# 15.9: Change of Variables in multiple integrals.



$$T: \underbrace{(u,v)}_{\text{old variables}} \longmapsto \underbrace{(x,y)}_{\text{new variables}}$$

we might express this as

$$x(u,v) = x \quad \& \quad y(u,v) = y.$$

Ex1: Find the image of the triangular region $S$ w/vertices $(0,0)$, $(1,1)$, & $(0,1)$ under the transformation $x = u^2$; $y = v$.

VOCAB:

image ... sort of like the range.

one-to-one ... No two pts have the same image.

Go thru the derivation of the Jacobian in the book

Dfn: The Jacobian of the transformation $T(u,v) \mapsto (x,y)$ is

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\cdot\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\cdot\frac{\partial y}{\partial u}$$

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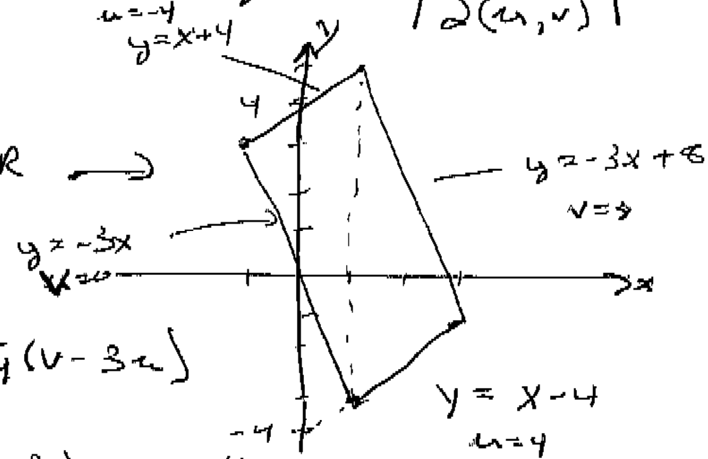
**Dfn:** Change of variables in Double Integrals

Suppose that $T$ is a $C^1$ transformation whose Jacobian is nonzero & that maps a region $S$ in the $uv$-plane onto a region $R$ in the $xy$-plane. Suppose that $f$ is cont. on $R$ & that $R$ & $S$ are type I or II plane regions. Suppose also that $T$ is 1-1, except perhaps on the boundary of $S$. Then

$$\iint_R f(x,y)\,dA = \iint_S f(x(u,v),\, y(u,v)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\,dv$$




already transformed

**Ex2:** $\iint_R 4x+8y\,dA$ where $R \longrightarrow$

and

$x = \frac{1}{4}(u+v)$ & $y = \frac{1}{4}(v-3u)$

$$\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{4}\cdot\frac{1}{4} - \frac{1}{4}\left(-\frac{3}{4}\right) = \frac{4}{16} = \frac{1}{4}$$

$$\iint \underbrace{(4x+8y)}\,\frac{1}{4}\,d\cancel{x}\,\cancel{dy}\,dy$$

$$\frac{1}{4}\cdot 4(u+v) + 8\left(\frac{1}{4}(v-3u)\right) = u+v+2v-6u$$
$$= -5u+3v$$

$$\frac{1}{4}\int_0^8\int_{-4}^4 (3v-5u)\,\cancel{dudv}\,du\,dv$$

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why...

$$\iint_R \frac{x-2y}{3x-y}\,dA$$

$$\iint_R (x+y)\,e^{x^2-y^2}\,dA$$

$u = x+y$
$v = x-y$

$$\iint_R \cos\left(\frac{y-x}{y+x}\right)dA$$

$$\iint_R e^{x+y}\,dA$$