15.2: Iterated Integrals.

Overview – f, a fcn of 2 variables is integrable over $R = [a,b] \times [c,d]$

– $\int_c^d f(x,y)\,dy$ means x is fixed and f is integrated WRT y.

– this is partial integration WRT y.

so $A(x) = \int_c^d f(x,y)\,dy$.

AND
$$\int_a^b A(x)\,dx = \int_a^b \left[\int_c^d f(x,y)\,dy\right] dx$$
$$= \int_a^b \int_c^d f(x,y)\,dy\,dx$$

this is called an iterated integral.

similarly
$$\int_c^d \left[\int_a^b f(x,y)\,dx\right] dy = \int_c^d \int_a^b f(x,y)\,dx\,dy.$$

NOTE: we work from the inside out.

Ex1: calculate $\int_0^1 \int_1^2 (4x^3 - 9x^2y^2)\,dy\,dx$.

and $\int_1^2 \int_0^1 (4x^3 - 9x^2y^2)\,dx\,dy$.

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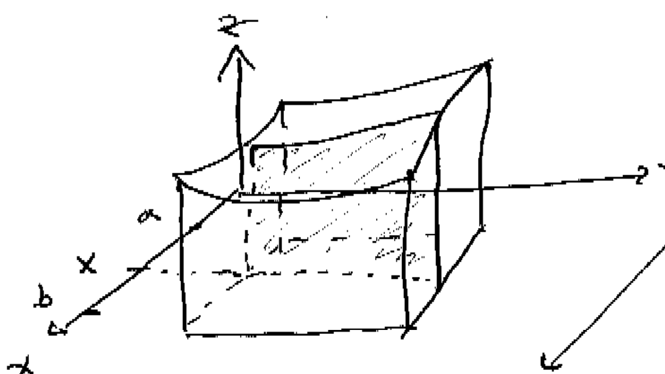
Fubini's thm: If f is cont on the rectangle $R = [a,b] \times [c,d]$, then

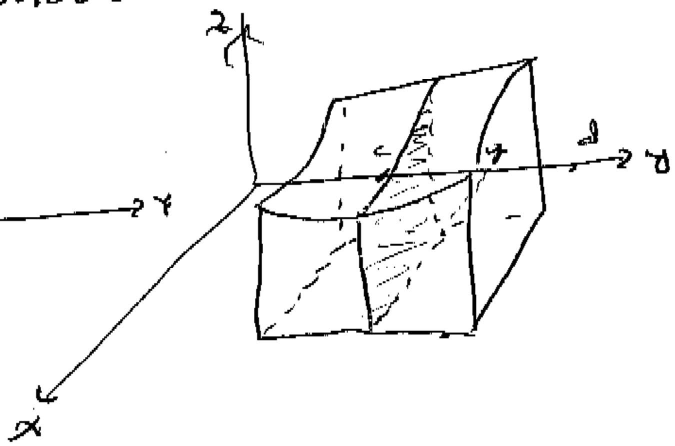
$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy.$$

more generally, this holds if f is bounded on R, f is discont only on a finite number of smooth curves, and the iterated integral exists.

Q: what if f is not bounded?

This result should be intuitive.





Ex2: Calculate $\iint_R x\cos(x+y)\,dA$ over
$R = [0, \pi/6] \times [0, \pi/3]$

a) WRT y then x

b) WRT x then y.

A handy trick:

$$\iint_R g(x)h(y)\,dA = \int_a^b g(x)\,dx \cdot \int_c^d h(y)\,dy$$

where $R = [a,b] \times [c,d]$

Q: why?

Look @ examples in text... see when this applies.

Ex 3: $\iint_R \frac{x}{1+xy}\,dA$ over $R = [0,1] \times [0,1]$

$$= \int_0^1 \int_0^1 \frac{x}{1+xy}\,dy\,dx$$

$$= \int_0^1 \left[\frac{x}{x} \ln|1+xy|\right]_0^1 dx$$

$$= \int_0^1 \ln|1+x|\,dx$$

$$= \int_0^1 \ln(1+x)\,dx$$

$$= (x+1)\ln(x+1) - (1+x)\Big|_0^1 \quad (2\ln(2) - 2) - (-1)$$

$\cancel{(-1)} =$