Section 9.8

Higher Order Derivatives

Part 1: Higher Order Derivatives

Example 1: Find the second derivative of $f(x) = 2x^{10} - 18x^5 - 12x^3 + 4$

$\Rightarrow f'(x) = 20x^9 - 90x^4 - 36x^2$

$\Rightarrow f''(x) = 180x^8 - 360x^3 - 72x$

Example 2: Find the second derivative of $y = 3x^2 - \sqrt[3]{x^2+1}$

$\Rightarrow \frac{dy}{dx} = 6x - \frac{1}{3}(x^2+1)^{-2/3} \cdot 2x$

$\Rightarrow \frac{d^2y}{dx^2} = 6 - \left[\frac{2}{3}(x^2+1)^{-2/3} + \frac{2}{3}x \cdot \left(\frac{-2}{3}\right)(x^2+1)^{-5/3} \cdot 2x\right]$

Example 3: Find the second derivative of $z = \sqrt{y^3 + y^2 - 2}$

$\Rightarrow z' = \frac{1}{2}(y^3 + y^2 - 2)^{-1/2} \cdot (3y^2 + 2y)$

$\Rightarrow z'' = \left[-\frac{1}{4}(y^3+y^2-2)^{-3/2} \cdot (3y^2+2y)\right] \cdot (3y^2+2y) +$

$\frac{1}{2}(y^3+y^2-2)^{-1/2} \cdot (6x+2)$

Example 4: Find $f''(x)$ if $f(x) = x^5 - x^{1/2}$

$$\Rightarrow \quad f'(x) = 5x^4 - \frac{1}{2}x^{-1/2}$$

$$\Rightarrow \quad f''(x) = 20x^3 + \frac{1}{4}x^{-3/2}$$

Example 5: Find $\frac{d^2 y}{d x^2}$ if $y = \sqrt{x+1}$ $= (x+1)^{1/2}$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2}(x+1)^{-1/2}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\frac{1}{4}(x+1)^{-3/2}$$

Example 6: Find $g^{(4)}(x)$ if $g(x) = x^6 - 15x^3$

$$\Rightarrow \quad g'(x) = 6x^5 - 45x^2$$

$$\Rightarrow \quad g''(x) = 30x^4 - 90x$$

$$\Rightarrow \quad g^{(3)}(x) = 120x^3 - 90$$

$$\Rightarrow \quad g^{(4)}(x) = 360x^2$$

Example 7: Find $h^{(3)}(x)$ if $h(x) = \frac{x^2}{x^2+1}$ (this is a pain)

$$\Rightarrow h'(x) = \frac{2x(x^2+1) - 2x \cdot x^2}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$$

$$\Rightarrow h''(x) = \frac{2(x^2+1)^2 - 2x \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4}$$

$$= \frac{2(x^2+1) - 8x^2}{(x^2+1)^3}$$

$$= \frac{2 - 6x^2}{(x^2+1)^3}$$

$$\Rightarrow h^{(3)}(x) = \frac{-12x(x^2+1)^3 - (2-6x^2) \cdot 3(x^2+1)^2 \cdot 2x}{(x^2+1)^6}$$

Part 2: nDeriv and *Mathematica*

Example 8: Use the numerical derivative to find $f''(x)$ if $f(x) = \frac{1}{\sqrt{x^2+7}}$

Calculator notes: To find the derivative of y1, you need to use:

y2 = nDeriv (y1(x), x, x)

and to find the second derivative:

y3 = nDeriv (nDeriv(y1, x, x), x, x)

To find the $\boxed{\text{nDeriv}}$ command, choose $\boxed{\text{MATH}} \to \boxed{8}$ and to find y2 (or y3), choose $\boxed{\text{VARS}} \to \boxed{Y-\text{VARS}} \to \boxed{\text{Function}} \to \boxed{2}$

or

y_3 = nDeriv(y2, x, x)

~~**Example 9**: There is a *Mathematica* example on the webpage.~~