Section 11.05

Applications

Part 1: Elasticity

Elasticity measures the responsiveness of demand to price changes. High elasticity means responsiveness is high while low elasticity means that demand is relatively unchanged by changes in price.

On a scale of 1 to 10, how elastic do you think demand is in the following markets?

- chocolate
- insulin (medicine for diabetics)
- clothing
- water
- cocaine
- automobiles
- cigerettes
- steel
- text books

Do you think that elasticity is constant for a given product? That is, do you think that the responsiveness of demand to price changes might depend upon the price in any way?

Elasticity of Demand: $\eta = -\frac{p}{q} \cdot \frac{dq}{dp} = -\frac{p}{q(p)} \cdot q'(p)$.

Note that η is the Greek letter "eta."

We have skipped over a topic called implicit differentiation. For that reason, skip over example 2 in the text.

Example 1: Find the elasticity of demand if demand is modeled by $2p+3q=150$ when $p=15,\ 37.5,$ and 45.

Vocabulary related to elasticity of demand:

If $\eta > 1$, we say that demand is elastic

If $\eta < 1$, we say that demand is inelastic

If $\eta = 1$, we say that demand is unitary elastic

Example 2: Demand is given by $p = \frac{1000}{(q+1)^2}$. Find η when $q = 19$.

Elasticity and Revenue:

$$R(p) = p \cdot q(p)$$

where $q(p)$ is the quantity demanded at a price p.

$$\begin{aligned} R'(p) &= p \cdot q'(p) + q(p) \\ &= q(p) \cdot \frac{p}{q(p)} \cdot q'(p) + q(p) \\ &= q(p)(-\eta) + q(p) \\ &= q(p)(1 - \eta) \end{aligned}$$

Summary:

If $\eta > 1$, then $R' < 0$ and a price increase will result in a revenue decrease and visa versa

If $\eta < 1$, then $R' > 0$ and a price increase will result in a revenue increase and visa versa

If $\eta = 1$, then $R' = 0$ and an increase in price will not result in a change in revenue. Revenue is optimized at this point.

Example 3: Given the demand function $p = 120 \sqrt[3]{125 - q}$, answer the following.

a.) Find $\eta(p)$

b.) Find the point (q, p) where $\eta = 1$

c.) Construct a sign diagram of R'

d.) Find the maximimum revenue

Part 2: Taxation in a competitive market

The goal of this section is to determine the level of taxation that will maximize tax revenue.

The picture:

So, rather than working with the supply function $S: \; p(q)$, work with the supply function after taxation $S: \; p(q) + t$. Also remember the simple formula that total tax is equal to $T = t \cdot q$.

Example 4: If the weekly demand function is $D: p = 200 - 2q^2$ and the supply function before taxation is $S: p = 20 + 3q$, what tax per item will maximize the total tax revenue and what is the maximum tax revenue that can be generated?

Ethical question: Is it important for a government to generate the maximum possible tax revenue? Why or why not?