Section 11.01

Derivatives of Logarithms

Part 1: Derivatives of Logs

If $f(x) = \ln(x)$, then $f'(x) = \frac{1}{x}$.

Example 1: Find the derivatives of the following:

a.) $y = 4x^7 - 2\ln(x)$

b.) $f(x) = 3x^4 \ln(x)$

c.) $g(x) = \frac{x^2}{\ln(x)}$

If $f(x) = \ln(u(x))$, then $f'(x) = \frac{u'(x)}{u(x)}$ (the chain rule)

Example 2: Find the derivatives of the following:

a.) $y = \ln(x^4)$

b.) $f(x) = \ln(x^3 - x + 7)$

c.) $z = \ln\left(\frac{2x^4}{(5x+7)^5}\right)$

Part 1: Derivatives with log rules

Logarithmic Rules: Let M, $N > 0$ and $p \in \mathbb{R}$ and $b > 0$ and $b \neq 1$.

1.) $\ln(e^x) = x$ (inverse function property)

2.) $e^{\ln(x)} = x, \ x > 0$ (inverse function property)

3.) $\ln(M \cdot N) = \ln(M) + \ln(N)$

4.) $\ln\left(\frac{M}{N}\right) = \ln(M) - \ln(N)$

5.) $\ln(M^p) = p \cdot \ln(M)$

6.) $\log_b(x) = \frac{\ln(x)}{\ln(b)}$ (change of base formula)

Example 2c revisited: Find the derivative of $z = \ln\left(\frac{2x^4}{(5x+7)^5}\right)$ using log rules.

Example 3: Find the derivatives of the following:

a.) $s = \ln(t^3(t^2 - 1))$

b.) $y = \ln\left(\sqrt[4]{\frac{3x+2}{x^2-5}}\right)$

c.) $f(x) = \ln\left(x^2(x^4 - x + 1)^{17}\right)$

Example 4: If the cost function for a product is $C(x) = 1500 + 200 \ln(2x + 1)$ where x is the number of units produced, then

a.) Find $\overline{MC}$

b.) Find and interpret $\overline{MC}(100)$

c.) Does $C(x)$ always increase (does this result make sense)?

Example 5: Between 1976 and 1998, the percent of moms who returned to work within one year of having a baby can be represented by $w(y) = 1.11 + 16.94 \ln(y)$ where y is in years since 1970. What is the expected rate of change of w ithis year (and what does this mean)?

Example 6: Find the following derivatives:

a.) $y = \log_4(x)$

b.) $y = \log_6(x^4 - 4x^3 + 1)$