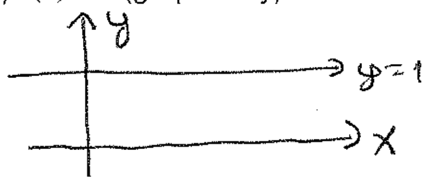
Section 9.4

Derivative Rules

Part 1: The Power Rule

Example 1: Find the derivatives of:

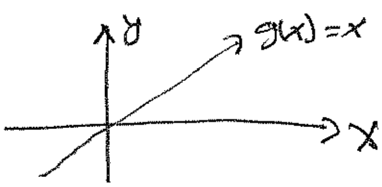
a.) $f(x) = 1$ (graphically)



slope $= 0$ so

$f'(x) = 0$

b.) $g(x) = x$ (graphically)



slope $= 1$ so

$g'(x) = 1$

c.) $h(x) = x^2$ (using the definition)

$$h'(x) = \lim_{h \to 0} \frac{h(x+h) - h(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} (2x + h)$$

$$= 2x$$

d.) $i(x) = x^3$ (using the definition)

$$i'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h \to 0} (3x^2 + 3xh + h^2)$$

$$= 3x^2.$$

e.) $j(x) = x^n$ (following the pattern from above)

fct	derivative
1	0
x^1	1
x^2	$2x$
x^3	$3x^2$
x^4	$4x^3$
$\vdots$	
x^n	nx^{n-1}

Derivative Rule: The power rule

If $f(x) = x^n$, where n is a real number, then $f'(x) = n \cdot x^{n-1}$

Example 2: Find the derivatives of:

a.) $f(x) = x^4$

$$f'(x) = 4x^3$$

b.) $g(x) = x^{-4}$

$$g'(x) = -4x^{-5}$$

Notation: We use a number of notations to refer to the derivative of $y = f(x)$. They are, but not limited to:
1.) $f'(x)$, read "f prime of x" or "the derivative of f with respect to x"
2.) f', read, "f prime"
3.) y', read, "y prime"
4.) $\frac{dy}{dx}$, read, "$d\,y\,d\,x$" or "$d\,y$ by $d\,x$."
5.) $\frac{d}{dx} f(x)$ or $\frac{d}{dx} f$, read "$d\,d\,x$ of f of x" or "$d\,d\,x$ of f"

Example 3: Find the derivatives of:

a.) $y = x^{\frac{2}{3}}$

$$y' = \frac{2}{3} x^{-1/3}$$

b.) $\frac{d}{dx} \sqrt{x}$

$$= \frac{d}{dx} x^{1/2}$$

$$= \frac{1}{2} x^{-1/2}$$

c.) If $y = \frac{1}{\sqrt[3]{x}}$, find $\frac{dy}{dx}$.

$$y = x^{-1/3}$$

$$\Rightarrow y' = -\frac{1}{3} x^{-4/3}$$

Example 4: Find the equation of the tangent line to $y = x^2$ when $x = 3$

pt: $(3, 9)$

slope: $y' = 2x \Big|_{x=3}$ 6

$\Rightarrow$ line: $y - 9 = 6(x - 3)$

Example 5: Derivatives with constants. Does the power rule still apply?

Find the derivative of $y = \pi x^7$.

$y' = 7\pi x^6$

Derivative Rule: The coefficient rule

If $f(x) = c \cdot u(x)$ where c is a constant and $u(x)$ is a differentiable function of x, then $f'(x) = c \cdot u'(x)$.

Example 6: Find:

a.) $\frac{d}{dx} 7 \sqrt[4]{x}$ $= \frac{d}{dx} 7x^{1/4}$

$= \frac{7}{4} x^{-3/4}$

b.) $(4x^5)'$ $= 20x^4$

c.) If $n = \frac{5}{v^{\frac{2}{3}}}$, find $\frac{dn}{dv}$

$n = 5v^{-2/3}$

$\Rightarrow \frac{dn}{dv} = -\frac{10}{3} v^{-5/3}$

Derivative Rule: sums and differences

If $f(x) = u(x) \pm v(x)$, where u and v are differentiable functions if x, then
$f'(x) = u'(x) \pm v'(x)$.

□ proof.

If $f(x) = u(x) + v(x)$

$$\Rightarrow \quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(u(x+h) + v(x+h)) - (u(x) + v(x))}{h}$$

$$= \lim_{h \to 0} \left[\frac{u(x+h) - u(x)}{h} + \frac{v(x+h) - v(x)}{h} \right]$$

since u & v are diff.

$$= \lim_{h \to 0} \frac{u(x+h) - u(x)}{h} + \lim_{h \to 0} \frac{v(x+h) - v(x)}{h}$$

$$= u'(x) + v'(x).$$

Part 2: Applications

Example 7: Suppose the revenue from the sale of x items is modeled by $R(x) = 300\,x - 0.02\,x^2$. (R in dollars)

a.) Find $\overline{\text{MR}}$ when $x = 40$.

$$\overline{MR} = R'(x)$$

$$= 300 - 0.04x$$

@ $x = 40$, $\overline{MR} = 300 - 1.6$

$$= 298.4$$

b.) Interpret the result from part (a.).

The 40th item sells for $298.40

Example 8: Suppose the cost from the sale of x items is
$C(x) = 40500 + 190\,x + 0.2\,x^2$.

a.) Find the average cost function $\overline{C}(x) = \frac{C(x)}{x}$

$$\overline{C} = \frac{40500 + 190x + 0.2x^2}{x}$$

$$= 40500x^{-1} + 190 + 0.2x$$

b.) Find the instantaneous ROC of the average cost function.

$$\overline{C}'(x) = -40500x^{-2} + 0.2$$

c.) When does the instantaneous ROC of the average cost function (from (b.)) equal zero?

solve $0.2 = 40500x^{-2}$

$$\Rightarrow x^2 = \frac{40500}{0.2}$$

$$\Rightarrow x = \pm\sqrt{40500/0.2}$$

$$= \pm \cancel{318.2}\ 450$$

d.) Find $\overline{MC}(x)$ and $\overline{C}(x)$ at the zero found in (c.).

$\overline{C}(\cancel{318.2}\ 450) = \cancel{444.6}\ 370$ so $\overline{C} = \overline{MC}$ when $\overline{C}' = 0$.

$C'(x) = 190 + 0.4x$

and $C'(\cancel{318.2}\ 450) = 370$