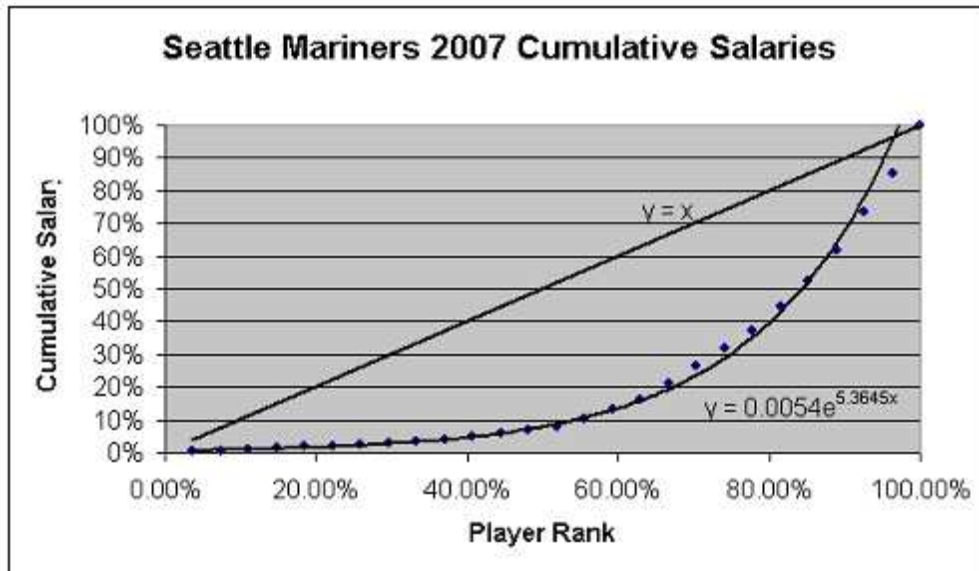


## Area between curves

### Part 1: The Lorenz Curve

In economics, the Lorenz curve is used to represent the inequality of income distribution among different groups in the population of a country. The curve is constructed by plotting the cumulative percent of families at or below a given income level and the cumulative percent of total personal income received by these families.

The curve below shows the Lorenz curve  $L(x)$  for the 27 players on the Seattle Mariner roster (as listed in USA Today).



**Part 2: Area between curves**

**Example 1:** Find the area between  $y = x^2 + 2$  and  $y = -x^2$  on  $[0, 2]$

**Example 2:** Find the area bounded between  $y = x^2$  and  $y = \sqrt{x}$

### Part 3: The Gini coefficient

Definition: The Gini Coefficient

We measure income distribution through the Gini coefficient which is defined as:

$$\frac{\text{area between } y=x \text{ and } L(x)}{\text{area below } y=x} = \frac{\int_0^1 [x - L(x)] dx}{1/2}$$
$$= 2 \int_0^1 [x - L(x)] dx$$

**Example 3:** The Lorenz curve for income distribution in the US in 1950 and 1970 are given. Find and compare the Gini coefficients.

$$1950 : y = 0.925 x^{1.891}$$

$$1970 : y = 0.920 x^{1.783}$$

Question: Which is better (an ethical question): a small or large Gini coefficient?

**Part 4: Average value of a function**

Definition: Average value of a function

The average value of a continuous function  $y = f(x)$  on  $[a, b]$  is:

$$\text{average value} = f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) dx$$

**Example 4:** The cost to produce  $x$  units of a product is  $C(x) = x^2 + 400x + 2000$ .

a.) Find the average cost of producing 1000 units (the cost per unit)

b.) Find the average value of the cost function on  $[0, 1000]$  (the cost per shipment).