Quiz 1 – Spring 2010
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Math 220

Name: KEY

Every good mathematician is at least half a philosopher, and every good philosopher is at least half a mathematician.

Gottlob Frege
(1848 - 1925)
German mathematician

No work = no credit

No Calculators

1.) (0 pts) The quote (see above) by Frege encourages you to branch out of pure mathematics and science

2.) (1 pt) Explain what it means for a system of linear equations to be inconsistent?

An inconsistent system has no solution.

3.) (4 pts) Solve the system of linear equations and express the solution in vector form. Use Gauss-Jordan Elimination.

$$\begin{aligned} -3x_1 - 5x_2 + 36x_3 &= 10 \\ -x_1 + 7x_3 &= 5 \\ x_1 + x_2 - 10x_3 &= -4 \end{aligned}$$

2/4 if a reasonable start.

$$\begin{bmatrix} -3 & -5 & 36 & 10 \\ -1 & 0 & 7 & 5 \\ 1 & 1 & -10 & -4 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 1 & -10 & -4 \\ -1 & 0 & 7 & 5 \\ -3 & -5 & 36 & 10 \end{bmatrix} \begin{matrix} R_1 + R_2 \to R_2 \\ 3R_1 + R_3 \to R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & -10 & -4 \\ 0 & 1 & -3 & 1 \\ 0 & -2 & 6 & -2 \end{bmatrix} 2R_2 + R_3 \to R_3$$

$$\begin{bmatrix} 1 & 1 & -10 & -4 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_1 - R_2 \to R_1$$

$$\begin{bmatrix} 1 & 0 & -7 & -5 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_1 &= -5 + 7x_3 \\ x_2 &= 1 + 3x_3 \\ x_3 &\ \text{arbitrary} \end{aligned}\right\} .5$$

$$\underbrace{x = \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}}_{.5}$$

4.) (4 pts) Solve the equation $Ax = b$ if $A = \begin{bmatrix} 4 & 8 & -4 \\ 3 & 6 & 5 \\ -2 & 1 & 12 \end{bmatrix}$ and $b = \begin{bmatrix} 4 \\ -13 \\ -17 \end{bmatrix}$ using Gauss-Jordan Elimination.

$$\begin{bmatrix} 4 & 8 & -4 & 4 \\ 3 & 6 & 5 & -13 \\ -2 & 1 & 12 & -17 \end{bmatrix} \quad \tfrac{1}{4}R_1 \to R_1$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 3 & 6 & 5 & -13 \\ -2 & 1 & 12 & -17 \end{bmatrix} \quad \begin{matrix} R_2 - 3R_1 \to R_2 \\ R_3 + 2R_1 \to R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 8 & -16 \\ 0 & 5 & 10 & -15 \end{bmatrix} \quad \begin{matrix} \tfrac{1}{8}R_2 \to R_3 \\ \tfrac{1}{5}R_3 \to R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & -2 \end{bmatrix} \quad \begin{matrix} R_1 + R_3 \to R_1 \\ R_2 - 2R_3 \to R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \quad R_1 - 2R_2 \to R_1$$

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$$

5.) (1 pt) Can a homogeneous system of linear equations be inconsistent? Explain your answer.

No - the zero vector/trivial solution always satisfies $Ax = \theta$.

6.) (3 pts) If $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 2 & -1 \end{bmatrix}$, find $A \cdot B$ and $B \cdot A$.

a.) $A \cdot B$ $\quad \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 2 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 1 \\ 7 & 10 & -7 \end{bmatrix}$

b.) $B \cdot A$ $\quad$ mismatched dimensions.