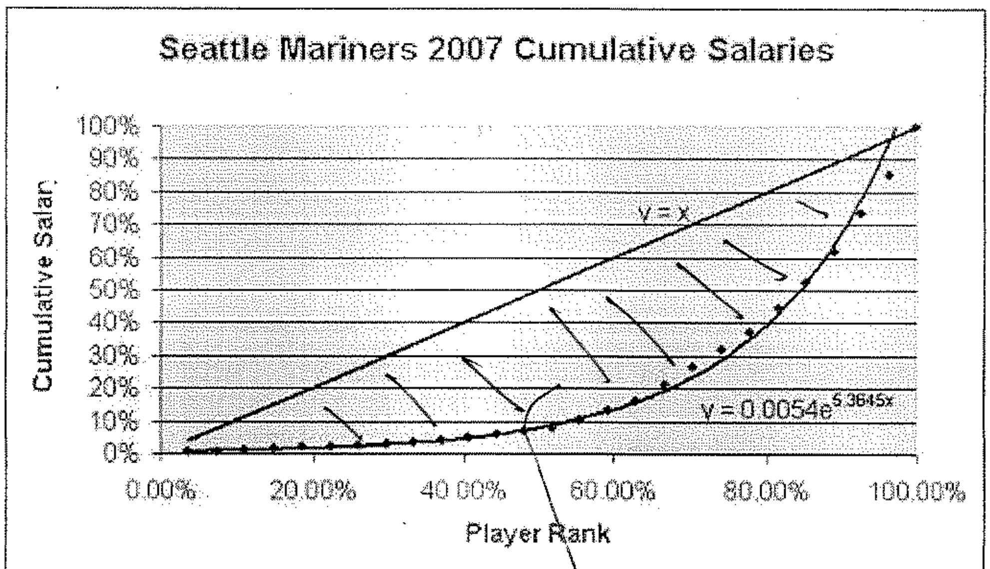
Section 13.03

Area between curves

Part 1: The Lorenz Curve

In economics, the Lorenz curve is used to represent the inequality of income distribution among different groups in the population of a country. The curve is constructed by plotting the cumulative percent of families at or below a given income level and the cumulative percent of total personal income received by these families.

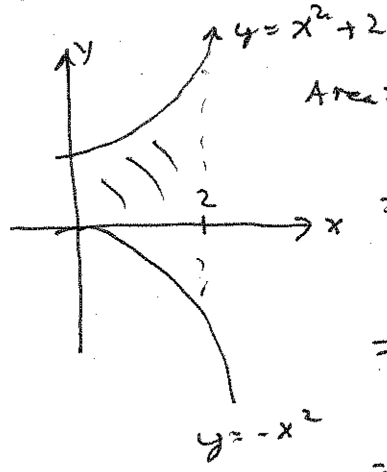
The curve below shows the Lorenz curve $L(x)$ for the 27 players on the Seattle Mariner roster (as listed in USA Today).



Area represents the disparity.

Part 2: Area between curves

Example 1: Find the area between $y = x^2 + 2$ and $y = -x^2$ on [0, 2]



$$\text{Area} = \int_0^2 [(x^2+2) - (-x^2)]\,dx$$

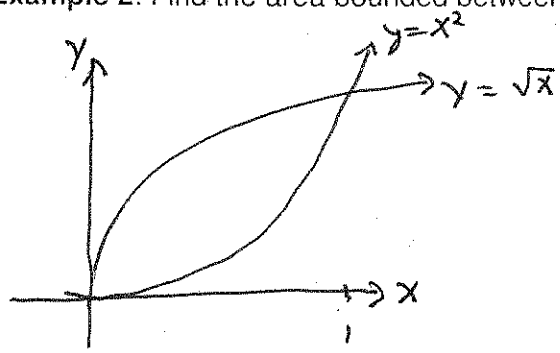
$$= \int_0^2 (2x^2 + 2)\,dx$$

$$= \left(\frac{2}{3}x^3 + 2x\right)\Big]_0^2$$

$$= \left(\frac{16}{3} + 4\right) - 0$$

$$= \frac{28}{3}$$

Example 2: Find the area bounded between $y = x^2$ and $y = \sqrt{x}$



$$\text{Area} = \int_0^1 (\sqrt{x} - x^2)\,dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{1}{3}x^3\right]_0^1$$

$$= \left(\frac{2}{3} - \frac{1}{3}\right) - 0$$

$$= \frac{1}{3}$$

Part 3: The Gini coefficient

Definition: The Gini Coefficient
We measure income distribution through the Gini coefficient which is defined as:

$$\frac{\text{area between } y=x \text{ and } L(x)}{\text{area below } y=x} = \frac{\int_0^1 [x-L(x)]\,dx}{1/2}$$

$$= 2\int_0^1 [x-L(x)]\,dx$$

$0 \leq$ Gini Coefficient ≤ 1

equal distribution of wealth

1 person owns everything.

Example 3: The Lorenz curve for income distribution in the US in 1950 and 1970 are given. Find and compare the Gini coefficients.

$$1950: y = 0.925\,x^{1.891}$$

$$1970: y = 0.920\,x^{1.783}$$

$$G_{1950} = 2\int_0^1 [x - 0.925\,x^{1.891}]\,dx$$

$$= 2\left[\frac{x^2}{2} - \frac{0.925}{2.891}x^{2.891}\right]_0^1$$

$$= 2\left[\left(\frac{1}{2} - \frac{0.925}{2.891}\right) - 0\right]$$

$$= 0.36$$

$G_{1970} = 0.34$ (found in a similar manner as above).

Question: Which is better (an ethical question): a small or large Gini coefficient?

It depends...

Part 4: Average value of a function

Definition: Average value of a function
The average value of a continuous function $y = f(x)$ on $[a, b]$ is:

$$\text{average value} = f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x)\, dx$$

Think $\dfrac{\text{Area}}{\text{base}}$ = Ave. ht.

Example 4: The cost to produce x units of a product is $C(x) = x^2 + 400x + 2000$.

a.) Find the average cost of producing 1000 units (the cost per unit)

$$\bar{C}(x) = \frac{C(x)}{x} = \frac{x^2 + 400x + 2000}{x}$$

AND $\bar{C}(1000) = \$1402/\text{unit}$

b.) Find the average value of the cost function on $[0, 1000]$ (the cost per shipment).

$$C_{\text{ave}} = \frac{1}{1000-0} \int_0^{1000} (x^2 + 400x + 2000)\, dx$$

$$= \frac{1}{1000} \left[\frac{x^3}{3} + 200x^2 + 2000x \right]_0^{1000}$$

$$= \$535{,}333/\text{shipment}.$$