4.4: Eigenvalues & the characteristic Poly.

$$A = \begin{bmatrix} 13 & -16 \\ 9 & -11 \end{bmatrix} \Rightarrow p(t) = (t-1)^2$$

$$B = \begin{bmatrix} -2 & -1 & 0 \\ 0 & 1 & 1 \\ -2 & -2 & -1 \end{bmatrix} \Rightarrow p(t) = -t(t+1)^2$$

Goal: (1) Find all scalars λ s.t. $A - \lambda I$ is singular.

(2) Given λ, find all $x \neq \theta$ s.t. $(A - \lambda I)x = \theta$.

If A $(n \times n)$, then $\det(A - tI)$ is a poly w/ degree n in t. We call it the characteristic poly for A.

Note that the roots of $p(t)$; where $p(t) = 0$ are the eigenvalues of A.

By the FToA

(a) An $(n \times n)$ matrix can have no more than n distinct eigenvalues.

(b) An $(n \times n)$ matrix always has @ least one eigenvalue (possibly complex)

Thm: Let A be an $(n \times n)$ matrix & let λ be an eigenvalue of A. Then:

prove it $\rightarrow$ (a) λ^k is an eigenvalue of A^k for $k = 2, 3, \ldots$

prove it $\rightarrow$ (b) If A is nonsingular, then $1/\lambda$ is an eigenvalue of A^{-1}.

(c) If α is any scalar, then $\lambda + \alpha$ is an eigenvalue of $A + \alpha I$.

Thm: Let A be $(n \times n)$. Then A & A^T have the same eigenvalues.

Thm: Let A be $(n \times n)$. then A is singular iff $\lambda = 0$ is an eigenvalue of A.
prove it $\rightarrow$

Thm: Let $T = (t_{ij})$ be an $(n \times n)$ triangular matrix. Then the eigenvalues of T are the diagonal entries.
prove it $\rightarrow$