So, the row or col. you expand along doesn't change the det.

<u>Tip</u>: expand along the row or col w/ the most zeros.

<u>Thm</u>: Let A & B be $(n \times n)$ matrices, then

$$\det(AB) = \det(A)\det(B)$$

<u>Thm</u>: Let A be an $(n \times n)$ matrix. Then A is singular iff $\det(A) = 0$.

<u>Thm</u>: Let $T = (t_{ij})$ be an $(n \times n)$ triangular matrix, then $\det(T) = t_{11} t_{22} \cdots t_{nn}$

Q: what is $\mathrm{Det}(I)$?

4.2: Determinants & the Eigenvalue Problem.

1. calculating determinants.

Let $A = \begin{bmatrix} 2 & -3 & 2 \\ -1 & -2 & 1 \\ 3 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 2 & 1 & 5 \\ 0 & 3 & 0 & 0 \\ 0 & 4 & 1 & 2 \\ 0 & 3 & 1 & 4 \end{bmatrix}$

calculate determinants 2 ways (2 expansions)

...

emphasize minor matrices & cofactors.

Dfn: Let $A = (a_{ij})$ be an $(n \times n)$ matrix. The $[(n-1) \times (n-1)]$ matrix that results from removing the r^{th} row & s^{th} col from A is called a minor matrix of A & is designated M_{rs}.

Dfn: The cofactor $A_{ij} = (-1)^{i+j} \det(M_{ij})$

Thm: Let $A = (a_{ij})$ be an $(n \times n)$ matrix w/ minor matrices M_{ij} & cofactors $A_{ij} = (-1)^{i+j} \det(M_{ij})$. Then

$$\det(A) = \sum_{j=1}^{n} a_{ij} A_{ij} \quad (i^{th} \text{ row})$$

$$= \sum_{i=1}^{n} a_{ij} A_{ij} \quad (j^{th} \text{ col.}).$$