

1.5

Division of Whole Numbers

OBJECTIVE A

To divide by a single digit with no remainder in the quotient

Division is used to separate objects into equal groups.

A store manager wants to display 24 new objects equally on 4 shelves. From the diagram we see that the manager would place 6 objects on each shelf.

The manager's division problem can be written as follows:

**Take Note**

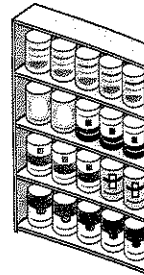
The **divisor** is the number that is divided into another number. The **dividend** is the number into which the divisor is divided. The result is the **quotient**.

Number of shelves
Divisor

$$\begin{array}{r} 6 \\ 4 \overline{)24} \end{array}$$

Number on each shelf
Quotient

Number of objects
Dividend



Note that the quotient multiplied by the divisor equals the dividend.

$$4 \overline{)24} \text{ because } \boxed{6} \text{ Quotient} \times \boxed{4} \text{ Divisor} = \boxed{24} \text{ Dividend}$$

$$9 \overline{)54} \text{ because } 6 \times 9 = 54$$

$$8 \overline{)40} \text{ because } 5 \times 8 = 40$$

Here are some important quotients and the properties of zero in division:

Properties of One in Division

Any whole number, except zero, divided by itself is 1.

$$8 \overline{)8} \quad 14 \overline{)14} \quad 10 \overline{)10}$$

Any whole number divided by 1 is the whole number.

$$1 \overline{)9} \quad 1 \overline{)27} \quad 1 \overline{)10}$$

Properties of Zero in Division

Zero divided by any other whole number is zero.

$$7 \overline{)0} \quad 13 \overline{)0} \quad 10 \overline{)0}$$

Division by zero is not allowed.

$$\begin{array}{r} ? \\ 0 \overline{)8} \end{array}$$

There is no number whose product with 0 is 8.

**Integrating Technology**

Enter 8 \div 0 $=$ on your calculator. An error message is displayed because division by zero is not allowed.

When the dividend is a larger whole number, the digits in the quotient are found in steps.

HOW TO 1 Divide $4\overline{)3192}$ and check.

$$\begin{array}{r} 7 \\ 4 \overline{) 3192} \\ \underline{-28} \\ 39 \end{array}$$

- Think $4\overline{)31}$.
- Subtract 7×4 .
- Bring down the 9.

$$\begin{array}{r} 79 \\ 4 \overline{) 3192} \\ \underline{-28} \\ 39 \\ \underline{-36} \\ 32 \end{array}$$

- Think $4\overline{)39}$.
- Subtract 9×4 .
- Bring down the 2.

$$\begin{array}{r} 798 \\ 4 \overline{) 3192} \\ \underline{-28} \\ 39 \\ \underline{-36} \\ 32 \\ \underline{-32} \\ 0 \end{array}$$

- Think _____.
- Subtract 8×4 .

Check:

$$\begin{array}{r} 798 \\ \times 4 \\ \hline 3192 \end{array}$$

The place-value chart can be used to show why this method works.

	HUNDREDS	TENS	ONES	
	7	9	8	
4)	3	1	9	2
-	2	8	0	0
		3	9	2
-	3	6	0	
		3	2	
-	3	2		
			0	

7 hundreds \times 4

9 tens \times 4

8 ones \times 4

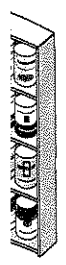
There are other ways of expressing division.

54 divided by 9 equals 6.

$$54 \div 9 \text{ equals } 6.$$

$$\frac{54}{9} \text{ equals } 6.$$

gram,



$\frac{1}{10}$

$\frac{0}{0}$

$\frac{0}{10}$

mber
ith 0

EXAMPLE 1Divide $7\overline{)56}$ and check.**Solution**

$$\begin{array}{r} 8 \\ 7\overline{)56} \end{array}$$

$$\text{Check: } 8 \times 7 = 56$$

YOU TRY IT 1Divide $9\overline{)63}$ and check.**Your solution****EXAMPLE 2**Divide $2808 \div 8$ and check.**Solution**

$$\begin{array}{r} 351 \\ 8\overline{)2808} \\ \underline{-24} \\ 40 \\ \underline{-40} \\ 08 \\ \underline{-8} \\ 0 \end{array}$$

$$\text{Check: } 351 \times 8 = 2808$$

YOU TRY IT 2Divide $4077 \div 9$ and check.**Your solution****EXAMPLE 3**Divide $7\overline{)2856}$ and check.**Solution**


$$\begin{array}{r} 408 \\ 7\overline{)2856} \\ \underline{-28} \\ 05 \\ \underline{-0} \\ 56 \\ \underline{-56} \\ 0 \end{array}$$

