Intro to Matrices & Systems of Eqts

(1) Linear Systems

$$\begin{cases} x + y = 7 \\ x - y = -3 \end{cases} \quad \text{we write as} \quad \begin{array}{l} x_1 + x_2 = 7 \\ x_1 - x_2 = -3 \end{array}$$

has the solution $(2, 5)$

$$x_1 - x_2 + x_3 = 3$$
$$2x_1 + x_2 - 4x_3 = -3$$

has solutions $(1, -1, 1)$ & $(2, 1, 2)$

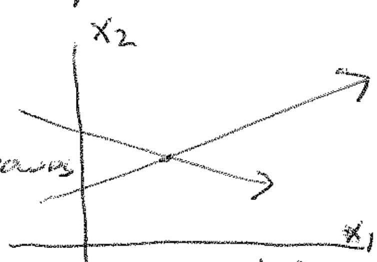
or $x_1 = x_3$ & $x_2 = 2x_3 - 3$ for any x_3.

$$3x_1 - 2x_2 = 7$$
$$6x_1 - 4x_2 = 12$$

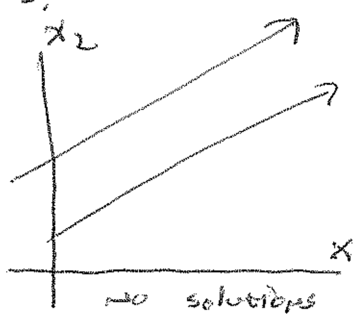
has no solution.

(2) Graphical solutions.

