Test 3
Dusty Wilson
Math 148

Name: Key

Television is something the Russians invented to destroy American education.

Paul Erdös (1913 - 1996)
Hungarian mathematician

No work = no credit

Warm-ups (1 pt each): $\sqrt{9} = \underline{3}$ $\qquad \frac{d}{dx}(3) = \underline{0}$ $\qquad \int 1\, dx = \underline{x + c}$

1.) (1 pt) According to Erdös, what was the purpose motivating the invention of the television?

2.) (4 pts) $\int (3x^2 - 4x + 2)\, dx$

$\underline{x^3 - 2x^2 + 2x + c}$

3.) (4 pts) $\int x^7 \cdot e^{x^8}\, dx = \frac{1}{8} \int 8x^7 e^{x^8} dx$

Let $u = x^8$

$du = 8x^7 dx$

$= \frac{1}{8} \int e^u du$

$= \frac{1}{8} e^u + c$

$\underline{\frac{1}{8} e^{x^8} + c}$

4.) (4 pts) Suppose that the marginal propensity to save is $0.2 - \frac{1}{\sqrt{2y+5}}$ (in billions of dollars) and that consumption is \$6 billion when disposable income is 0. Find the national consumption function.

$\frac{dc}{dy} = 1 - \frac{ds}{dy} = \left(1 - \left(0.2 - \frac{1}{\sqrt{2y+5}}\right)\right) = 0.8 + \frac{1}{\sqrt{2y+5}}$

$c(y) = 0.8y + \frac{1}{2} \int \frac{2\,dy}{\sqrt{2y+5}}$ Let $u = 2y+5$, $du = 2dy$.

$= 0.8y + \frac{1}{2} \int u^{-1/2} du$

$= 0.8y + \frac{1}{2} \cdot 2u^{\frac{1}{2}} + k$

$= 0.8y + \sqrt{2y+5} + k$

$c(0) = 6 = \sqrt{5} + k \Rightarrow k = 6 - \sqrt{5}$

$\underline{c(y) = 0.8y + \sqrt{2y+5} + \underbrace{(6 - \sqrt{5})}_{3.76}}$

5.) (4 pts) $\int \frac{x}{\sqrt{x^2-5}}\,dx$ $= \frac{1}{2}\int u^{-\frac{1}{2}}\,du$

Let $u = x^2-5$ $= \frac{1}{2}\cdot 2u^{\frac{1}{2}} + c$

$du = 2x\,dx$

$= \sqrt{x^2-5} + c$

6.) (4 pts) $\int 4x\left(x^2-1\right)^7 dx$ $= \frac{4}{2}\int u^7\,du$

Let $u = x^2-1$

$= \frac{4}{16}u^8 + c$

$du = 2x\,dx$

$= \frac{1}{4}(x^2-1)^8 + c$

7.) (4 pts) $\int \frac{3x^2}{x^3-1}\,dx$ $= \int \frac{1}{u}\,du$

Let $u = x^3-1$ $= \ln|x^3-1| + c.$

$du = 3x^2\,dx$

8.) (8 pts) A firm knows that the marginal costs of producing x units is $\overline{MC} = 3x + 20$, there is marginal revenue of $\overline{MR} = 44 - 5x$, and that the cost to produce 2 units is \$146. Find the optimal profit (or loss) for the firm and give your recommendation for how they should proceed.

$\overline{MP} = (44 - 5x) - (3x + 20)$

$= 24 - 8x$

$\overline{MP} = 0$ when $x = 0.$

$\to P = \int (24 - 8x)\,dx$

$= 24x - 4x^2 + k$

$= 24x - 4x^2 - 100$

$c = \int (3x + 20)\,dx$

$= \frac{3x^2}{2} + 20x + k$

$c(2) = 6 + 40 + k = 146$

$k = 100$

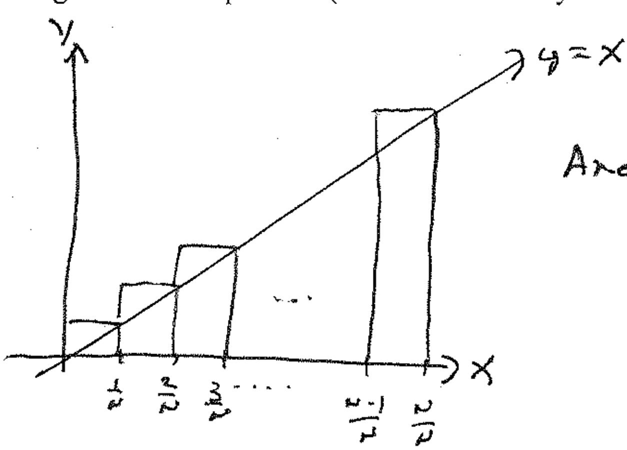
AND $P(3) = 72 - 36 - 100$

$= -64$

Shut down!

The best you can do is lose \$64 when 3 units are produced/sold.

