LESSON 10.3 – COMPLEX NUMBERS

Here's what you'll learn in this lesson:

Complex Number System

a. Definition of complex numbers

b. Powers of i

c. Operations on complex numbers

OVERVIEW

Prior to the 19th century, mathematicians dismissed the idea that the square root of a negative number had any meaning—one mathematician even claimed that thinking about these quantities involved "putting aside…mental tortures." However, people eventually began accepting these values, which they called complex numbers, as important. Today, for example, engineers use complex numbers to describe certain properties of electrical circuits.

In this lesson, you will learn about complex numbers. You will learn how to add, subtract, multiply, and divide complex numbers. You will also see how you can use the discriminant of a quadratic equation to determine if the equation has complex solutions.

EXPLAIN

COMPLEX NUMBER SYSTEM

Summary

The Number i

Sometimes when solving a quadratic equation such as $x^2 + 1 = 0$, you cannot find a real number x which solves the equation.

To solve such an equation, mathematicians defined $\boldsymbol{i}$ as:

$$i = \sqrt{-1}$$

The number i is called an imaginary number. If you were to square both sides, you would get $i^2 = -1$.

You can use i to help you rewrite negative roots.

For example:

$$\sqrt{-4} = \sqrt{(-1)(4)} = \sqrt{4}i = 2i$$

$$\sqrt{-9} = \sqrt{(-1)(9)} = \sqrt{9}i = 3i$$

$$\sqrt{-16} = \sqrt{(-1)(16)} = \sqrt{16}i = 4i$$

In general, if k is a positive number:

$$\sqrt{-k} = \sqrt{(-1)k} = \sqrt{k}i = i\sqrt{k}$$

Notice that in the term $\sqrt{k}i$, the i is outside the radical sign.

You can use the imaginary number i to solve a quadratic equation that cannot be solved using only real numbers.

For example, to solve $x^2 = -4$:

1. Use the square root property. $\quad x = \pm\sqrt{-4}$
2. Use $i = \sqrt{-1}$ to simplify the answer.

$$= \pm\sqrt{(-1)(4)}$$
$$= \pm 2\sqrt{-1}$$
$$= \pm 2i$$

So $x = \pm 2i$ are the solutions of $x^2 = -4$.

Make sure when you do problems like this that you first write the radicals in terms of i. If you don't and you do the multiplication first, you get the wrong answer:

$$\sqrt{-9} \cdot \sqrt{-16} \neq \sqrt{(-9)(-16)}$$
$$\neq \sqrt{144}$$
$$\neq 12$$

But $\sqrt{-9} \cdot \sqrt{-16} = -12$, not 12.

Here is another example.

To find $\sqrt{-9} \cdot \sqrt{-16}$:

1. Rewrite both radicals in terms of i. $= 3i \cdot 4i$
   $= 12i^2$
2. Rewrite i^2 as –1. $= 12(-1)$
   $= -12$

So $\sqrt{-9} \cdot \sqrt{-16} = -12$.

Powers of i

Look at the powers of i below:

$i^1 = i$

$i^2 = -1$

$i^3 = i^2 \cdot i = (-1) \cdot i = -i$

$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$

$i^5 = i^4 \cdot i = 1 \cdot i = i$

This pattern, i, –1, $-i$, 1, i, ..., continues. You can use this to rewrite powers of i.

To simplify powers of i when the power is greater than or equal to 4:

1. Rewrite the power as a product using as many factors of i^4 as possible.
2. Rewrite i^4 as 1.
3. Simplify.

For example, to simplify i^{33}:

1. Rewrite i^{33} as a product using factors of i^4. $= (i^4)^8 \cdot i$
2. Rewrite i^4 as 1. $= (1)^8 \cdot i$
3. Simplify. $= 1 \cdot i$
   $= i$

So $i^{33} = i$.

Complex Numbers

A complex number is a number that can be written in the form:

$a + bi$

where a and b are real numbers. The real number a is called the real part of the complex number. The real number b is called the imaginary part of the complex number.

For example, these are complex numbers:

$3 + 4i$	$a = 3, b = 4$
$2 + 17i$	$a = 2, b = 17$
$-6 + 3i$	$a = -6, b = 3$
$4 - 9i$	$a = 4, b = -9$
$-5 - 11i$	$a = -5, b = -11$

Do you see why b = –9 in this example? The expression 4 – 9i can be rewritten as 4 + (–9)i .