- Think $7\overline{)5}$. Place 0 in quotient.
- Subtract 0×7 .
- Bring down the 6.

$$\text{Check: } 408 \times 7 = 2856$$

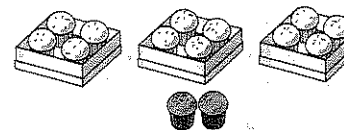
YOU TRY IT 3Divide $9\overline{)6345}$ and check.**Your solution**

Solutions on pp. S2-

OBJECTIVE BTo divide by a single digit with a remainder in the quotient 

Sometimes it is not possible to separate objects into a whole number of equal groups.

A baker has 14 muffins to pack into 3 boxes. Each box holds 4 muffins. From the diagram, we see that after the baker places 4 muffins in each box, there are 2 left over. The 2 is called the **remainder**.



The baker's division problem could be written

$$\begin{array}{r} \text{Divisor} \quad \longrightarrow 3 \overline{)14} \\ \text{(Number of boxes)} \quad \quad \quad -12 \\ \hline \quad \quad \quad \quad \quad \quad 2 \end{array}$$

Quotient
 (Number in each box)
Dividend
 (Total number of objects)
Remainder
 (Number left over)

The answer to a division problem with a remainder is frequently written

$$\begin{array}{r} \quad \quad \quad 4 \text{ r}2 \\ 3 \overline{)14} \end{array}$$

Note that

4	3	+	2	=	14
Quotient	× Divisor		Remainder		Dividend

EXAMPLE 4

Divide $4 \overline{)2522}$ and check.

Solution

$$\begin{array}{r} \quad \quad \quad 630 \text{ r}2 \\ 4 \overline{)2522} \\ \underline{-24} \\ \quad 12 \\ \underline{-12} \\ \quad \quad 02 \\ \underline{\quad \quad -0} \\ \quad \quad \quad 2 \end{array}$$

- Think $4 \overline{)2}$. Place 0 in quotient.
- Subtract 0×4 .

Check: $(630 \times 4) + 2 =$
 $2520 \quad + 2 = 2522$

YOU TRY IT 4

Divide $6 \overline{)5225}$ and check.

Your solution

EXAMPLE 5

Divide $9 \overline{)27,438}$ and check.

Solution

$$\begin{array}{r} \quad \quad \quad 3,048 \text{ r}6 \\ 9 \overline{)27,438} \\ \underline{-27} \\ \quad 43 \\ \underline{-36} \\ \quad \quad 78 \\ \underline{-72} \\ \quad \quad \quad 6 \end{array}$$

- Think $9 \overline{)4}$.
- Subtract 0×9 .

Check: $(3048 \times 9) + 6 =$
 $27,432 \quad + 6 = 27,438$

YOU TRY IT 5

Divide $7 \overline{)21,409}$ and check.

Your solution

OBJECTIVE C**To divide by larger whole numbers**

When the divisor has more than one digit, estimate at each step by using the first digit the divisor. If that product is too large, lower the guess by 1 and try again.

HOW TO 2 Divide $34 \overline{)1598}$ and check.

$$\begin{array}{r} 5 \\ 34 \overline{) 1598} \\ \underline{-170} \end{array}$$

- Think $5 \overline{)34}$.
- Subtract 5×34 .

$$\begin{array}{r} 4 \\ 34 \overline{) 1598} \\ \underline{-136} \\ 238 \end{array}$$

- Subtract 4×34 .

170 is too large. Lower the guess by 1 and try again.

$$\begin{array}{r} 47 \\ 34 \overline{) 1598} \\ \underline{-136} \\ 238 \\ \underline{-238} \\ 0 \end{array}$$

- Think $3 \overline{)23}$.
- Subtract 7×34 .

$$\begin{array}{r} \text{Check:} \\ 47 \\ \times 34 \\ \hline 188 \\ 141 \\ \hline 1598 \end{array}$$

**Tips for Success**

One of the key instructional features of this text is the Example/You Try It pairs. Each Example is completely worked. You are to solve the You Try It problems. When you are ready, check your solution against the one in the Solutions section. The solution for You Try It 6 below is on page S3 (see the reference at the bottom right of the You Try It). See *AIM for Success* at the front of the book.

