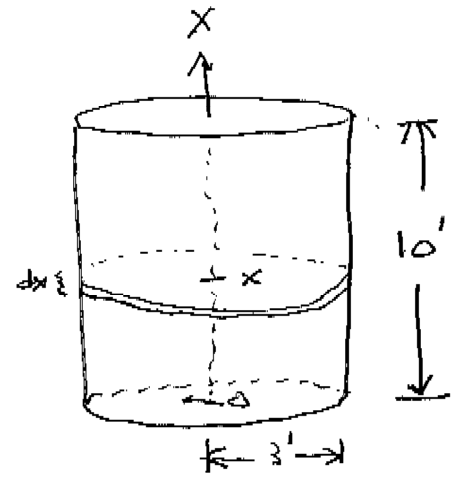
Ex 3: Find the work required to pump all the water out of the top of a completely filled tank (see pic) The density of water is 62.5 lbm/ft³



$A(x) = 9\pi \quad ft^2$

$v(x) = 9\pi\, dx \quad ft^3$

$m(x) = 9\pi \cdot 62.5\, dx \quad lbm$

$f(x) = 9\pi(62.5)(32)\, dx \quad \frac{lbm \cdot ft}{s^2}$

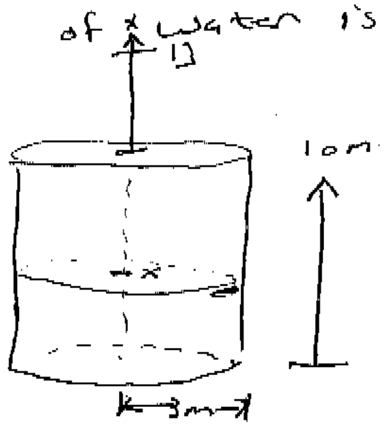
$= 9\pi\,(62.5)\,dx \quad \frac{slug \cdot ft}{s^2}$

$w(x) = 9\pi(62.5)(10-x)\, dx \quad \frac{slug \cdot ft^2}{s^2}$

$$W = \int_0^{10} 9\pi(62.5)(10-x)\, dx$$

$$= \underline{\qquad\qquad} \quad \frac{slug \cdot ft^2}{s^2}$$

Ex 4: Find the work required to pump all the water out of the tank to a height 3 m above the top of the tank. The density of water is 1000 kg/m³.



$A(x) = 9\pi \quad m^2$

$v(x) = 9\pi\, dx \quad m^3$

$m(x) = 9000\pi\, dx \quad kg$

$f(x) = 90000\pi\, dx \quad \frac{kg \cdot m}{s^2} = N$

$w(x) = 90000\pi(13-x)\, dx \quad N \cdot m$

$$W = \int_0^{10} 90000\pi(13-x)\, dx \quad N \cdot m.$$

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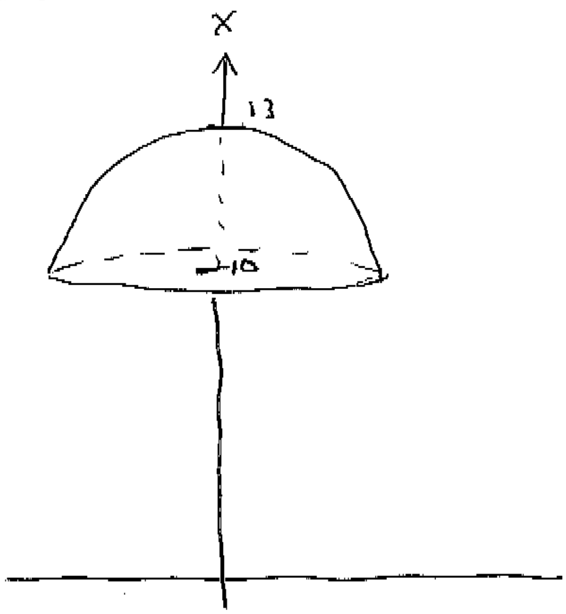
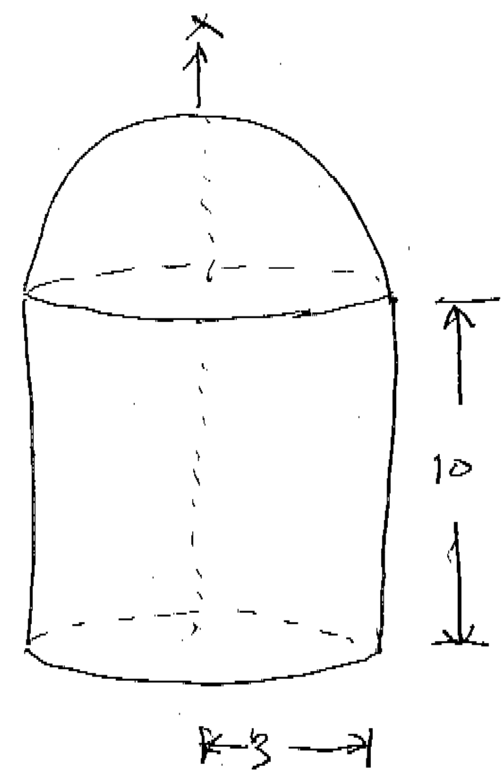
Ex5: Empty the tank



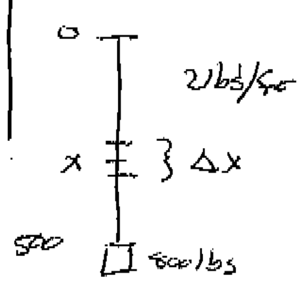
If the work required to stretch a spring 1 ft beyond its natural length is 12 ft-lbs, how much work is needed to stretch it 9 in. beyond its natural length?

$12 = \int_0^1 kx\,dx \Rightarrow k = 24$

Ex6: Empty the tank.



A cable that weighs 2 lb/ft is used to lift 800 lb of coal up a mine shaft 500 feet deep. Find the work done



$W = 800(500) + \int_0^{500} 2x\,dx$