A complex number whose imaginary part is 0 is a real number.

For example, these are real numbers:

$3 = 3 + 0i$	$a = 3, b = 0$
$-7 = -7 + 0i$	$a = -7, b = 0$
$\frac{5}{8} = \frac{5}{8} + 0i$	$a = \frac{5}{8}, b = 0$

Notice that b = 0 in each of these examples.

A complex number whose real part is 0 is called a pure imaginary number.

For example, these are pure imaginary numbers:

$6i = 0 + 6i$	$a = 0, b = 6$
$19i = 0 + 19i$	$a = 0, b = 19$
$-2i = 0 + (-2i)$	$a = 0, b = -2$

Notice that a = 0 in each of these examples.

Two complex numbers are equal if their real parts are equal and their imaginary parts are equal.

For example,

$$5 + (8 - 2)i = [4 - (-1)] + 6i$$

Here, the real parts are equal. $5 = [4 - (-1)]$

And the imaginary parts are equal. $(8 - 2) = 6$

As another example,

$$(3 + 4) - 2i \neq 7 + 2i$$

Here the real parts are equal. $(3 + 4) = 7$

But the imaginary parts are **not** equal. $-2 \neq 2$

Remember that for two complex numbers to be equal, **both** the real parts and the imaginary parts of the complex numbers must be equal.

Complex Conjugates

When the real parts of two complex numbers are equal and the imaginary parts only differ in sign, the complex numbers are called complex conjugates.

For example, here are some pairs of complex numbers that are complex conjugates:

$7 + 2i$ and $7 - 2i$

$-3 + 6i$ and $-3 - 6i$

$12 - 8i$ and $12 + 8i$

And here are some pairs of complex numbers that are **not** complex conjugates:

$4 + 7i$ and $7 - 4i$

$3 + 5i$ and $-3 + 5i$

$7 + 2i$ and $-7 - 2i$

In general, the pairs of complex numbers

$a + bi$ and $a - bi$

are called complex conjugates.

Adding and Subtracting Complex Numbers

To add two complex numbers, just add their real parts and add their imaginary parts.

In general,

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

For example, to find $(2 + 7i) + (6 + 3i)$:

1. Add the real parts and add the imaginary parts. $= (2 + 6) + (7 + 3)i$
$= 8 + 10i$

So $(2 + 7i) + (6 + 3i) = 8 + 10i$.

As another example, to find $(9 - 4i) + (-3 + 5i)$:

1. Add the real parts and add the imaginary parts. $= [9 + (-3)] + [(-4) + 5]i$
$= 6 + i$

So $(9 - 4i) + (-3 + 5i) = 6 + i$.

To subtract two complex numbers, just subtract their real parts and subtract their imaginary parts as indicated.

In general,

$(a + bi) - (c + di) = (a - c) + (b - d)i$

For example, to find $(8 + 2i) - (5 + 4i)$:

1. Subtract the real parts and subtract the imaginary parts. $= (8 - 5) + (2 - 4)i$ $= 3 - 2i$

So $(8 + 2i) - (5 + 4i) = 3 - 2i$.

When subtracting complex numbers, make sure you subtract both the real and imaginary parts.

As another example, to find $(11 - 7i) - (3 - 13i)$:

1. Subtract the real parts and subtract the imaginary parts. $= (11 - 3) + [-7 - (-13)]i$ $= 8 + 6i$

So $(11 - 7i) - (3 - 13i) = 8 + 6i$.

Multiplying Complex Numbers

You multiply complex numbers in much the same way as you multiply polynomials.

Don't forget that $i^2 = -1$.

For example, to find $2i \cdot 7i$:

1. Multiply the real parts. Multiply the imaginary parts.

$$\begin{aligned} &= 2 \cdot i \cdot 7 \cdot i \\ &= 2 \cdot 7 \cdot i \cdot i \\ &= 14 \cdot i^2 \\ &= 14 \cdot (-1) \\ &= -14 \end{aligned}$$

So $2i \cdot 7i = -14$.

