Derivative Rules

Part 1: The Power Rule

Example 1: Find the derivatives of:

a.) $f(x) = 1$ (graphically)

b.) $g(x) = x$ (graphically)

c.) $h(x) = x^2$ (using the definition)

d.) $i(x) = x^3$ (using the definition)

e.) $j(x) = x^n$ (following the pattern from above)

Derivative Rule: The power rule

If $f(x) = x^n$, where n is a real number, then $f'(x) = n \cdot x^{n-1}$

Example 2: Find the derivatives of:

a.) $f(x) = x^4$

b.) $g(x) = x^{-4}$

Notation: We use a number of notations to refer to the derivative of $y = f(x)$.
They are, but not limited to:
1.) $f'(x)$, read "f prime of x" or "the derivative of f with respect to x"
2.) f', read, "f prime"
3.) y', read, "y prime"
4.) $\frac{dy}{dx}$, read, "$d\,y\,d\,x$" or "$d\,y$ by $d\,x$."
5.) $\frac{d}{dx} f(x)$ or $\frac{d}{dx} f$, read "$d\,d\,x$ of f of x" or "$d\,d\,x$ of f"

Example 3: Find the derivatives of:

a.) $y = x^{\frac{2}{3}}$

b.) $\frac{d}{dx} \sqrt{x}$

c.) If $y = \frac{1}{\sqrt[3]{x}}$, find $\frac{dy}{dx}$.

Example 4: Find the equation of the tangent line to $y = x^2$ when $x = 3$

Example 5: Derivatives with constants. Does the power rule still apply?

Find the derivative of $y = \pi x^7$.

Derivative Rule: The coefficient rule

If $f(x) = c \cdot u(x)$ where c is a constant and $u(x)$ is a differentiable function of x, then $f'(x) = c \cdot u'(x)$.

Example 6: Find:

a.) $\frac{d}{dx} 7 \sqrt[4]{x}$

b.) $(4 x^5)'$

c.) If $n = \frac{5}{v^{\frac{2}{3}}}$, find $\frac{dn}{dv}$

Derivative Rule: sums and differences

If $f(x) = u(x) \pm v(x)$, where u and v are differentiable functions if x, then $f'(x) = u'(x) \pm v'(x)$.

□ <u>proof</u>.

Part 2: Applications

Example 7: Suppose the revenue from the sale of x items is modeled by $R(x) = 300\,x - 0.02\,x^2$.

a.) Find $\overline{MR}$ when $x = 40$.

b.) Interpret the result from part (a.).

Example 8: Suppose the cost from the sale of x items is
$C(x) = 40500 + 190\,x + 0.2\,x^2$.

a.) Find the average cost function $\overline{C}(x) = \frac{C(x)}{x}$

b.) Find the instantaneous ROC of the average cost function.

c.) When does the instantaneous ROC of the average cost function (from (b.)) equal zero?

d.) Find $\overline{MC}(x)$ and $\overline{C}(x)$ at the zero found in (c.).