Vector spaces

$$\mathbb{R}^n = \left\{ x \mid x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \text{ and } x_1, \dots, x_n \in \mathbb{R} \right\}$$

vector addition & scalar mult. are the same as w/matrices.

Thm: $x, y, z \in \mathbb{R}^n$ and $a, b \in \mathbb{R}$, then:

closure

$x + y \in \mathbb{R}^n$

$ax \in \mathbb{R}^n$

addition

$x + y = y + x$

$x + (y+z) = (x+y) + z$

$\theta \in \mathbb{R}^n$ & $x + \theta = x \quad \forall\, x \in \mathbb{R}^n$.

$\forall\, x \in \mathbb{R}^n,\ \exists\, -x \in \mathbb{R}^n$ s.t. $x + (-x) = \theta$

multiplication

$a(bx) = (ab)x$

$a(x+y) = a(x+y)$

$(a+b)x = ax + bx$

$1x = x \quad \forall\, x \in \mathbb{R}^n$.

These properties hold for all of $\mathbb{R}^n$, but they also hold for some subsets of $\mathbb{R}^n$. These are special subsets & get the name <u>Subspace</u>

Thm: A subset W of $\mathbb{R}^n$ is a subspace of $\mathbb{R}^n$ iff the conditions below are met

(1) $\theta \in W$

(2) $x + y \in W$ when $x, y \in W$

(3) $ax \in W$ when $x \in W$ & $a \in \mathbb{R}$.

Q: what would a proof look like?

Determine if the set is a subspace.

ex1: $W = \{x \mid x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $x_1 = x_2$ ~~&~~ OR $x_1 = -x_2\}$

ex2: $W = \{x \mid x_1$ & $x_2 \in \mathbb{Q}\}$.

ex3: $W = \{x \mid x_1^2 + x_2 = 1\}$

ex4: $W = \{x \mid x_1 = 2x_3$ & $x_2 = -x_3\}$.

ex4: $W = \{x \mid x_2 = x_3 = 0\}$