3.5: Dimension

Thm: Let W be a subspace of $\mathbb{R}^n$, and let $B = \{w_1, \dots, w_p\}$ be a spanning set for W containing p vectors. Then any set of $p+1$ or more vectors in W is linearly dependent.

Cor: Let W be a subspace of $\mathbb{R}^n$, and let $B = \{w_1, \dots, w_p\}$ be a basis for W containing p vectors. Then every basis for W contains p vectors.

Dfn: Let W be a subspace of $\mathbb{R}^n$. If W has a basis $B = \{w_1, \dots, w_p\}$ of p vectors, then we say that W is a subspace of dimension p, and we write $\dim(W) = p$.

Thm: Let W be a subspace of $\mathbb{R}^n$ w/ $\dim(W) = p$

(1) Any set of $(p+1)$ or more vectors in W is L.D

(2) Any set of fewer than p vectors in W does not span W.

(3) Any set of p L.I. vectors in W is a basis for W.

(4) Any set of p vectors that spans W is a basis for W.

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Dfn: $\text{Dim}(\mathcal{N}(A))$ is the nullity of A

Dfn: $\text{Dim}(\mathcal{R}(A))$ = the rank of A.

Thm: If A $(m \times n)$ is a matrix then the rank of A is equal to the rank of A^T.

Cor: If A $(m \times n)$ is a matrix, then the row space & the column space of A have the same dimension.

Note: If A is an $(m \times n)$ matrix...

$$n = \text{rank}(A) + \text{nullity}(A)$$

Thm: An $(m \times n)$ sys. of lin. eqts $Ax = b$ is consistent iff $\text{rank}(A) = \text{rank}([A|b])$

Thm: An $(n \times n)$ matrix A is nonsingular iff $\text{rank}(A) = n$.

examples for 3.5 on Dimension.

3.4 ex 3:

A basis for the range of $A = \begin{bmatrix} 1 & -2 & 1 \\ 1 & -2 & 0 \\ -1 & 2 & 2 \end{bmatrix}$

Two methods for finding a basis for the ~~[illegible]~~ subspace W where $W = sp\{A_1, A_2, \ldots, A_n\}$ where $A = [A_1, \ldots, A_n]$

(1) solve $Ax = \theta$ and discard vectors corresponding to arbitrary variables.

(2) Use row ops to transform A^T into B^T which is in echelon form. The nonzero columns of B are a basis for W.

3.4 ex 4: Two ways to find a basis for the vectors $v_1, \ldots, v_5$ given yesterday.