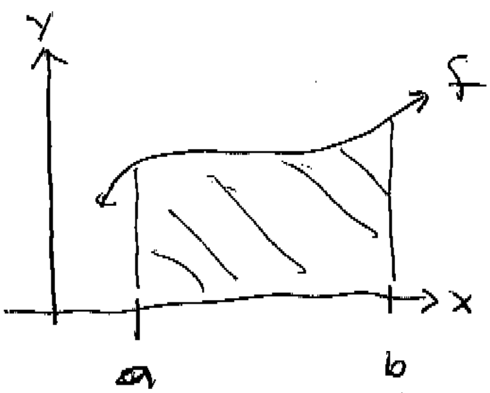
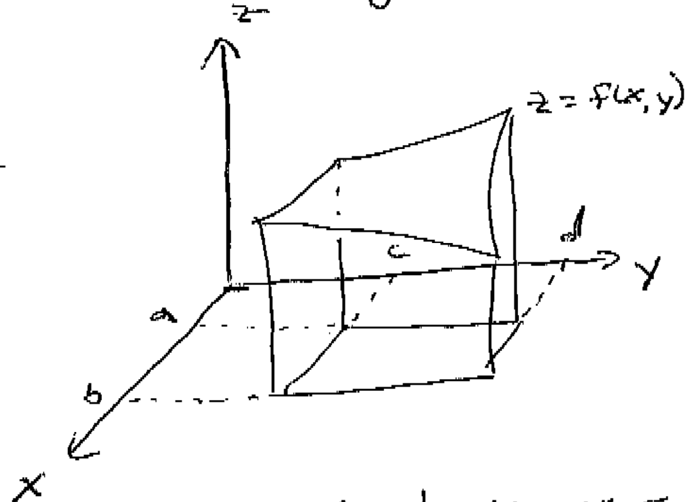
15.1: Double Integrals over rectangles

This section is analogous to sec 5.2 on the definite Integral.





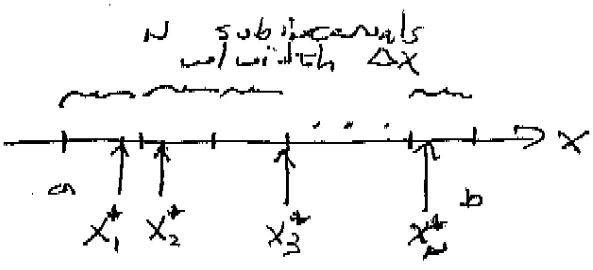
Find the AREA (if $f > 0$)

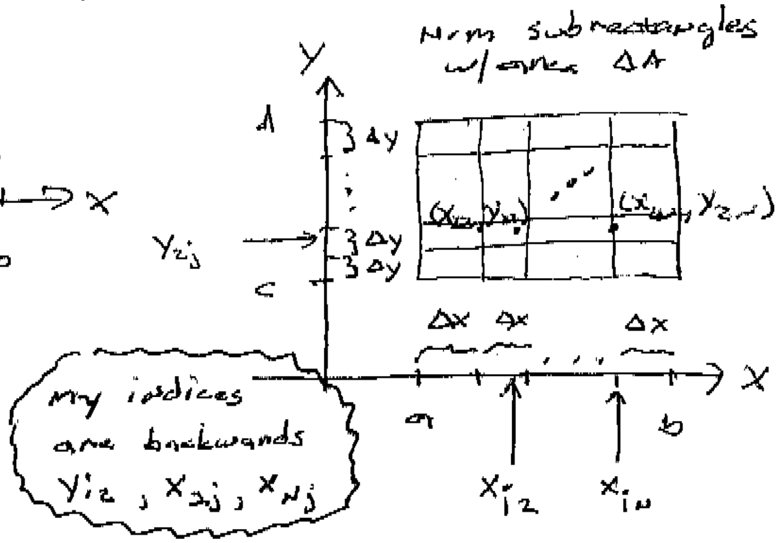
Find the VOLUME (if $f > 0$).