As another example, to find $4i(9 - 5i)$:

1. Use the distributive property, then multiply.

$$\begin{aligned} &= 4i \cdot 9 - 4i \cdot 5i \\ &= 36i - 20i^2 \\ &= 36i - 20(-1) \\ &= 36i + 20 \\ &= 20 + 36i \end{aligned}$$

So $4i(9 - 5i) = 20 + 36i$.

As a third example, to find $(8 + 3i)(5 - 4i)$:

1. Use the FOIL method and simplify.

$$\begin{aligned} &= 8 \cdot 5 - 8 \cdot 4i + 3i \cdot 5 - 3i \cdot 4i \\ &= 40 - 32i + 15i - 12i^2 \\ &= 40 - 17i - 12(-1) \\ &= 40 - 17i + 12 \\ &= 52 - 17i \end{aligned}$$

So $(8 + 3i)(5 - 4i) = 52 - 17i$.

In general,

$$
\begin{aligned}
(a + bi)(c + di) &= ac + adi + bci + bdi^2 \\
&= ac + adi + bci + bd(-1) \\
&= ac + adi + bci - bd \\
&= ac - bd + adi + bci \\
&= (ac - bd) + (ad + bc)i
\end{aligned}
$$

Dividing Complex Numbers

You can also divide complex numbers.

To divide complex numbers:

1. Rewrite the division problem as a fraction.
2. Multiply the numerator and denominator of the fraction by the complex conjugate of the denominator.
3. Simplify.

For example, to find $(7 + 5i) \div (3 + 2i)$:

1. Rewrite as a fraction. $\frac{7 + 5i}{3 + 2i}$
2. Multiply by the complex conjugate. $= \frac{7 + 5i}{3 + 2i} \cdot \frac{3 - 2i}{3 - 2i}$

   $= \frac{(7 + 5i)(3 - 2i)}{(3 + 2i)(3 - 2i)}$
3. Simplify. $= \frac{21 - 14i + 15i - 10i^2}{9 - 4i^2}$

   $= \frac{21 + i - 10(-1)}{9 - 4(-1)}$

   $= \frac{21 + i + 10}{9 + 4}$

   $= \frac{31 + i}{13}$

   $= \frac{31}{13} + \frac{1}{13}i$

So $(7 + 5i) \div (3 + 2i) = \frac{31 + i}{13} = \frac{31}{13} + \frac{1}{13}i$.

In general,

$$
\begin{aligned}
(a + bi) \div (c + di) &= \frac{a + bi}{c + di} \\
&= \frac{a + bi}{c + di} \cdot \frac{c - di}{c - di} \\
&= \frac{(a + bi)(c - di)}{(c + di)(c - di)} \\
&= \frac{(a + bi)(c - di)}{c^2 + d^2}
\end{aligned}
$$

Quadratic Equations

When you used the quadratic formula to find solutions of a quadratic equation, you learned that if the discriminant is negative, the equation has no real solutions. But now you can find solutions that are complex numbers.

For example, to solve $x^2 + 2x + 5 = 0$:

1. Find the values for a, b, and c.

   $a = 1,\ b = 2,\ c = 5$

2. Substitute these values into the quadratic formula.

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}$$
$$= \frac{-2 \pm \sqrt{4 - 20}}{2}$$
$$= \frac{-2 \pm \sqrt{-16}}{2}$$
$$= \frac{-2 \pm 4i}{2}$$
$$= -1 \pm 2i$$

So the solutions of $x^2 + 2x + 5 = 0$ are $x = -1 \pm 2i$.

Notice that these two solutions, $x = -1 + 2i$ and $x = -1 - 2i$, are complex conjugates. If one solution of a quadratic equation $ax^2 + bx + c = 0$ (a, b, and c real numbers) is a complex number, the other solution is its complex conjugate.

You can solve the quadratic equation $ax^2 + bx + c = 0$ using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. You can use the sign of the discriminant, $b^2 - 4ac$, to determine the nature of the solutions.

Since the discriminant of $x^2 + 2x + 5 = 0$ is –16, the equation has two imaginary solutions.

