Curve sketching with asymptotes

Part 1: More curve sketching

Recall the manner in which we found limits in the early sections of chapter nine. That is the foundation for how we determine horizontal and vertical asymptotes (even if we do not always show the work required to determine the asympotes).

> Horizontal aysmptotes: The function $y = f(x)$ has a horizontal asymptote at $y = b$ if $\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$.

Recall that a rational function can be represented as the ratio of two polynomials. That is, it is a "fraction" where the numerator and denominator or both polynomial functions.

> Vertical asymptotes of a rational function: The graph of the rational function $h(x) = \frac{f(x)}{g(x)}$ has a vertical asymptote at $x = c$ if $g(c) = 0$ and $f(c) \neq 0$ (after simplification).

Example 1: Find all asymptotes (horizontal and vertical) of $f(x) = \frac{3x+2}{x-4}$

simplified ✓

(1) V A @ $x = 4$

(2) $\lim_{x\to\pm\infty} \frac{3x+2}{x-4} = \lim_{x\to\pm\infty} \frac{3 + \frac{2}{x}}{1 - \frac{4}{x}} = 3$

(3) H. A. @ $y = 3$.

Example 2: Find all asymptotes (horizontal and vertical) of $g(x) = \frac{x^2-1}{x+1}$

$$g(x) = \frac{x^2-1}{x+1} = \frac{(\cancel{x+1})(x-1)}{\cancel{x+1}} = x-1 \quad \text{(simplified)} \checkmark$$

(1) V.A. None (there is a hole when $x=-1$)

(2) H.A. None.

Example 3: Find all asymptotes (horizontal and vertical) of $h(x) = \frac{x+2}{x^2-4}$

$$h(x) = \frac{x+2}{x^2-4} = \frac{\cancel{x+2}}{(\cancel{x+2})(x-2)} = \frac{1}{x-2} \quad \text{(simplified} \checkmark)$$

(1) V.A. @ $x=2$ (hole @ $x=-2$)

(2) $\lim\limits_{x \to \pm\infty} \frac{1}{x-2} = 0$

$\Rightarrow$ H.A. @ $y=0$.

Curve sketching process:

1.) Find the domain (sign diagram of the function)

2.) Determine vertical asymptotes

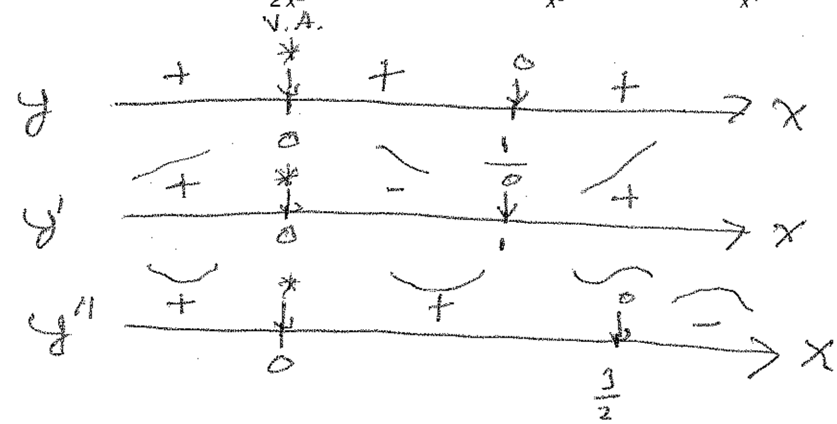
3.) Determine horizontal asymptotes

4.) Find relative extremes (sign diagram of the derivative)

5.) Find points of inflection (sign diagram of the second derivative)

