Applications of the Derivative

Part 1: Business Applications

Example 1: If the cost in dollars to produce x items is given by $C(x) = 50 + 48x + x^3$, find the marginal cost function $\overline{MC}(x)$.

$\Rightarrow \overline{MC}(x) = 48 + 3x^2$

Example 2: Suppose that the cost function for a commodity is $C(x) = 300 + 6x + \frac{1}{20}x^2$. (in dollars).

a.) Find and interpret $\overline{MC}(8)$.

$\overline{MC}(x) = 6 + \frac{1}{10}x$

AND $\overline{MC}(8) = 6.8$

The cost to produce the 8th item is about \$6.80.

b.) Find and interpret $C(8) - C(7)$. $= 351.2 - 344.45$

$= 6.75$

The exact cost to produce the 8th item was \$ 6.75.

Example 3: Suppose that the total revenue function for a commodity is $R(x) = 36x - 0.01x^2$. (in \$)

a.) Find and interpret $R(100)$ = 3500

The total revenue from the sale of 100 units is \$3500

b.) Find and interpret $\overline{MR}(100)$

$\overline{MR}(x) = 36 - 0.02x$

$\Rightarrow \overline{MR}(100) = 34.$

The approx revenue from the sale of the 100th item is \$34.

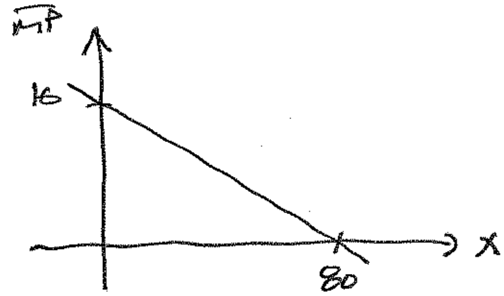
c.) Find and interpret $R(100) - R(99)$ = 34.01

The exact revenue from the sale of the 100th item was \$34.01

Example 4: If the profit from the sale of *x* items is given by
$P(x) = 16\,x - 0.1\,x^2 - 100$

a.) Graph $\overline{\text{MP}}(x)$

$\overline{MP}(x) = 16 - 0.2x$



b.) What level of production and sales will give a marginal profit of zero?

$\overline{MP} = 0$ when $x = 80$
(80 items are produced).

c.) At what level of production is the profit maximized?

$P(x)$ is maxed when 80 items are produced/sold.

(see the vertex of $P(x)$).

Example 5: If the daily of cost per unit associated with producing a product by the Caterpillar (CAT) Corp is $10 + 2x$ and if the price for each unit is \$50 on the competitive market, what is the maximum daily profit that can be expected from this product?

$C(x) = x(10+2x)$

$R(x) = 50x$

$\Rightarrow P(x) = 50x - 2x^2 - 10x$

$= -2x^2 + 40x$

$\Rightarrow P'(x) = -4x + 40$

$P'(x) = 0$ when $x = 10$. So...

$P(10) = 200$ is the max profit.