Assuming that our functions are well chosen & the region of integration is partitioned correctly.

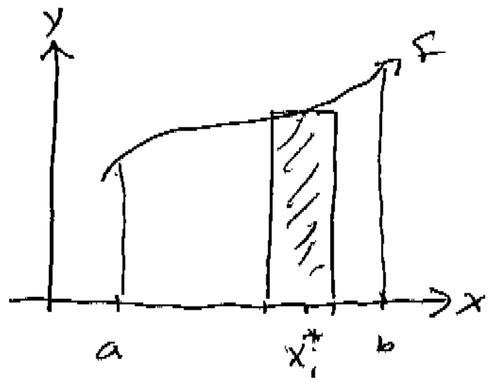
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \qquad \iint_R f(x,y)\,dA = \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$$

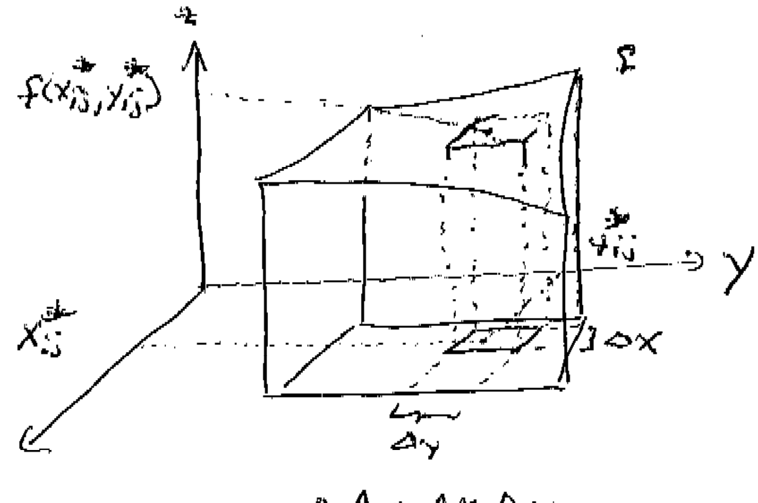
we can break up our domains





Then, we approx the area/volume w/ rectangle/boxes





$\Delta A = \Delta x \Delta y.$

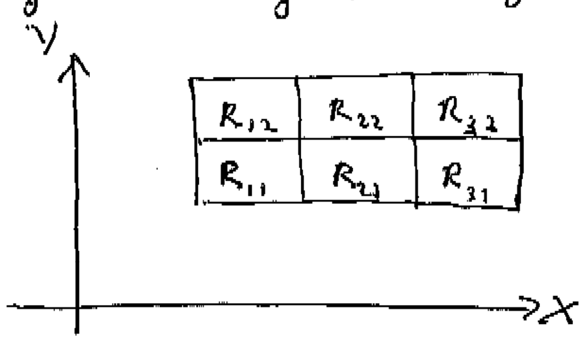
evaluating the limit (if it exists) increases the number of rectangles/boxes $\longrightarrow \infty$ and, the limit provides us w/the exact area/volume.

Reading double sums

$$\sum_{i=1}^{3} \sum_{j=1}^{2} A_{ij} \Delta A = \sum_{i=1}^{3} (A_{i1} + A_{i2}) \Delta A$$

$$= \Big[(A_{11} + A_{12}) + (A_{21} + A_{22}) + (A_{31} + A_{32}) \Big] \Delta A$$

Labeling rectangular regions



the 1st index gives "x" & the second the "y"

Ex1: (A) Estimate the volume below $z = x + 2y^2$ over $R = [0,2] \times [0,4]$ w/ Riemann sums and $m = n = 2$. Use sample points in the lower left.

(B) Use the midpoint rule to estimate the same volume.

(c) use the midpoint rule to estimate the average height of the fct.

Ex2: Find $\iint_R \sin(x+y)\,dA$ by first identifying the volume as a solid where $R = [0, \pi] \times [0, \pi]$