The phrases below are used to indicate the operation of division. An example is shown to the right of each phrase.

the quotient of	the quotient of 9 and 3	$9 \div 3$
divided by	6 divided by 2	$6 \div 2$

EXAMPLE 6

Find 7077 divided by 34 and check.

Solution

$$\begin{array}{r} 208 \text{ r}5 \\ 34 \overline{) 7077} \\ \underline{-68} \\ 27 \\ \underline{-0} \\ 277 \\ \underline{-272} \\ 5 \end{array}$$

- Think $34 \overline{)27}$.
- Place 0 in quotient.
- Subtract 0×34 .

$$\begin{array}{l} \text{Check: } (208 \times 34) + 5 = \\ 7072 + 5 = 7077 \end{array}$$

YOU TRY IT 6

Divide $4578 \div 42$ and check.

Your solution

EXAMPLE 7

Find the quotient of 21,312 and 56 and check.

Solution

$$\begin{array}{r}
 380 \text{ r}32 \\
 56 \overline{)21,312} \\
 \underline{-168} \\
 451 \\
 \underline{-448} \\
 32 \\
 \underline{-0} \\
 32
 \end{array}$$

• Think $5 \overline{)21}$.
 4×56 is too large.
 Try 3.

$$\begin{array}{r}
 \text{Check: } (380 \times 56) + 32 = \\
 21,280 + 32 = 21,312
 \end{array}$$

YOU TRY IT 7Divide $18,359 \div 39$ and check.**Your solution****EXAMPLE 8**Divide $427 \overline{)24,782}$ and check.**Solution**

$$\begin{array}{r}
 58 \text{ r}16 \\
 427 \overline{)24,782} \\
 \underline{-2135} \\
 3432 \\
 \underline{-3416} \\
 16
 \end{array}$$

$$\begin{array}{r}
 \text{Check: } (58 \times 427) + 16 = \\
 24,766 + 16 = 24,782
 \end{array}$$

YOU TRY IT 8Divide $534 \overline{)33,219}$ and check.**Your solution****EXAMPLE 9**Divide $386 \overline{)206,149}$ and check.**Solution**

$$\begin{array}{r}
 534 \text{ r}25 \\
 386 \overline{)206,149} \\
 \underline{-1930} \\
 1314 \\
 \underline{-1158} \\
 1569 \\
 \underline{-1544} \\
 25
 \end{array}$$

$$\begin{array}{r}
 \text{Check: } (534 \times 386) + 25 = \\
 206,124 + 25 = 206,149
 \end{array}$$

YOU TRY IT 9Divide $515 \overline{)216,848}$ and check.**Your solution**

ESTIMATION**Estimating the Quotient of Two Whole Numbers**

Calculate $36,936 \div 54$. Then use estimation to determine whether the quotient is reasonable.

Divide to find the exact quotient.

$$36,936 \div 54 = 684$$

To estimate the quotient, round each number so that it contains one nonzero digit. Then divide. The estimated answer is 800, which is close to the exact quotient 684.

$$36,936 \div 54 \approx 40,000 \div 50 = 800$$

OBJECTIVE D

To solve application problems

The **average** of several numbers is the sum of all the numbers divided by the number of those numbers.

$$\text{Average test score} = \frac{81 + 87 + 80 + 85 + 79 + 86}{6} = \frac{498}{6} = 83$$

HOW TO 3

The table at the right shows what an upper-income family can expect to spend to raise a child to the age of 17 years. Find the average amount spent each year. Round to the nearest dollar.

Expenses to Raise a Child	
Housing	\$89,580
Food	\$35,670
Transportation	\$32,760
Child care/education	\$26,520
Clothing	\$13,770
Health care	\$13,380
Other	\$30,090

Source: Department of Agriculture, *Expenditures on Children by Families*

Strategy

To find the average amount spent each year:

- Add all the numbers in the table to find the total amount spent during the 17 years.
- Divide the sum by 17.

