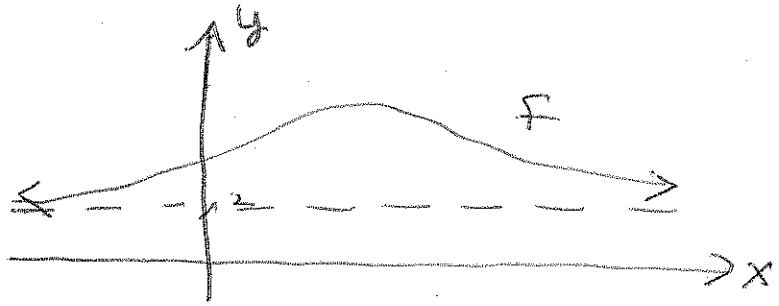
Limits at Infinity

Section 9.2

Part 1: Graphical limits at infinity

■ **Example 1: A Review**

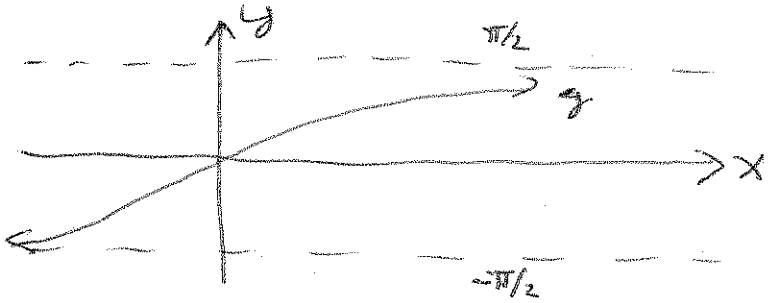
(a.)



i) $\lim_{x \to \infty} f(x) = 2$

ii) $\lim_{x \to -\infty} f(x) = 2$

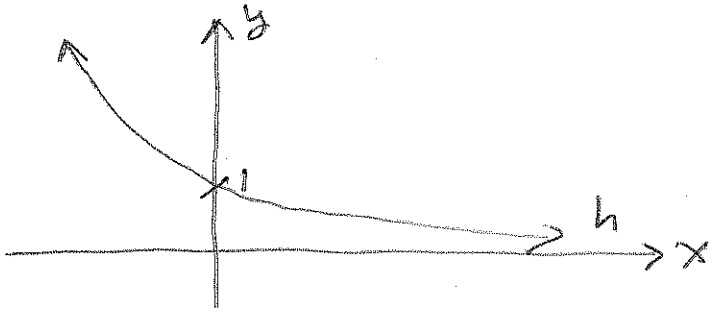
(b.)



i) $\lim_{x \to \infty} g(x) = \pi/2$

ii) $\lim_{x \to -\pi/2} g(x) = -\pi/2$

(c.)



i) $\lim_{x \to \infty} h(x) = 0$

ii) $\lim_{x \to -\infty} h(x)$ DNE OR ∞

Part 2: Limits at infinity algebraically

■ Example 2: Simple Algebraic Examples

(a.) Find $\lim_{x\to\pm\infty} \frac{1}{x}$ $= 0$

(b.) Find $\lim_{x\to\pm\infty} c$ $= c$

(c.) Find $\lim_{x\to\pm\infty} x$ $= \pm\infty$

■ Example 3: Find the following limits analytically

(a.) Find $\lim_{x\to-\infty} \frac{3}{x+2}$ $= \lim_{x\to-\infty} \frac{3}{x} = 0$

(b.) Find $\lim_{x\to\infty} \frac{4x^2+2}{x^2-7}$

$$= \lim_{x\to\infty} \frac{4 + \frac{2}{x^2}}{1 - \frac{7}{x^2}}$$

$$= \frac{4}{1}$$

$$= 4$$

(c.) Find $\lim_{x\to-\infty} \frac{5x^3-4x}{2-3x^3}$

$$= \lim_{x\to-\infty} \frac{5 - \frac{4}{x^2}}{\frac{2}{x^3} - 3}$$

$$= -\frac{5}{3}$$

(d.) Find $\lim_{x\to-\infty} \frac{3x^6-2x}{4x^2+7x}$

$$= \lim_{x\to-\infty} \frac{3x^4 - \frac{2}{x}}{4 + \frac{7}{x}} \qquad 3x^4 - \frac{2}{x} \longrightarrow \infty, \quad 4 + \frac{7}{x} \longrightarrow 4$$

$$= \infty \text{ OR DNE.}$$

■ Example 4: Sales application

The sales volume S (in \$1000's) is related to advertising expenditures d (also in \$1000's) according to $S(d) = \frac{35d}{7+d}$. What would happen to sales if there was an infinite advertising budget?

$$\lim_{d\to\infty} S(d) = \lim_{d\to\infty} \frac{35d}{7+d}$$

$$= \lim_{d\to\infty} \frac{35}{\frac{7}{d} + 1}$$

$$= \frac{35}{1}$$

$$= 35$$

Sales would plateau @ \$35,000.

