Quiz 3 – Spring 2010
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Math 220

Name: key

No work = no credit

Calculators Allowed

His [Gram's] brilliance and scientific training together with his practical skills made his contributions to pure and applied mathematics very significant.

Regarding Jorgen Gram
(1850 - 1916)
Durch mathematician

1.) (0 pts) The quote above is by Hieronymous Georg Zeuthen and speaks of the work of Jorgen Gram – one of the mathematicians for whom the Gram-Schmidt orthogonalization algorithm is named.

2.) (1 pt) If A is an $(m \times n)$ matrix, then (fill in the blanks):

$$N(A) \subset \mathbb{R}^{n} \text{ and } R(A) \subset \mathbb{R}^{m}$$

$[\ ]_{m\times n}[\ ]_{n\times 1} = [\ ]_{m}$

3.) (2 pts) What is a basis?

a minimal spanning set for W,
a L.I. set of vectors that span W,

4.) (6 pt) Suppose A is an $(m \times n)$ matrix and the equation $Ax = \theta$ has non-trivial solutions.

a.) What can be said of the columns of A?

n cols,
the cols are L.D.

b.) What can be said of the null space of A?

it has dim ≥ 1
dim $\leq n$.

c.) What can be said of the rank of A?

since rank + nullity = n
$\Rightarrow$ rank $< n$

5.) (4 pts) Let $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 5 & 7 & 1 \\ 1 & 0 & 1 & 3 \end{bmatrix}$.

a.) Find the null space of A.

solve $[A \mid 0]$

$x_1 = -x_3 - 3x_4$

$x_2 = -x_3 + x_4$

x_3 & x_4 arb.

$$N(A) = \{\vec{x} \mid \vec{x} = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix} \text{ and } x_3, x_4 \in \mathbb{R}\}$$

b.) Find a basis for the range of A.

method 1

discard A_3 & A_4

$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

method 2

non zero cols in $(\text{ref}(A^T))$

$\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$ & $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$

method 3

$$\begin{bmatrix} 1 & 2 & 3 & 1 & b_1 \\ 2 & 5 & 7 & 1 & b_2 \\ 1 & 0 & 1 & 3 & b_3 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} 1 & 2 & 3 & 1 & b_1 \\ 0 & 1 & 1 & -1 & -2b_1 + b_2 \\ 0 & 0 & 0 & 0 & -5b_1 + 2b_2 + b_3 \end{bmatrix}$$

$b_3 = 5b_1 - 2b_2$

$\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$ & $\begin{bmatrix} 0 \\ +1 \\ -2 \end{bmatrix}$

c.) Find the nullity and rank of A.

Rank: 2 and Nullity: 2

6.) (1 pt) If you could attend one professional event from the past or present (personal, sporting, dramatic, academic, political, …) what would it be?

The resurrection.

7.) (4 pts) Give at least one use we have found for the transpose of a matrix.

- Determine if matrices are symmetric
- scalar product.
- Find the range.