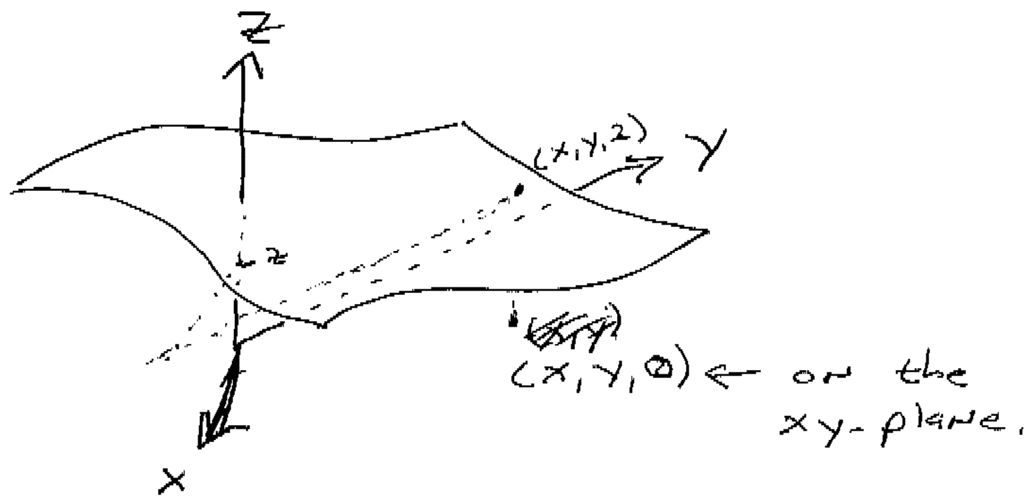
14.1: Fcts of several Variables

Dfn: A fct of two variables assigns each ordered pair (x,y) in its domain a unique output in its range.

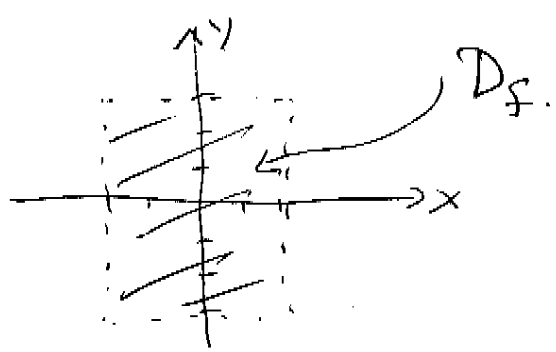
$$f : (x,y) \longmapsto z$$

Graphically, our fct is represented by a surface in $\mathbb{R}^3$ (assuming $x, y, z \in \mathbb{R}$)



Ex 1: Find the domain of $f(x,y) = \sqrt{4-x^2} + \sqrt{9-x^2}$

$4 - x^2 \geq 0 \Rightarrow -2 \leq x \leq 2$ also $-3 \leq y \leq 3$



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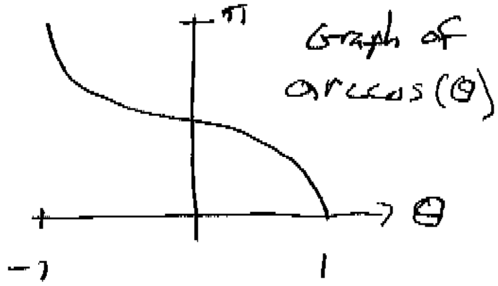
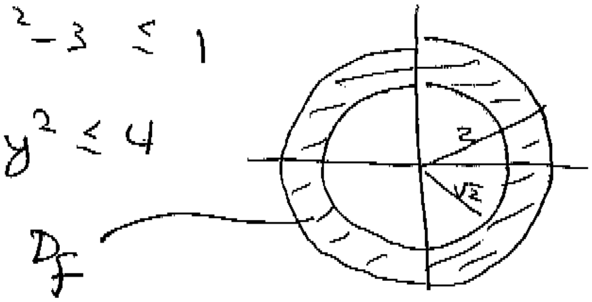
Ex2: Find the domain of $f(x,y) = \arccos(x^2+y^2-3)$



Graph of $\arccos(\theta)$

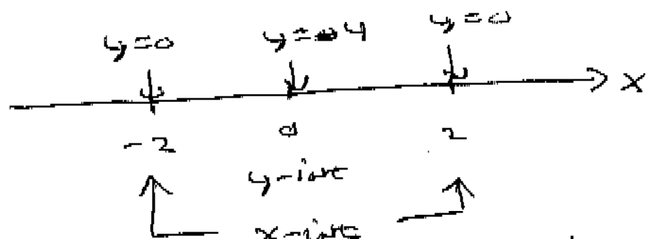
$-1 \le x^2+y^2-3 \le 1$

$\Rightarrow 2 \le x^2+y^2 \le 4$



Graphs are sweet, but not always easy to obtain (model the ocean's floor).