Answers to Sample Problems

1\.
a. $(2 + 12) + (7 - 4)i$

b. $14 + 3i$

2\.
a. $(8 - 5) + [-3 - (-2)]i$

b. $3 - i$

3\.
a. $21 - 42i + 12i - 24i^2$

b. $45 - 30i$

4\.
c. $\frac{21 + 3i}{5}$ or $\frac{21}{5} + \frac{3}{5}i$

5\.
a. 2

3

7

b. $\frac{-3 \pm i\sqrt{47}}{4}$

c. $\frac{-3 + i\sqrt{47}}{4}, \frac{-3 - i\sqrt{47}}{4}$

or $-\frac{3}{4} + \frac{\sqrt{47}}{4}i, -\frac{3}{4} - \frac{\sqrt{47}}{4}i$

Sample Problems

1. Find: $(2 + 7i) + (12 - 4i)$

   ☐ a. Add the real parts and add the imaginary parts. = ____________

   ☐ b. Simplify. = ____________

2. Find: $(8 - 3i) - (5 - 2i)$

   ☐ a. Subtract the real parts and subtract the imaginary parts. = ____________

   ☐ b. Simplify. = ____________

3. Find: $(7 + 4i)(3 - 6i)$

   ☐ a. Multiply using FOIL. = ________________

   ☐ b. Simplify. = ________________

4. Find: $(3 + 9i) \div (1 + 2i)$

   ✔ a. Rewrite as a fraction. $= \frac{3 + 9i}{1 + 2i}$

   ✔ b. Multiply the numerator and denominator by $(1 - 2i)$. $= \frac{3 + 9i}{1 + 2i} \cdot \frac{1 - 2i}{1 - 2i}$

   ☐ c. Simplify. = ____________

5. Solve for x: $2x^2 + 3x + 7 = 0$

   ☐ a. Find the values for a, b, and c.
   $a =$ _____
   $b =$ _____
   $c =$ _____

   ☐ b. Substitute the values for a, b, and c into the quadratic formula, and simplify.
   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
   $x =$ ____________

   ☐ c. Write the solutions. $x =$ _________ or $x =$ _________

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Complex Number System

1. Circle the complex numbers below that are equal to $7 - 4i$.

   $4i - 7$ $7 - (2 - 6)i$

   $-4i + 7$ $7 - 3i - i$

   $6 + 1 - 4i$

2. Circle the pairs below that are complex conjugates of each other.

   $2 + 3i$ and $2 - 3i$ $9 + 7i$ and $7 + 9i$

   $5 + 3i$ and $3 - 5i$ $4 + 6i$ and $4 - 6i$

   $8 + i$ and $-8 - i$

3. Simplify each expression below.

   a. $\sqrt{-16}$

   b. $\sqrt{-25}$

   c. $\sqrt{-16} + \sqrt{-25}$

   d. $\sqrt{-16} \cdot \sqrt{-25}$

   e. i^{20}

   f. i^{47}

In problems (4) – (12), perform the indicated operation.