Solution

$$\begin{array}{r} 89,580 \\ 35,670 \\ 32,760 \\ 26,520 \\ 13,770 \\ 13,380 \\ + 30,090 \\ \hline 241,770 \end{array} \quad \begin{array}{l} \text{Sum of all} \\ \text{the costs} \end{array}$$

$$\begin{array}{r} 14,221 \\ 17 \overline{) 241,770} \\ \underline{-17} \\ 71 \\ \underline{-68} \\ 37 \\ \underline{-34} \\ 37 \\ \underline{-34} \\ 30 \\ \underline{-17} \\ 13 \end{array}$$

- When rounding to the nearest whole number, compare twice the remainder to the divisor. If twice the remainder is less than the divisor, drop the remainder. If twice the remainder is greater than or equal to the divisor, add 1 to the units digit of the quotient.

- Twice the remainder is $2 \times 13 = 26$. Because $26 > 17$, add 1 to the units digit of the quotient.

The average amount spent each year to raise a child to the age of 17 is \$14,222.



Michelle D. Bridwell/PhotoEdit, Inc.

EXAMPLE 10

Ngan Hui, a freight supervisor, shipped 192,600 bushels of wheat in 9 railroad cars. Find the amount of wheat shipped in each car.

Strategy

To find the amount of wheat shipped in each car, divide the number of bushels (192,600) by the number of cars (9).

Solution

$$\begin{array}{r} 21,400 \\ 9 \overline{) 192,600} \\ \underline{-18} \\ 12 \\ \underline{-9} \\ 36 \\ \underline{-36} \\ 0 \end{array}$$

Each car carried 21,400 bushels of wheat.

EXAMPLE 11

The used car you are buying costs \$11,216. A down payment of \$2000 is required. The remaining balance is paid in 48 equal monthly payments. What is the monthly payment?

Strategy

To find the monthly payment:

- Find the remaining balance by subtracting the down payment (2000) from the total cost of the car (11,216).
- Divide the remaining balance by the number of equal monthly payments (48).

Solution

$$\begin{array}{r} 11,216 \\ - 2,000 \\ \hline 9,216 \\ \text{Remaining balance} \end{array} \qquad \begin{array}{r} 192 \\ 48 \overline{) 9216} \\ \underline{-48} \\ 441 \\ \underline{-432} \\ 96 \\ \underline{-96} \\ 0 \end{array}$$

The monthly payment is \$192.

YOU TRY IT 10

Suppose a Michelin retail outlet can store 270 tires on 15 shelves. How many tires can be stored on each shelf?

Your strategy**Your solution****YOU TRY IT 11**

A soft-drink manufacturer produces 12,600 cans of soft drink each hour. Cans are packed 24 to a case. How many cases of soft drink are produced in 8 hours?

Your strategy**Your solution**

1.5 EXERCISES**OBJECTIVE A** To divide by a single digit with no remainder in the quotient

For Exercises 1 to 20, divide.

1. $4\overline{)8}$

2. $3\overline{)9}$

3. $6\overline{)36}$

4. $9\overline{)81}$

5. $7\overline{)49}$

6. $5\overline{)80}$

7. $6\overline{)96}$

8. $6\overline{)480}$

9. $4\overline{)840}$

10. $3\overline{)690}$

11. $7\overline{)308}$

12. $7\overline{)203}$

13. $9\overline{)6327}$

14. $4\overline{)2120}$

15. $8\overline{)7280}$

16. $9\overline{)8118}$

17. $3\overline{)64,680}$

18. $4\overline{)50,760}$

19. $6\overline{)21,480}$

20. $5\overline{)18,050}$

21. What is 7525 divided by 7?

22. What is 32,364 divided by 4?



23. If the dividend and the divisor in a division problem are the same number, what is the quotient?

For Exercises 24 to 27, use the relationship between multiplication and division to complete the multiplication problem.

24. $\underline{\hspace{1cm}} \times 7 = 364$

25. $8 \times \underline{\hspace{1cm}} = 376$

26. $5 \times \underline{\hspace{1cm}} = 170$

27. $\underline{\hspace{1cm}} \times 4 =$

OBJECTIVE B To divide by a single digit with a remainder in the quotient

For Exercises 28 to 50, divide.

