Section 9.5

# The Product and Quotient Rules

## Part 1: The Product Rule

**Example 1**: How would we approach $\frac{d}{dx}(3x^4 - 2x^3 + 7)(x^3 + 2x^2 + 4x - 9)$ using the techniques of the previous section?

**Derivative Rule**: The Product Rule

If $f(x) = u(x) \cdot v(x)$, where $u$ and $v$ are differentiable functions of $x$, then $f'(x) = u'(x) \cdot v(x) + v'(x) \cdot u(x)$.

This can be memorized as, $(u \cdot v)' = u'v + v'u$.

**Example 1 revisited**: Find $\frac{d}{dx}(3x^4 - 2x^3 + 7)(x^3 + 2x^2 + 4x - 9)$ using the product rule (Hint: What are $u$ and $v$?)

**Example 2**: Find $y'$ if $y = (x+3)(x^2 - 2x)$.

**Example 3** (for you): Find $\frac{ds}{dt}$ if $s = (t^2+1)(t^3-1)$

**Example 4**: Differentiate $f(x) = (7x^6 - 5x^4 + 2x^2 - 1)(4x^9 + 3x^7 - 5x^2 + 3x)$, but do not simplify your result.

**Example 5**: Differentiate $y = \left(\sqrt[5]{x} - 2\sqrt[4]{x} + 1\right)\left(x^3 - \frac{5}{x} + 3x^{-8}\right)$, but do not simplify your result.

**Example 6**: An agency will give tours for groups of at least 25. The cost is \$300/person for groups of 25. The cost is reduced by \$10 (per person) for each person over 25.

a.) Find $R(n)$ (the revenue function)

b.) Find $D_R$ (the domain)

c.) Find $\overline{\text{MR}}$

d.) Find and interpret $\overline{\text{MR}}(30)$

## Part 2: The Quotient Rule

**Derivative Rule**: The quotient rule

If $f(x) = \frac{u(x)}{v(x)}$, where $u$ and $v$ are differentiable functions of $x$ with $v(x) \neq 0$, then $f'(x) = \frac{u'(x) \cdot v(x) - v'(x) \cdot u(x)}{[v(x)]^2}$.

This can be memorized as, $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$.

**Example 7**: If $f(x) = \frac{x^3 + 2x}{x^2 - 7}$, find $f'$

**Example 8**: Find $\frac{d}{dx} \frac{1 - 2x^2}{x^4 - 2x^2 + 5}$

**Quiz - Just for you**

a.) Write down the product rule

b.) Write down the quotient rule

c.) What two pieces of information do you need to find the equation of a tangent line? And, what formula is most helpful for writing the equation of the line?

**Example 9**: Find the equation of the tangent line to $y = (4x^2 + 4x + 1)(7 - 2x)$ when $x = 1$.

Example 10: Experimental evidence has shown that the concentration of injected adrenaline $x$ is related to the response $y$ of a muscle according to the equation $y = \frac{x}{a+b\,x}$ where $a$ and $b$ are constants. Find the ROC of response with respect to (WRT) concentration.