4a. $(5 + 7i) + (2 + 4i)$

b. $(6 + i) + (8 + 3i)$

5a. $(10 + 6i) + (7 - 4i)$

b. $(8 + 9i) - (5 + 2i)$

c. $3i(11 - 4i)$

6a. $(2 + 7i)(14 - i)$

b. $(4 - 12i)(5 - 3i)$

c. $(5 + 6i) \div (2 + 7i)$

7a. $(6 + 11i)(6 - 11i)$

b. $(7 + 12i) \div (9 - 4i)$

c. Solve for x: $x^2 + 4x + 8 = 0$

8a. $(3 + 10i) \div (8 - 5i)$

b. $(9 + 5i) \div (7 - 2i)$

c. Solve for x: $x^2 + 6x + 15 = 0$

9a. $(2 + 4i)(5 - 6i)$

b. $(1 + 2i) \div (1 - 2i)$

10a. Solve for x: $x^2 - 4 = 0$

b. Solve for x: $x^2 + 4 = 0$

c. $(1 - 2i) \div (1 + 2i)$

11a. $(2i + 7)(3 - 8i)$

b. $(6 + 5i) \div (1 - 4i)$

c. Solve for x: $x^2 + 2x + 9 = 0$

12a. $(4 + 7i) \div (3 + 5i)$

b. Solve for x: $x^2 - 4x + 11 = 0$

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Complex Number System

1. Simplify: $\sqrt{-169}$
2. Simplify: $-\sqrt{-144}$
3. Simplify: $\sqrt{16} - \sqrt{-25}$
4. Simplify: $-\sqrt{9} + \sqrt{-16}$
5. Simplify: i^{125}
6. Simplify: i^{234}
7. Simplify: i^{100}
8. Simplify: $\sqrt{-7} \cdot \sqrt{-16}$
9. Simplify: $\sqrt{-5} \cdot \sqrt{-10}$
10. Simplify: $\sqrt{-6} \cdot \sqrt{-4}$
11. Find: $(7 + \sqrt{-9}) + (12 - \sqrt{-36})$
12. Find: $(10 - \sqrt{-121}) - (3 + \sqrt{-25})$
13. Find: $(5 + \sqrt{-4}) + (8 - \sqrt{-49})$
14. Find: $5(3 - 6i) + 3(7 + 2i)$
15. Find: $4(8 + 3i) - 2(7 + 9i)$
16. Find: $3(4 + 3i) - 6(1 + 2i)$
17. Find: $(3 + 7i)(2 + 3i)$
18. Find: $(4 - 7i)(6 - 9i)$
19. Find: $(2 - 5i)(6 + 4i)$
20. $(4 - 7i)(4 + 7i)$
21. $(8 + 3i)(8 - 3i)$
22. Find: $(5 - 3i)(5 + 3i)$
23. Find: $(4 + 5i) \div (2 + 7i)$
24. Find: $(3 - 2i) \div (4 + i)$
25. Find: $(5 - 2i) \div (3 + 4i)$
26. Solve for x: $x^2 - 5x + 8 = 0$
27. Solve for x: $x^2 + 8x + 18 = 0$
28. Solve for x: $x^2 + 6x + 11 = 0$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Find:

   a. $(7 + 2i) + (4 + 6i)$

   b. $(7 + 2i) - (4 + 6i)$

2. Circle the expressions below that are equal to $8 + 4i$.

   $(2i)(4i + 2)$

   $3i + 6i - i$

   $-8i^2 - 4i^3$

   $\sqrt{-64}$

3. Find: $(3 + \sqrt{-16}) + (2 + \sqrt{-9})$

4. Find: $(4 + 2i)(3 + 5i)$

5. Find: $(5 + 3i)(5 - 3i)$

6. Find: $2 \div (4 + 7i)$

7. Circle the expressions below that are equal to i.

   i^4

   i^{37}

   $(i^5)^{10}$

   $-i^3$

8. Use the quadratic formula to find the solutions of each equation below.

   a. $x^2 + 3x + 7 = 0$

   b. $x^2 - 5x + 9 = 0$

   c. $3x^2 + 2x + 1 = 0$

TOPIC 10 CUMULATIVE ACTIVITIES

CUMULATIVE REVIEW PROBLEMS

These problems combine all of the material you have covered so far in this course. You may want to test your understanding of this material before you move on to the next topic, or you may wish to do these problems to review for a test.

1. Factor: $3b + 6bx + 10x + 5$

2. Solve for x: $|3x + 4| = 10$

3. Find: $(xy^3 - 5x^2y + 11xy - 1) + (4xy^3 - 7 - x^2y + 3xy)$

4. Solve $x^2 - 11x + 30 = 0$ for x.

5. Graph the line that passes through the point (–2, –4) with slope 1.

6. Factor: $5x^2 + 9xy - 2y^2$

7. Solve $\frac{1}{x-3} = \frac{x}{4}$ for x.

8. Simplify the expressions below.

   a. $\sqrt{\frac{81x^4}{16}}$

   b. $\left(9^{\frac{5}{2}}\right)(3^{-2})$

   c. $\sqrt[3]{x^6}$

9. Solve $y^2 + 1 = -3$ for y.

10. Find:

    a. $\frac{a^2 \cdot c}{a^5 \cdot b^6 \cdot c^4}$

    b. $6x^0 \cdot y^4$

    c. $(7a^0)^2$

11. Solve for x: $|x + 5| \le 7$

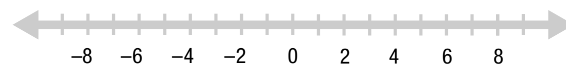
12. Factor: $x^2 - 14x + 13$

13. Find:

    a. $(7 + 4i)(1 - 5i)$

    b. $(6 - 2i)(6 + 2i)$

14. Solve $-1 < 2x - 3 \le 4$ for x, then graph its solution on the number line below.



15. Find the equation of the line that passes through the point (11, –4) that has slope $-\frac{6}{5}$:

    a. in point-slope form.

    b. in slope-intercept form.

    c. in standard form.