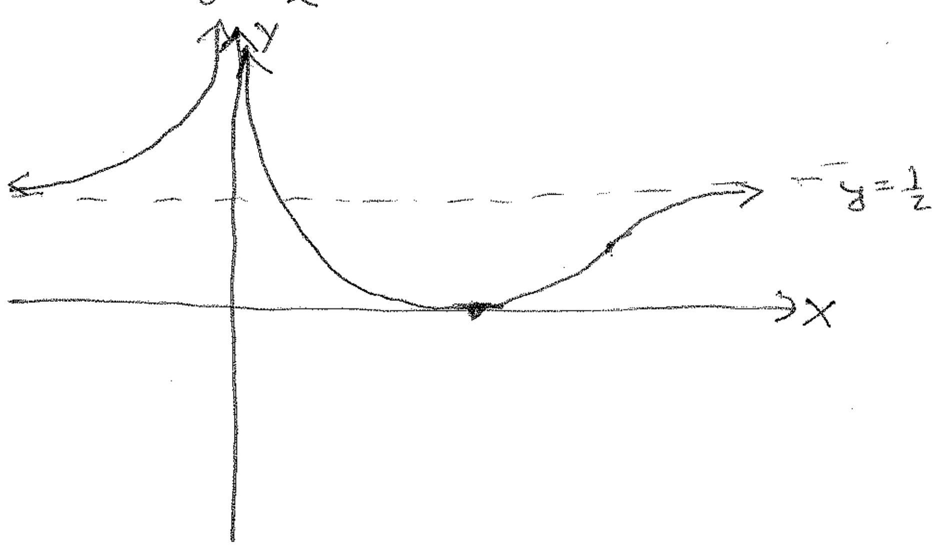
6.) Sketch the graph

Example 4: Sketch $y = \frac{(x-1)^2}{2x^2}$ given that $y' = \frac{(x-1)}{x^3}$ and $y'' = \frac{3-2x}{x^4}$



V.A @ $x = 0$

H.A @ $y = \frac{1}{2}$



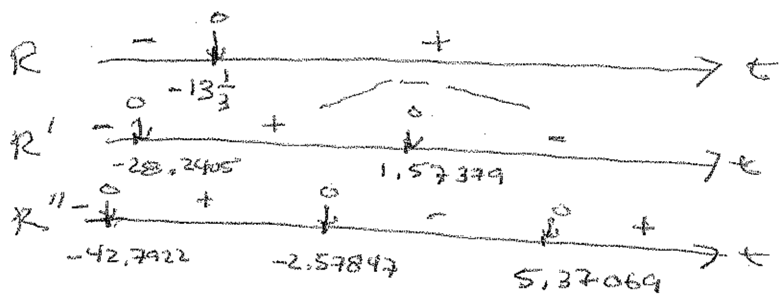
Example 5: Suppose a film (say, Spiderman III) has weekly revenue of $R(t) = \frac{150\,(0.3\,t+4)}{0.09\,t^2+4}$, $t \geq 0$ where the revenue is given in ~~mi~~llions of dollars. 100,000 ~~dollars~~ dollars,

a.) Sketch a graph of $R(t)$ given that

$$R'[t] = -\frac{4.05\,(t-1.57379)\,(t+28.2405)}{(4.+0.09\,t^2)^2}$$

and

$$R''[t] = \frac{0.729\,(t+-5.37069)\,(t+2.57847)\,(t+42.7922)}{(4.+0.09\,t^2)^3}$$



$R(1.57379) = 158.85$

b.) Find and interpret the maximum of R.

weekly rev. is maxed after 11 days @ 15,885 million.

wk 5.37

c.) Four weeks after the marginal revenue begins to increase, a film is pulled from the theater. Twelve weeks later, the DVD is released. How long after the initial release will this film be released on DVD.

17.37 wks.

Example 6: An entrepreneur starts new companies and sells them. Suppose the annual profit of one of her start ups is given by $P(x) = 22 - \frac{1}{2}x - \frac{18}{x+1}$. If she sells before profits begin to decline, when should she sell?

$$P'(x) = -\frac{1}{2} + \frac{18}{(x+1)^2}$$

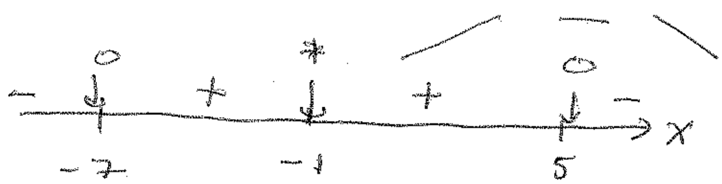
Solve $P' = 0 \Rightarrow \frac{1}{2} = \frac{18}{(x+1)^2}$

$\Rightarrow (x+1)^2 = 36$

$\Rightarrow x+1 = \pm 6$

$\Rightarrow x = -1 \pm 6$

$\Rightarrow x = -7$ or $x = 5$



Sell before 5 yrs are up.