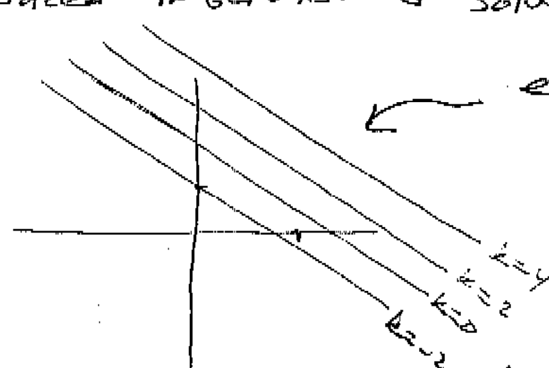
One tool we use to understand fcts is the contour plot. By analogy, if we had $y = x^2 - 4$, we might ask when $y = 0$ or $y = -4$.



Notice that we fixed y (the output) and solved for the inputs.

Ex3: Construct a contour plot of $g(x,y) = x + 2y - 7$. We must fix $z = k$ @ equally spaced intervals & solve $k = x+2y-7$

k	contour line
-2	$y = (5-x)/2$
0	$y = (7-x)/2$
2	$y = (9-x)/2$
4	$y = (11-x)/2$



equally spaced parallel lines.

And the surface is a... plane.

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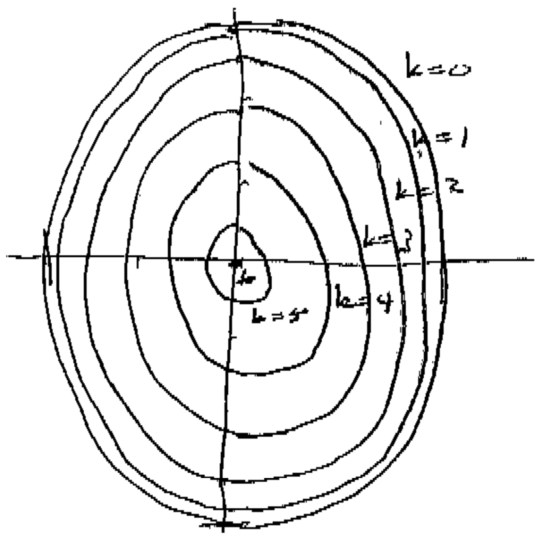
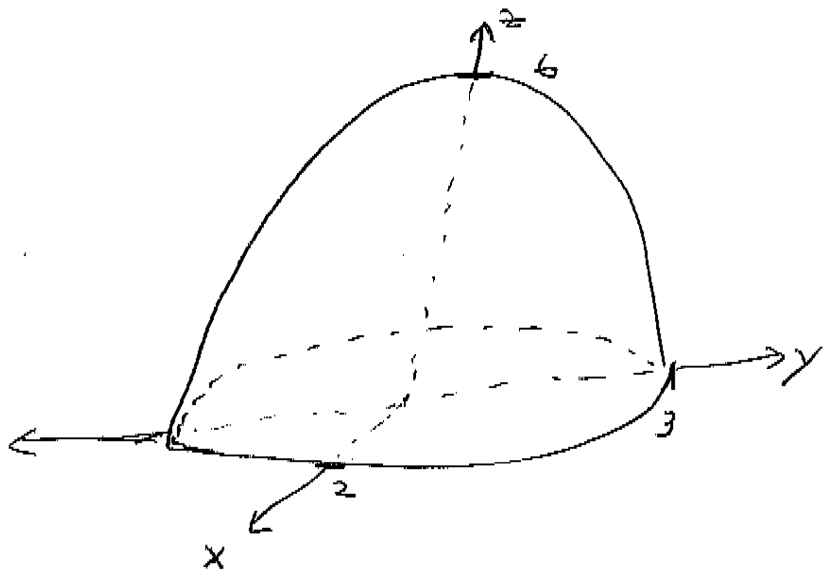
Ex 4: sketch a contour map of $h(x,y) = \sqrt{36-9x^2-4y}$

k	
<0	not possible
0	solve $0 = 36-9x^2-4y^2 \Rightarrow 1 = \frac{x^2}{4} + \frac{y^2}{9}$
1	solve $1 = 36-9x^2+4y^2 \Rightarrow 1 = \frac{x^2}{35/9} + \frac{y^2}{35/4}$
2	solve $4 = 36-9x^2+4y^2 \Rightarrow 1 = \frac{x^2}{32/9} + \frac{y^2}{32/4}$
$\vdots$	
6	$(0,0)$



what does this look like?