16. Solve for x: $2(5 + x) = 2x - 4$

17. Evaluate the expression $5xy + 1 - 4xy^3 - x^2$ when $x = 3$ and $y = -2$.

18. Find:

    a. $3 \div (2 + 5i)$

    b. $(8 + 4i) \div (6 - 3i)$

19. Solve $2y^2 - 17y + 21 = 0$ for y.

20. Graph the line $x - \frac{4}{3}y = 6$.

21. Find: $\frac{1}{2^{-3} + 4^{-2}}$

22. Factor: $3x^2y + 36xy + 108y$

23. Simplify this expression: $w^2xy - w^2x + 4w^2xy + y - 5w^2x$

24a. Find the slope of the line through the points (–14, 8) and (–2, 5).

b. Write the equation of the line passing through the point (–4, 6) that is parallel to the line in (a).

25. Find the slope of the line through the points (28, 4) and (–19, –12).

26. Rewrite the expressions below using only positive exponents. Then simplify.

a. $\dfrac{3^{\frac{4}{3}}x^{-\frac{3}{2}}}{3^{-\frac{2}{3}}x^{-\frac{5}{2}}}$

b. $\dfrac{2^{\frac{1}{3}}x^{-\frac{4}{3}}}{2^{-\frac{1}{9}}x^{\frac{2}{3}}}$

27. Solve $\frac{5}{x} + \frac{7}{6} = \frac{4}{x-4}$ for x.

28. Solve $5y^2 - 16y + 4 = 0$ for y.

29. Solve for x: $|2x + 7| - 4 = 8$

30. Factor: $12xy + 18y - 2x - 3$

31. Find: $\left(\frac{2}{5}x^2 - 3x\right)\left(5x - \frac{7}{3}\right)$

32. For what values of x is the expression $\frac{2}{x^2 - 25}$ undefined?

33. Solve $3x^2 + 7x + 4 = 0$ for x.

34. Solve $\frac{3}{2x} + \frac{1}{6x} = \frac{4}{5}$ for x.

35. Factor: $25y^2 - 30y + 9$

36. Solve for x: $|2x - 3| > 6$

37. Circle the true statements.

$2\frac{6}{7} + 5\frac{1}{4} = 7\frac{7}{11}$

$|11 - 5| = 6$

$\frac{5}{3} \div \frac{2}{3} = \frac{10}{9}$

$8 + 4(3 + 6) = 12(3 + 6)$

The GCF of 64 and 81 is 1.

If $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then $16 \notin S$.

38. Evaluate the expression $5a^3 + 4a^2b - b^2$ when $a = -3$ and $b = -2$.

39. Find: $(2a^4 + 5b^2)(2a^4 - 5b^2)$

40. Rewrite the expressions below without using radicals or exponents.

a. $\sqrt{-81}$

b. $\sqrt[3]{-64}$

c. $\sqrt{0.09}$

d. $4^{\frac{3}{2}}$

41. Solve $16x^2 - 16x + 1 = 0$ for x.

42. Factor: $5x^2 + 9xy - 2y^2$

43. Solve $-17 \leq 6y + 7 < 17$ for y.

44. Find:

a. $(3 + 5i) + (4 - 7i)$

b. $(8 - 2i) - (3 - 6i)$

45. Find the x- and y-intercepts of the line $4x - 3y = 5$.

46. Solve for a: $\frac{6}{a-2} + 1 = \frac{1}{a}$

47. Factor: $x^4 - y^2$

48. Find: $(6x + y)^2$

49. Solve $3x^2 - 5x - 22 = 0$ for x.

50. Find:

a. $7^3 \cdot 7^5$

b. $(5x^4)^3$

c. $\frac{b^{21}}{b^{24}}$