Echelon Form & Gauss-Jordan Elimination

(1) Review:

Given system of eqts — linear

↓

Augmented matrix

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Reduced matrix — method of Gauss-Jordan elimination
Goal 1: row echelon form
Goal 2: reduced row echelon form.

↓

Reduced sys of eqts

↓

Solution

unique, ∞ # } consistent
no solution } inconsistent

(2) Recognizing forms.

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 7 & 0 & 9 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Dfn: Row echelon form

(A) all 0 rows @ bottom

(B) left most non zero entry in a row is 1.

(C) if the $(i+1)^{st}$ row contains non-zero entries, then the 1st nonzero entry is in a column to the right of the 1st nonzero entry in the i^{th} row.

Dfn: Reduced Row Echelon Form

a matrix that is in row echelon form is in RREF provided that the 1st non-zero entry in any row is the only non-zero entry in its column.

(3) Interpreting a system in RREF corresponding to the augmented matrix of a linear system.

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

consistent w/ unique solution
$x_1 = 2$; $x_2 = 3$; $x_3 = 4$

$$\begin{bmatrix} 0 & 1 & -4 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

inconsistent

$$\begin{bmatrix} 1 & -2 & 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

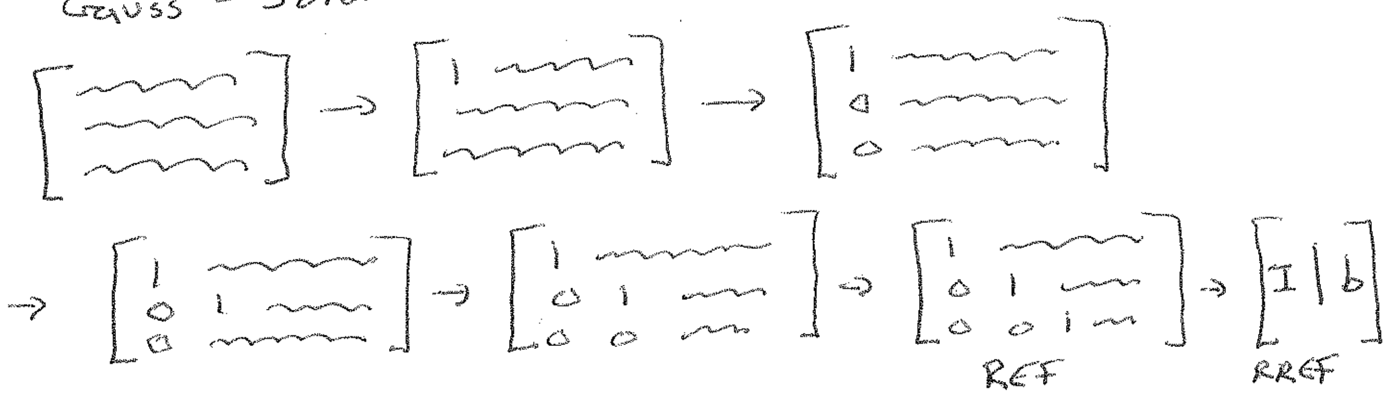
consistent w/ ∞ solutions.
$x_1 = 3 + 2x_2 - 2x_5$
$x_3 = 2 + x_5$
$x_4 = -4 - 3x_5$
x_2 & x_5 are arbitrary.

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

consistent w/ ∞ solutions.
$x_1 = 3$; $x_3 = 0$
x_2 is arbitrary.

Q: How do you recognize an inconsistent & how does this benefit you?

(4) Gauss - Jordan elimination.



example

Solve

$$x_2 + x_3 + x_4 - 2x_5 = 1$$
$$x_1 + 2x_2 + 3x_3 - x_4 + x_5 = 0$$
$$-x_1 + 3x_2 + 4x_3 + x_4 + x_5 = 1$$

$$\begin{bmatrix} 0 & 1 & 1 & 1 & -2 & 1 \\ 1 & 2 & 3 & -1 & 1 & 0 \\ -1 & 3 & 4 & 1 & 1 & 1 \end{bmatrix} \quad \begin{array}{l} R_1 \leftrightarrow R_2 \\ R_2 + R_3 \to R_3 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 3 & -1 & 1 & 0 \\ 0 & 1 & 1 & 1 & -2 & 1 \\ 0 & 5 & 7 & 0 & 2 & 1 \end{bmatrix} \quad R_3 - 5R_2 \to R_3$$

$$\begin{bmatrix} 1 & 2 & 3 & -1 & 1 & 0 \\ 0 & 1 & 1 & 1 & -2 & 1 \\ 0 & 0 & 2 & -5 & 12 & -4 \end{bmatrix} \quad \begin{array}{l} R_1 - 2R_2 \to R_1 \\ \frac{1}{2}R_3 \to R_3 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 1 & -3 & +5 & -2 \\ 0 & 1 & 1 & 1 & -2 & 1 \\ 0 & 0 & 1 & \frac{-5}{2} & 6 & -2 \end{bmatrix} \quad \begin{array}{l} R_1 - R_3 \to R_1 \\ R_2 - R_3 \to R_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 & -1 & 0 \\ 0 & 1 & 0 & 7/2 & -8 & 3 \\ 0 & 0 & 1 & -5/2 & 6 & -2 \end{bmatrix}$$

$$x_1 = \frac{1}{2}x_4 + x_5$$
$$x_2 = 3 - \frac{7}{2}x_4 + 8x_5$$
$$x_3 = -2 + \frac{5}{2}x_4 - 6x_5$$

x_4 & x_5 are arbitrary.

Show the process on the calculator.

Steps in Gauss-Jordan Elimination

step 1: locate the first column w/ a non-zero entry

step 2: if necessary, interchange the 1st row w/ another so that the ~~upper~~ 1st ~~left~~ nonzero column has a nonzero entry in the 1st row.

step 4: add appropriate multiples of row one to each of the remaining rows so that every entry below the leading 1 in row one is a zero.

step 3: If a denotes the leading nonzero entry in row one, multiply row one by $\frac{1}{a}$.

step 5: Temporarily ignore row one & repeat steps 1-4 on the sub matrix that remains. Stop when the resulting matrix is in REF.

step 6: Backwards substitute by adding multiples of each nonzero row to the rows above in order to zero all entries above the leading 1.

(5) example. (app to curve fitting).

Find the quadratic thru $(1,6), (2,9), (3,16)$

soln: $y = 2x^2 - 3x + 7$,