7.1 #46

$$\int \sin^{2n} x \, dx = \int \sin^{2n-1} x \cdot \sin x \, dx$$

$$u = \sin^{2n-1} x$$

$$du = (2n-1) \sin^{2(n-1)} x \cdot \cos x$$

$$dv = \sin x \, dx$$

$$v = -\cos x$$

$$= \left[-\sin^{2n-1} x \cos x + \int (2n-1) \sin^{2(n-1)} x \cos^2 x \, dx \right]_0^{\pi/2}$$

$$(2n-1) \int \sin^{2(n-1)} x \, (1 - \sin^2 x) \, dx$$

$$I = \int \sin^{2n} x \, dx = -\sin^{2n-1} x \cos x + (2n-1) \int \sin^{2(n-1)} x \, dx - (2n-1) \underbrace{\int \sin^{2n} x \, dx}_{I}$$

cyclic

$$\Rightarrow 2n \int \sin^{2n} x \, dx = \Big[\underbrace{-\sin^{2n-1} x \cos x}_{= 0 \text{ after FToC 2}} + (2n-1) \int \sin^{2(n-1)} x \, dx \Big]_0^{\pi/2}$$

$$\Rightarrow \int_0^{\pi/2} \sin^{2n} x \, dx = \frac{2n-1}{2n} \int_0^{\pi/2} \sin^{2(n-1)} x \, dx$$

$$= \frac{2n-1}{2n} \cdot \frac{2(n-1)-1}{2(n-1)} \cdot \ldots \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \int_0^{\frac{\pi}{2}} \sin^0 x \, dx$$

$$= \frac{2n-1}{2n} \cdot \frac{2n-3}{2(n-1)} \cdot \ldots \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \int_0^{\frac{\pi}{2}} 1 \, dx$$

$$= \frac{1 \cdot 3 \cdot \ldots \cdot (2n-3) \cdot (2n-1)}{2 \cdot 4 \cdot \ldots \cdot (2n-2)\,(2n)} \frac{\pi}{2}$$