Test 3
Dusty Wilson
Math 148

Name: key

Television is something the Russians invented to destroy American education.

Paul Erdös (1913 - 1996)
Hungarian mathematician

No work = no credit

Warm-ups (1 pt each): $\sqrt{4} =$ 2 $\qquad \frac{d}{dx}(4) =$ 0 $\qquad \int 4\,dx =$ $4x + c$

1.) (1 pt) According to Erdös, what was the purpose motivating the invention of the television?

To destroy American education.

2.) (4 pts) $I = \int \left(2x^5 - \frac{5}{x} + \sqrt{x} + e^{3x} \right) dx$

3.) (4 pts) $\int_1^5 4x^2\, dx$

$$= \frac{4}{3}\left[x^3\right]_1^5$$

$$= \frac{4}{3}(125 - 1)$$

$$= \frac{4}{3}(124)$$

$\frac{1}{3}x^6 - 5\ln|x| + \frac{2}{3}x^{3/2} + \frac{1}{3}e^{3x} + c$ $\qquad \frac{496}{3}$

4.) (4 pts) If consumption is \$8 billion when income is 0, and if the marginal propensity to consume is $\frac{dC}{dy} = 0.3 + \frac{0.2}{\sqrt{y}}$ (in billions of dollars), find the national consumption function.

$$C(y) = \int 0.3 + 0.2y^{-1/2}\, dy$$

$$= 0.3y + 2(0.2)\, y^{1/2} + C_1$$

$$= 0.3y + 0.4\sqrt{y} + C_1$$

$C(y) = 0.3y + 0.4\sqrt{y} + 8$

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5.) (4 pts) $\int \frac{2x}{\sqrt{x^2-5}}\,dx \quad = \int \frac{du}{\sqrt{u}}$

Let $u = x^2 - 5$

$du = 2x\,dx$

$= \int u^{-1/2}\,du$

$= 2u^{1/2} + c$

$\underline{2\sqrt{x^2-5} + c}$

6.) (4 pts) $\int_0^2 3x^2\left(x^3+1\right)^4 dx \quad = \int_1^9 u^4\,du$

Let $u = x^3 + 1$

$du = 3x^2\,dx$

$u(0) = 1$

$u(2) = 9$

$= \left[\frac{u^5}{5}\right]_1^9$

$= \frac{1}{5}(9^5 - 1)$

$= \frac{59048}{5}$

$\underline{11{,}809.6}$

7.) (4 pts) $\frac{1}{3}\int \frac{5x^2\cdot 3}{x^3-1}dx \quad = \frac{5}{3}\int \frac{du}{u}$

Let $u = x^3 - 1$

$du = 3x^2\,dx$

$= \frac{5}{3}\ln|u| + c$

$\underline{\frac{5}{3}\ln|x^3-1| + c}$

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8.) (8 pts) The marginal cost of producing a product is $50+2x$, where x represents the number of units produced per week. If the marginal revenue from the sale of x units is \$174 and if the fixed costs of production are \$4,300, answer the following.

i.) (1 pt) How many units should the firm produce and sell each week to maximize its profit?

$174 = 50 + 2x$

$124 = 2x$

$x = 62$

62 units

ii.) (1 pt) Find the profit function.

$p(x) = \int 174 - (50 + 2x)\, dx$

$= \int (124 - 2x)\, dx$

$= 124x - x^2 + k$

$p(x) = 124x - x^2 - 4300$

iii.) (2 pts) Find and interpret the value of the profit function when evaluated at the optimal level of production (found in (a.)).

$P(62) = -456$. The best the firm can do is lose \$456 when 62 units are produced / sold.

9.) (4 pts) What does the definite integral represent?

Area under the curve

10.) (4 pts) Verify the formula: $\int \ln(x)\, dx = x\ln(x) - x + C$.

$\frac{d}{dx}(x\ln x - x + c) = \ln x + \frac{x}{x} - 1$

$= \ln x$ ✓

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11.) (4 pts) Estimate the area under $f(x) = xe^x$ on $[0,1]$ with Simpson's rule using four subdivisions of equal width. Show enough work to convince me you knew what you were doing. Give your answer to 5 decimal places.

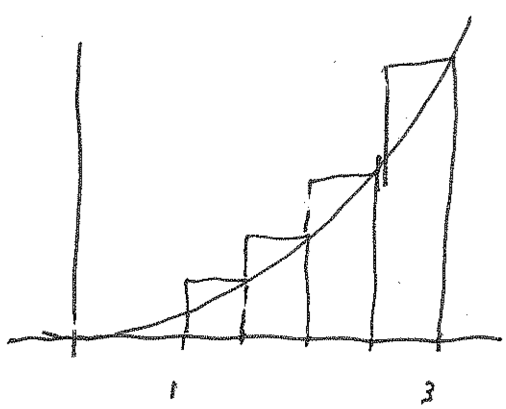
x	F(x)
0	0
.25	.32101
.5	.82436
.75	1.5878
1	2.7183

$S_4 = \frac{1}{3}(.25)\left(1(0) + 4(.32101) + 2(.82436) + 4(1.5878) + 1(2.7183)\right)$

$= 1.00019$

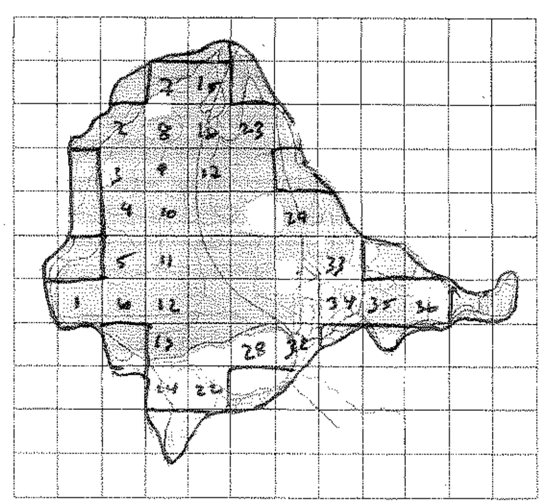
$T_4 = 1.02308$

12.) (4 pts) Neatly sketch a graph showing how you would use 4 subintervals of equal with and right endpoints to approximate the area under $y = x^2$ on the interval $[1,3]$. No calculations are required.



13.) (4 pts) The given map shows Stanley Park. If each square represents 100 square yards, approximate the area of the park to within 300 square yards.

4800 yds^2



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