28. $4\overline{)9}$

29. $2\overline{)7}$

30. $5\overline{)27}$

31. $9\overline{)88}$

32. $3\overline{)40}$

33. $6\overline{)97}$

34. $8\overline{)83}$

35. $5\overline{)54}$

36. $7\overline{)632}$

37. $4\overline{)365}$

38. $4\overline{)921}$

39. $7\overline{)845}$

40. $8\overline{)1635}$

41. $5\overline{)1548}$

42. $7\overline{)9432}$

43. $7\overline{)8124}$

44. $3\overline{)5162}$

45. $5\overline{)3542}$

46. $8\overline{)3274}$

47. $4\overline{)15,301}$

48. $7\overline{)43,500}$

49. $8\overline{)72,354}$


50. $5\overline{)43,542}$

51. What is 45,738 divided by 4? Round to the nearest ten.

52. What is 37,896 divided by 9? Round to the nearest hundred.

53. What is 3572 divided by 7? Round to the nearest ten.

54. What is 78,345 divided by 4? Round to the nearest hundred.

 55. True or false? When a three-digit number is divided by a one-digit number, the quotient can be a one-digit number.

OBJECTIVE C To divide by larger whole numbers

For Exercises 56 to 83, divide.

56. $27\overline{)96}$

57. $44\overline{)82}$

58. $42\overline{)87}$

59. $67\overline{)93}$

60. $41\overline{)897}$

61. $32\overline{)693}$

62. $23\overline{)784}$

63. $25\overline{)772}$

64. $74\overline{)600}$

65. $92\overline{)500}$

66. $70\overline{)329}$

67. $50\overline{)467}$

68. $36\overline{)7225}$

69. $44\overline{)8821}$

70. $19\overline{)3859}$

71. $32\overline{)9697}$

72. $88\overline{)3127}$

73. $92\overline{)6177}$

74. $33\overline{)8943}$

75. $27\overline{)4765}$

76. $22\overline{)98,654}$

77. $77\overline{)83,629}$

78. $64\overline{)38,912}$

79. $78\overline{)31,434}$

$$80. 206 \overline{)3097} \qquad 81. 504 \overline{)6504} \qquad 82. 654 \overline{)1217} \qquad 83. 546 \overline{)2344}$$

84. Find the quotient of 5432 and 21.

85. Find the quotient of 8507 and 53.

86. What is 37,294 divided by 72?

87. What is 76,788 divided by 46?

88. Find 23,457 divided by 43. Round to the nearest hundred.

89. Find 341,781 divided by 43. Round to the nearest ten.



90. True or false? If the remainder of a division problem is 210, then the divisor was less than 210.



For Exercises 91 to 102, use a calculator to divide. Then use estimation to determine whether the quotient is reasonable.

$$91. 76 \overline{)389,804}$$

$$92. 53 \overline{)117,925}$$

$$93. 29 \overline{)637,072}$$

$$94. 67 \overline{)738,072}$$

$$95. 38 \overline{)934,648}$$

$$96. 34 \overline{)906,304}$$

$$97. 309 \overline{)876,324}$$

$$98. 642 \overline{)323,568}$$

$$99. 209 \overline{)632,016}$$

$$100. 614 \overline{)332,174}$$

$$101. 179 \overline{)5,734,444}$$

$$102. 374 \overline{)7,712,228}$$

OBJECTIVE D To solve application problems



Insurance The table at the right shows the sources of insurance claims for losses of laptop computers in a recent year. Claims have been rounded to the nearest ten thousand dollars. Use this information for Exercises 103 and 104.

103. What was the average monthly claim for theft?

104. For all sources combined, find the average claims per month.

Source	Claims
Accidents	\$560,000
Theft	\$300,000
Power surge	\$80,000
Lightning	\$50,000
Transit	\$20,000
Water/flood	\$20,000
Other	\$110,000

Source: Safeware, The Insurance Company

Work Hours The table at the right shows, for different countries, the average number of hours per year that employees work. Use this information for Exercises 105 and 106. Use a 50-week year. Round answers to the nearest whole number.

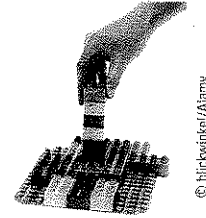
Country	Average Number of Hours per Year
Britain	1731
France	1656
Japan	1889
Norway	1399
United States	1966

Source: International Labor Organization



105. What is the average number of hours worked per week by employees in Britain?
106. On average, how many more hours per week do employees in the United States work than employees in France?

107. Coins The U.S. Mint estimates that about 114,000,000,000 of the 312,000,000,000 pennies it has minted over the last 30 years are in active circulation. That works out to how many pennies in circulation for each of the 300,000,000 people living in the United States?



© iStockphoto/Alamy

108. Toy Sales Every hour, 25,200 sets of Legos® are sold by retailers worldwide. (Source: *Time*, February 11, 2008) How many sets of Legos are sold each second by retailers worldwide?