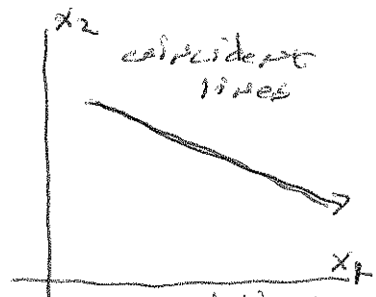
2 eqts
2 unknowns



one solution

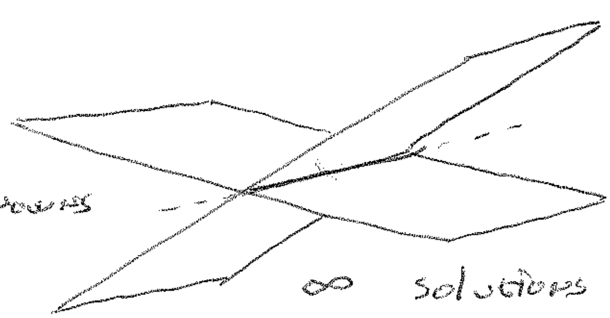


no solutions

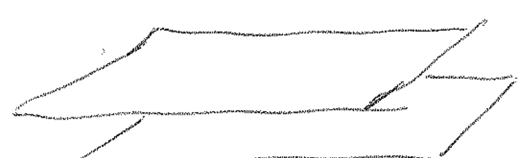


∞ solutions

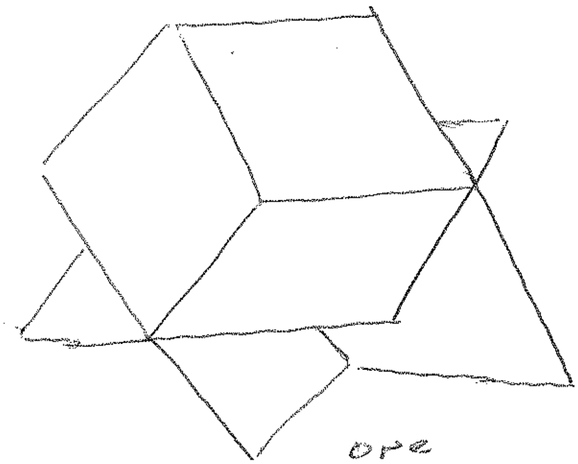
2 eqts
3 unknowns



∞ solutions

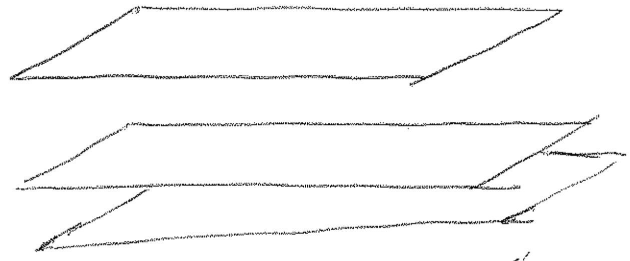


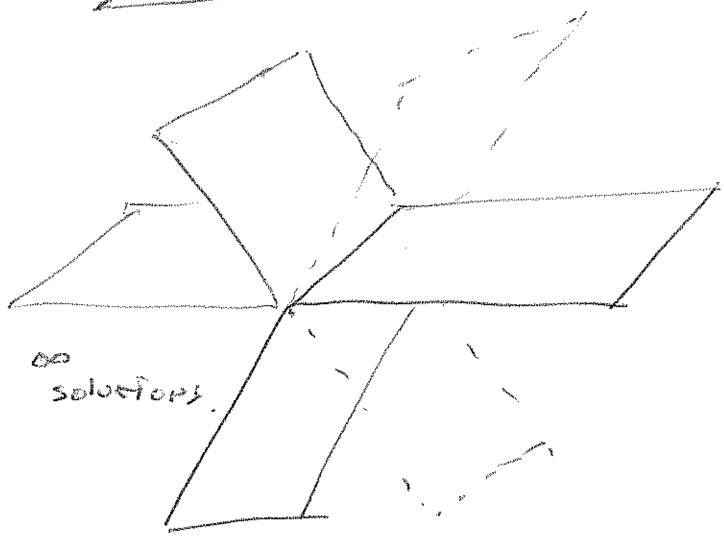
no solutions.



one solution

no solution





∞ solutions

3 eqns
3 unknowns

(3) General case

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\
\vdots \qquad\qquad\qquad\qquad & \quad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_n
\end{aligned}
$$

(a) one solution
(b) ∞ solutions } consistent system

(c) no solution } inconsistent system

(4) matrices.

We begin by relating matrix theory to linear systems. Later we will learn of apps/uses ind. of eqts.

(a)
$$\begin{aligned} x_1 + x_2 &= 7 \\ x_1 - x_2 &= -3 \end{aligned} \Rightarrow \begin{bmatrix} 1 & 1 & 7 \\ 1 & -1 & -3 \end{bmatrix}$$

augmented matrix

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad b = \begin{bmatrix} 7 \\ -3 \end{bmatrix} \qquad [A \mid b]$$

coefficient matrix — constants

(b)
$$\begin{aligned} x_1 - x_2 + x_3 &= 3 \\ 2x_1 + x_2 - 4x_3 &= -3 \end{aligned}$$

Goal: Use matrices to solve systems of linear equations.

Dfn: Two systems are equivalent provided they have the same solution set.

Thm: The following elementary ops can be performed w/o changing the solution set.

Eqts	Matrices
(i) interchange eqt	(i) interchange rows.
(ii) mult. eqt. by a non-zero scalar	(ii) mult. row by a non-zero scalar.
(iii) add a const mult. of one eqt to another.	(iii) add a const. mult. of one row to another

$$
\begin{aligned}
x_2 + x_3 &= 4 \\
x_1 - x_2 + 2x_3 &= 1 \\
2x_1 + x_2 - x_3 &= 6
\end{aligned}
$$

$$\begin{bmatrix} 0 & 1 & 1 & 4 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & -1 & 6 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 1 & 4 \\ 2 & 1 & -1 & 6 \end{bmatrix} \quad R_3 - 2R_1 \to R_3$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 3 & -5 & 4 \end{bmatrix} \quad R_3 - 3R_2 \to R_3$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & -8 & -8 \end{bmatrix} \quad -\frac{1}{8}R_3 \to R_3$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{matrix} R_1 - 2R_3 \to R_1 \\ R_2 - R_3 \to R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_1 + R_2 \to R_1$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$x_1 = 2$

$x_2 = 3$ solution

$x_3 = 1$

This is an example of Gauss-Jordan elimination - our topic tomorrow.

<u>Corollary</u>: Suppose $[A|b]$ & $[C|d]$ are augmented matrices, each representing a different system of eqts. If $[A|b]$ & $[C|d]$ are row equivalent, then the two systems are also equivalent.