Part 3: Continuity at a point

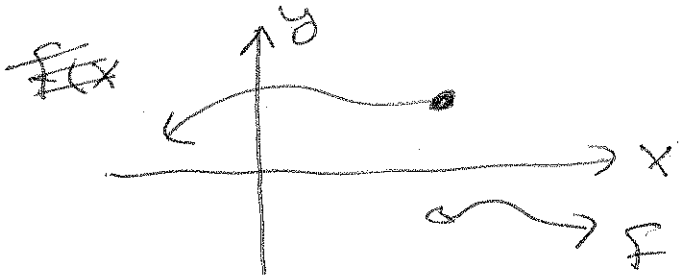
■ Definition: Continuity at a point

The function f is continuous at c if $\lim_{x \to c} f(x) = f(c)$.

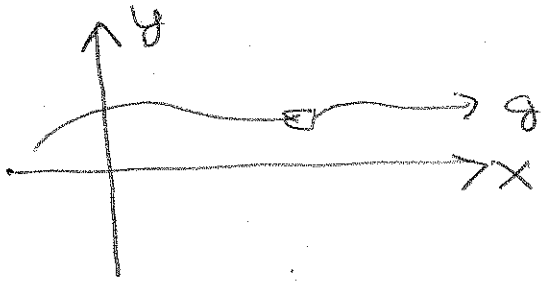
If the LHS DNE and/or the RHS is undefined and/or the LHS ≠ RHS, then we say that the function is discontinous.

■ Examples of discontinuity

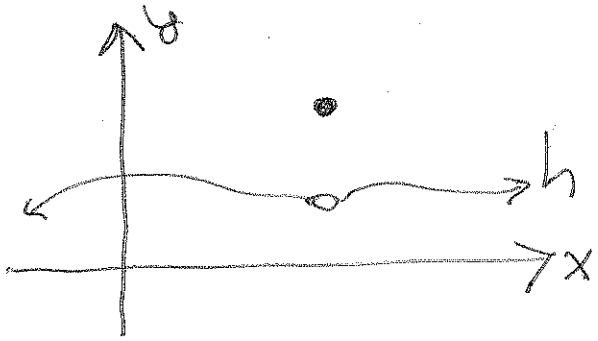
(a.) A function where the limit DNE



(b.) A function that is undefined at a point



(c.) A function where the limit does not equal the function value at a point



■ Example 5: If or when are the following functions discontinuous?

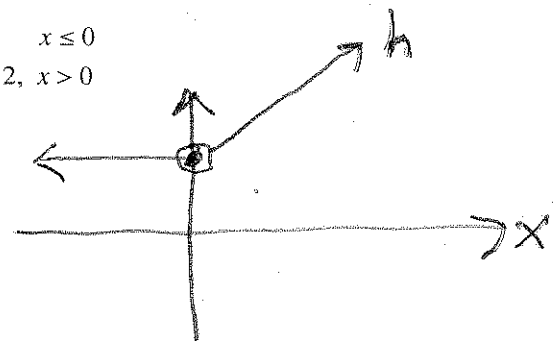
(a.) $f(x) = \frac{x^2-4}{x-2}$

since $f(2)$ is undefined, f is discontinuous @ $x = 2$

(b.) $g(x) = \frac{x^2+5x-6}{x+1}$

g is discont. @ $x = -1$

(c.) $h(x) = \begin{cases} 2, & x \le 0 \\ x+2, & x > 0 \end{cases}$



h is continuous.

(d.) $f(x) = \begin{cases} x^2+1, & x \le 1 \\ 2x^2-1, & x > 1 \end{cases}$

$f(1) = 2$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2x^2 - 1) = 1$

since the for. value does not equal the limit on the RHS... f is discont. @ $x = 1$.

■ Example 4 revisited: Sales application

When is the sales function $S(d) = \frac{35\,d}{7+d}$ discontinuous? What does this mean in context?

It is discontinuous when $d = -7$.
This value of d ($d < 0$) is
meaningless in context.

■ Real world examples of piecewise defined functions

- cell phones and calling cards

- income tax

- postage rates

- utility rates (electricity, natural gas, water, etc)

- rental cars