Quiz 1 – Spring 2010
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Math 220

Name: Key

As for everything else, so for a mathematical theory: beauty can be perceived but not explained.

Arthur Cayley
(1821 - 1895)
English mathematician

No work = no credit

Calculators Allowed

1.) (0 pts) Why linear algebra? See the quote above by Cayley.

2.) (1 pt) What math classes have you taken at Highline? Please circle courses that you have repeated.

61 81 85 79 89 91 97 107 111 112/148
141 142 151 152 153 254 220 291

3.) (3 pts) Carefully give the conditions that must be satisfied for a subset W of $\mathbb{R}^n$ to be a subspace.

(1) $\theta \in W$

(2) $x + y \in W$ for all $x, y \in W$

(3) $ax \in W$ for all $x \in W$ & $a \in \mathbb{R}$

4.) (4 pts) Give an algebraic representation for the null space of $A = \begin{bmatrix} -3 & -5 & 36 \\ -1 & 0 & 7 \\ 1 & 1 & -10 \end{bmatrix}$.

Solve $Ax = \theta$

$$\mathrm{rref}([A|\theta]) = \begin{bmatrix} 1 & 0 & -7 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$N(A) = \{x \mid x_1 = 7x_3 \text{ and } x_2 = 3x_3\}.$$

$$x = \begin{bmatrix} 7x_3 \\ 3x_3 \\ x_3 \end{bmatrix}$$

5.) (4 pts) Show that the vector $b = \begin{bmatrix} 4 \\ -13 \\ -17 \end{bmatrix}$ is in the range of $A = \begin{bmatrix} 4 & 8 & -4 \\ 3 & 6 & 5 \\ -2 & 1 & 12 \end{bmatrix}$.

solve $Ax = b$ by rref([A|b]) $\Rightarrow \vec{x} = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$

check $\begin{bmatrix} 4 & 8 & -4 \\ 3 & 6 & 5 \\ -2 & 1 & 12 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ -13 \\ -17 \end{bmatrix}$

Thus $\exists$ x s.t. $Ax = b$ & so $b \in R(A)$.

-1 unless range $= \mathbb{R}^3$ & for $Ax = b$

6.) (5 pts) W is a subset of $\mathbb{R}^3$. Determine if $W = \{\vec{x} \in \mathbb{R}^3 \mid x_3 = 4x_2 = 5x_1\}$ is a subspace of $\mathbb{R}^3$.

(1) $\vec{0} \in W$ since $0 = 4(0) = 5(0)$

(2) let $x, y \in W$

$\Rightarrow x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{5}x_3 \\ \frac{1}{4}x_3 \\ x_3 \end{bmatrix}$ and $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{5}y_3 \\ \frac{1}{4}y_3 \\ y_3 \end{bmatrix}$

$\Rightarrow x + y = \begin{bmatrix} \frac{1}{5}x_3 + \frac{1}{5}y_3 \\ \frac{1}{4}x_3 + \frac{1}{4}y_3 \\ x_3 + y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{5}(x_3 + y_3) \\ \frac{1}{4}(x_3 + y_3) \\ x_3 + y_3 \end{bmatrix} \in W.$

(3) Let $x \in W$ & $a \in \mathbb{R}$.

$\Rightarrow x = \begin{bmatrix} \frac{1}{5}x_3 \\ \frac{1}{4}x_3 \\ x_3 \end{bmatrix}$

$\Rightarrow ax = \begin{bmatrix} \frac{1}{5}(ax_3) \\ \frac{1}{4}(ax_3) \\ ax_3 \end{bmatrix} \in W$

Therefore W is a subspace.