Ex 5: The matching game (p794)

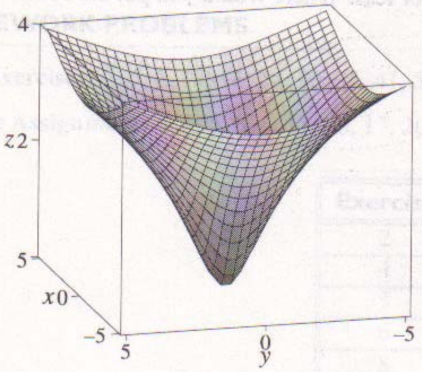
Ex 6: Dali's Target (p795).

The Matching Game

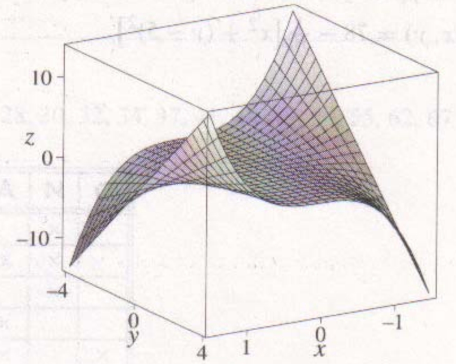
Match each function with its graph. Give reasons for your choices.

1. $f(x,y) = \dfrac{1}{x+1} + \sin y$ **2.** $f(x,y) = \sqrt{4 - x^2 - y^2}$ **3.** $f(x,y) = \cos\left(x + y^2\right)$

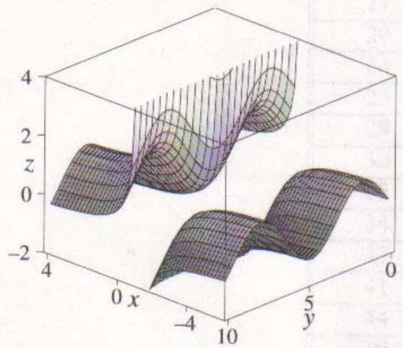
4. $f(x,y) = \ln\left(x^2 + y^2 + 1\right)$ **5.** $f(x,y) = x^2\sqrt{y}$ **6.** $f(x,y) = x^3 y$



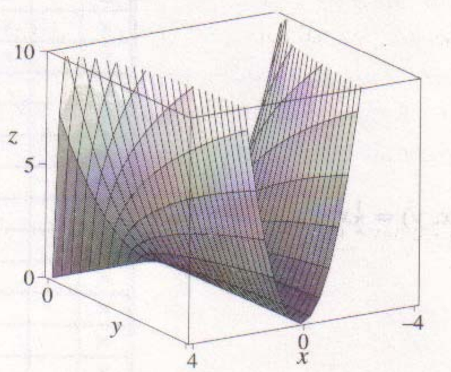
I



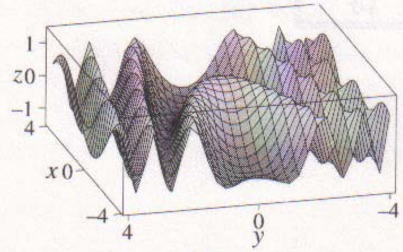
II



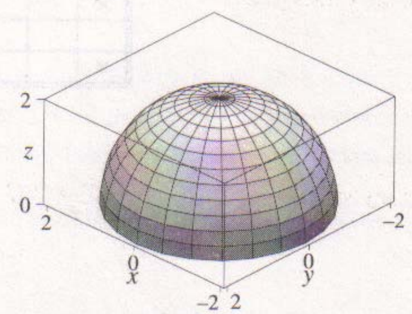
III



IV



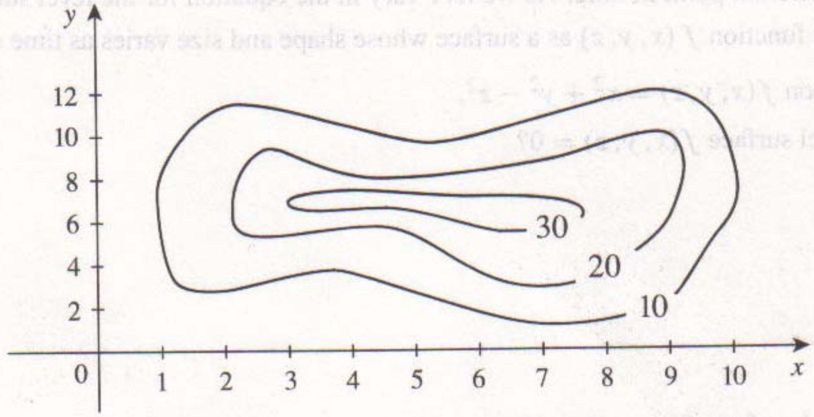
V



VI

Dali's Target

Consider the following contour map of a continuous function $f(x, y)$:



1. For approximately what values of y is it true that $10 \le f(5, y) \le 30$?

2. What can you estimate $f(2, 4)$ to be, and why?

3. Do we have any good estimates for $f(5, 8)$? Explain.

4. How many values y satisfy $f(7, y) = 20$?

5. How many values of x satisfy $f(x, 8) = 20$?