9.) (4 pts) Approximate the area under the line $y = x$ on $[0,1]$ using n rectangles of equal width with right hand end points. (It is not necessary to use Σ notation).



$$\text{Area} \approx \frac{1}{n}\cdot\frac{1}{n} + \frac{1}{n}\cdot\frac{2}{n} + \cdots + \frac{1}{n}\cdot\frac{n}{n}$$

$$= \frac{1}{n^2}(1 + 2 + 3 + \cdots + n)$$

$$= \frac{1}{n^2}\cdot\frac{n(n+1)}{2}$$

$$\text{Area} = \lim_{n\to\infty} \frac{n(n+1)}{2n^2}$$

$$= \frac{1}{2}$$

10.) (4 pts) What does the indefinite integral represent (what is another name for it)?

A fct.

antiderivative.

11.) (4 pts) Verify the formula: $\int 4x \cdot \ln(x)\,dx = 2x^2 \ln(x) - x^2 + C$.

$$\frac{d}{dx}\left(2x^2 \ln x - x^2 + c\right)$$

$$= 4x \ln x + \frac{2x^2}{x} - 2x$$

$$= 4x \ln x \checkmark$$

12.) (4 pts) Find the value of the sum $\sum_{i=1}^{1000}(5i-2)$ using summation formulas.

$$3 + 8 + 13 + \ldots + 4998$$

$$= \frac{1000(3+4998)}{2}$$

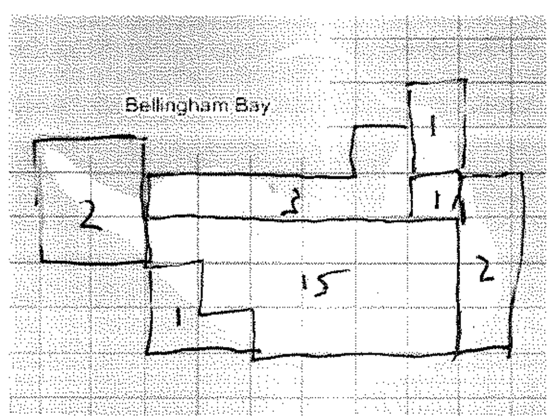
$$= 2{,}500{,}500$$

$$= 5\sum_{i=1}^{1000} i - \sum_{i=1}^{1000} 2$$

$$= 5 \cdot \frac{1000(1001)}{2} - 2(1000)$$

$$= 2{,}500{,}500$$

13.) (4 pts) The given map shows a small island in Bellingham Bay (Bellingham is a few miles to the east of the island). If each square represents 100 square yards, approximate the area of the island to within 300 square yards.



2400 yds^2

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Warm-ups (1 pt each): $\sqrt{9}=$_____ $\frac{d}{dx}(3)=$_____ $\int 1\,dx=$_____

1.) (1 pt) According to Erdös, what was the purpose motivating the invention of the television?

2.) (4 pts) $\int\left(3x^2-4x+2\right)dx$

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4.) (4 pts) $\int_1^4\left(x^2-2x+3\right)dx$

$$= \left[\frac{x^3}{3} - x^2 + 3x\right]_1^4$$

$$= \left(\frac{64}{3} - 16 + 12\right) - \left(\frac{1}{3} - 1 + 3\right)$$

$$= 21 - 4 + -2$$

$$= 15$$

____

16

5

5.) (4 pts) $\int \frac{x}{\sqrt{x^2-5}}\,dx$

6.) (4 pts) $\int_0^1 2x\left(x^2-1\right)^7 dx$

$$= \left.\frac{(x^2-1)^8}{8}\right]_0^1$$

Let $u = x^2-1$

$du = 2x\,dx$

$\int u^7\,du$

$$\left.\frac{1}{8}(x^2-1)^8\right]_0^1$$

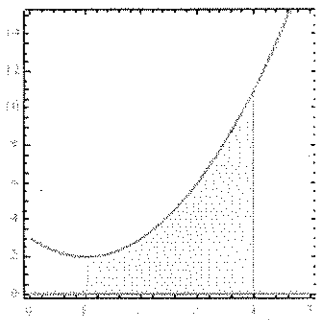
$$\frac{1}{8}\left(0^8-(-1)^8\right)$$

$-\frac{1}{8}$

7.) (4 pts) $\int \frac{3x^2}{x^3-1}\,dx$

$\overline{12}$

10.) (4 pts) Express the area under $f(x)=(x-1)^2+2$ on the interval $[1,4]$ as a definite integral (picture below).



$\int_1^4 [(x-1)^2+2]\,dx \quad = 15$

11.) (4 pts) Verify the formula: $\int 4x \cdot \ln(x)\,dx = 2x^2\ln(x) - x^2 + C$.

12.) (4 pts) Find the value of the sum $\sum_{i=1}^{1000}(5i-2)$ using summation formulas.

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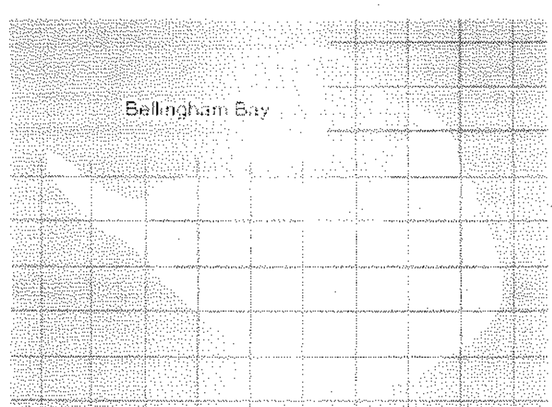

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