Test 1
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Math 148

Name: KEY

It is rare to find learned men who are clean, do not stink and have a sense of humour.

Duchess of Orléans

No work = no credit

Warm-ups (1 pt each): $(-3)^2 =$ 9 $\qquad -3^2 =$ -9 $\qquad \frac{2}{0} =$ und.

1.) (1 pt) The quote above is attributed to the Duchess of Orléans regarding Gottfried Leibniz (the co-discover/inventor of calculus). What special qualities did Leibniz have? Use complete English sentences.

Leibniz took baths. !! ☺

2.) (16 pts) Find the derivatives of the following: (Simplification is optional).

a.) (4 pts) $y = 3 - 5x^8 - \frac{1}{x^2} + \sqrt[4]{x} + \frac{1}{x}$

$-x^{-2} + x^{1/4} - x^{-1}$

$y' = -40x^7 + x^{-3} + \frac{1}{4}x^{-3/4} + x^{-2}$

b.) (4 pts) $y(x) = \underbrace{(3x^5 + x^7 - 2x)}_{u} \cdot \underbrace{(5 - 4x^2)}_{v}$

$y'(x) = (15x^4 + 7x^6 - 2)(5 - 4x^2) + (-8x)(3x^5 + x^7 - 2x)$

c.) (4 pts) $z = \frac{3x^2 + x}{2x^4 + x^3}$ $\leftarrow u$, $\leftarrow v$

$$\frac{dz}{dx} = \frac{(6x+1)(2x^4 + x^3) - (8x^3 + 3x^2)(3x^2 + x)}{(2x^4 + x^3)^2}$$

d.) (4 pts) $f(x) = 8\sqrt{4x^5 - 3x}$ $= 8(4x^5 - 3x)^{1/2}$

$f'(x) = 8 \cdot \frac{1}{2}(4x^5 - 3x)^{-1/2} \cdot (20x^4 - 3)$

3.) (4 pts) Find $f''(x)$ if $f(x)=\sqrt[3]{x}-7x^4$.

$$= x^{1/3} - 7x^4$$

$$f'(x) = \frac{1}{3}x^{-2/3} - 28x^3$$

$$f''(x) = -\frac{2}{9}x^{-5/3} - 74x^2$$

4.) (4 pts) If $f(n)=\underbrace{(n-n^3)}_{u}\cdot\underbrace{(4+5n^2)^7}_{v}$, find $\dfrac{df}{dn}$

$$f'(n) = (1-3n^2)(4+5n^2)^7 + 7(4+5n^2)^6\cdot(10n)(n-n^3)$$

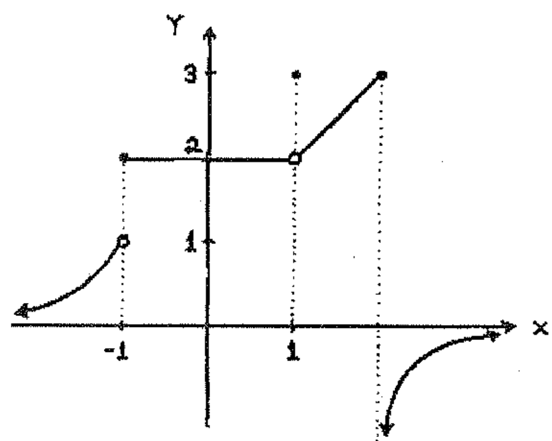
5.) (5 pts) Given the function $f(x)$ shown in the graph to the right, evaluate the following:

a.) $f(-1)=$ 1

b.) $\lim_{x\to -1^+} f(x) =$ 2

c.) $\lim_{x\to -1^-} f(x) =$ 1

d.) $\lim_{x\to -1} f(x) =$ DNE



e.) Is $f(x)$ continuous at $x=1$?
Explain why or why not using the definition of continuity?

No. $f(1) = 3 \neq 2 = \lim_{x\to 1} f(x)$.

6.) (3 pt) Consider the table of values for $y = f(x)$. Suppose that $f(2)$ is undefined.

x	$f(x)$
1.9	6.849
1.99	7.8805
1.999	7.988
⋮	⋮
2	undefined
⋮	⋮
2.001	8.012
2.01	8.1205
2.1	9.251

a.) Estimate $\lim_{x \to 2} f(x)$

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b.) Based upon the given information, is $f(x)$ continuous at $x = 2$? Explain your answer.

No, f(2) is undefined.

c.) Use the table to calculate the average rate of change of $f(x)$ on the interval $[1.99, 2.10]$

$$\text{Ave ROC} = \frac{f(2.10) - f(1.99)}{2.10 - 1.99} = \frac{8.1205 - 7.8805}{.11} = 2.18$$

7.) (4 pts) Use the definition of the derivative to find $f'(x)$ if $f(x) = 3x - 5x^2$

$$f'(x) = \lim_{h \to 0} \frac{[3(x+h) - 5(x+h)^2] - [3x - 5x^2]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x} + 3h - 5(x^2 + 2xh + h^2) - \cancel{3x} + 5x^2}{h}$$

$$= \lim_{h \to 0} \frac{3h - \cancel{5x^2} - 10xh - 5h^2 + \cancel{5x^2}}{h}$$

$$= \lim_{h \to 0} (3 - 10x - 5h)$$

$$= 3 - 10x$$

$f'(x) = 3 - 10x$

8.) (4 pts) Consider the function: $g(x) = \dfrac{3x-6}{2x-4} = \dfrac{3(x-2)}{2(x-2)}$

a.) $g(2) =$ und.

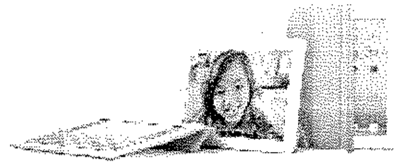
b.) $\lim_{x \to 2} g(x) =$ 3/2

c.) $\lim_{x \to -\infty} g(x) =$ 3/2

d.) $\lim_{x \to +\infty} g(x) =$ 3/2

9.) (7 pts) The cost C in dollar from the production of x of the all new iPad's is:

$$C(x) = 400x - 0.01x^2 + 19,000,000$$



a.) (1 pt) Find and interpret $C(0)$.

The fixed costs are $19,000,000

b.) (2 pts) Calculate $\overline{MC}(x)$. $= 400 - 0.02x$

$\overline{MC}(x) = 400 - 0.02x$

c.) (2 pt) Interpret $\overline{MC}(10,000)$ using complete English sentences.

The cost of producing the 10000th item is about $200.

d.) (2 pts) Find and interpret $C(10,000) - C(9,999)$ using complete English sentences.

The exact cost of producing the 10,000th item is $200.01

10.) (4 pts) Consider the function $g(x) = x^2$.

a.) Find $g(3.1)$

$g(3.1) = 9.61$

b.) Find the equation of the tangent line when $x = 3$

$g'(x) = 2x \Big|_{x=3}$ 6 (slope)

$g(3) = 9$ (pt)

$y - 9 = 6(x - 3)$

c.) Find the y value on the tangent line from (b.) corresponding to $x = 3.1$.

$y - 9 = (6)[(3.1) - 3)] \Rightarrow y = 9.6$

d.) Compare the results of (a.) and (c.). and explain the similarity of the results.

The results are similar because the tangent line approxs $g(x)$ near $x=3$.