In the News

Holiday Mail Delivery

The U.S. Postal Service expects to deliver 20 billion pieces of mail between Thanksgiving and Christmas this year.

Source: www.usps.com

109. U.S. Postal Service There are 114 households in the United States. Use the information in the news clipping at the right to determine, on average, how many pieces of mail each household will receive between Thanksgiving and Christmas this year. Round to the nearest whole number.

110. Arlington National Cemetery There are approximately 10,200 funerals each year at Arlington National Cemetery. (Source: www.arlingtoncemetery.org) Calculate the average number of funerals each day at Arlington National Cemetery. Round to the nearest whole number.



© 2008 Jupiterimages

Arlington National Cemetery

111. Which problems below require division to solve?
- Four friends want to share a restaurant bill of \$45.65 equally. Find the amount that each friend should pay.
 - On average, Sam spends \$30 a week on gas. Find Sam's average yearly expenditure for gas.
 - Emma's 12 phone bills for last year totaled \$660. Find Emma's average monthly phone bill.

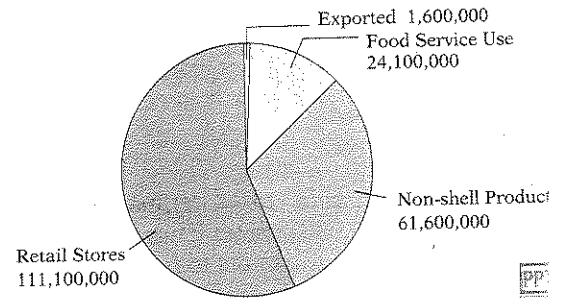
Applying the Concepts

112. **Wages** A sales associate earns \$374 for working a 40-hour week. Last week the associate worked an additional 9 hours at \$13 an hour. Find the sales associate's total pay for last week's work.

- 113. Payroll Deductions** Your paycheck shows deductions of \$225 for savings, \$98 for taxes, and \$27 for insurance. Find the total of the three deductions.

Dairy Products The topic of the graph at the right is the eggs produced in the United States in a recent year. It shows where the eggs that were produced went or how they were used. Use this table for Exercises 114 and 115.

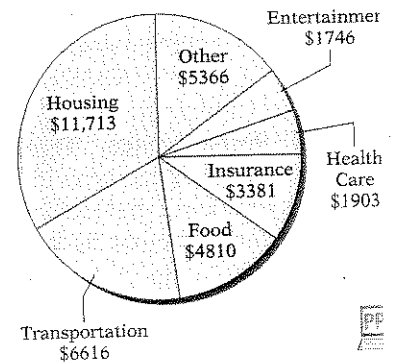
- 114.** Use the graph to determine the total number of cases of eggs produced during the year.
- 115.** How many more cases of eggs were sold by retail stores than were used for non-shell products?



Eggs Produced in the United States (in cases)
Source: American Egg Board

Finance The graph at the right shows the annual expenditures, in a recent year, of the average household in the United States. Use this information for Exercises 116 to 118. Round answers to the nearest whole number.

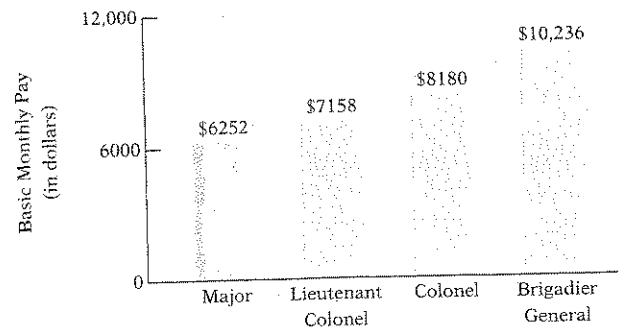
- 116.** What is the total amount spent annually by the average household in the United States?
- 117.** What is the average monthly expense for housing?
- 118.** What is the difference between the average monthly expense for food and the average monthly expense for health care?



Average Annual Household Expense
Source: Bureau of Labor Statistics Consumer Expenditure Survey

The Military The graph at the right shows the basic monthly pay for Army officers with over 20 years of service. Use this graph for Exercises 119 and 120.

- 119.** What is a major's annual pay?
- 120.** What is the difference between a colonel's annual pay and a lieutenant colonel's annual pay?



Basic Monthly Pay for Army Officers
Source: Department of Defense

- 121. Finances** You purchase a used car with a down payment of \$2500 and monthly payments of \$195 for 48 months. Find the total amount paid for the car.