Test 4
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Math 111 – Fall 2009

Name: key

It appears to me that if one wishes to make progress in mathematics, one should study the masters and not the pupils.

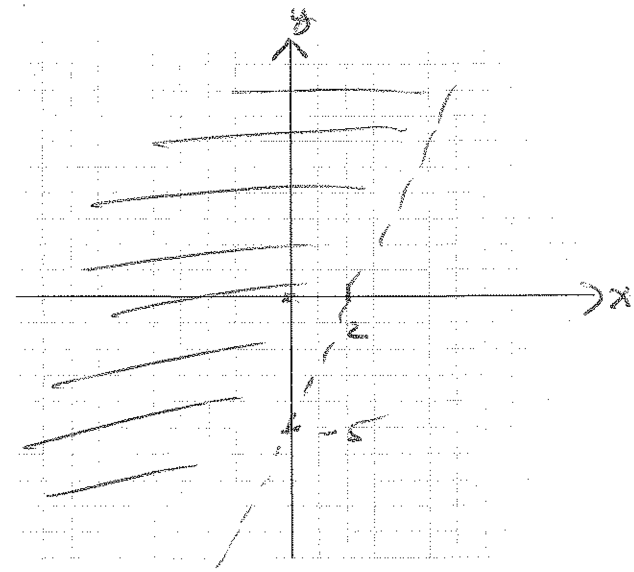
Niels Abel (1802 - 1829)
Norse mathematician

No work = no credit

Closed Book & Closed Notes

Warm-ups (1 pt each) $e^0 =$ 1 $\frac{e}{0} =$ undefined $\frac{0}{e} =$ 0

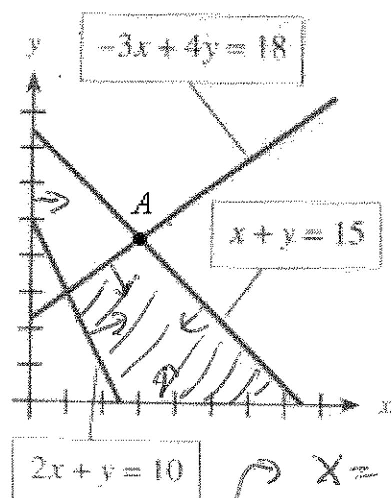
1.) (4 pts) Carefully graph the inequality $10 > 5x - 2y$



2.) (4 pts) The graph of the boundary equations for the system of inequalities is shown.

$$\begin{cases} -3x + 4y \le 18 \\ 2x + y \ge 10 \\ x + y \le 15 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

a.) Locate the solution region and clearly shade it.



b.) Find the coordinates of point A.

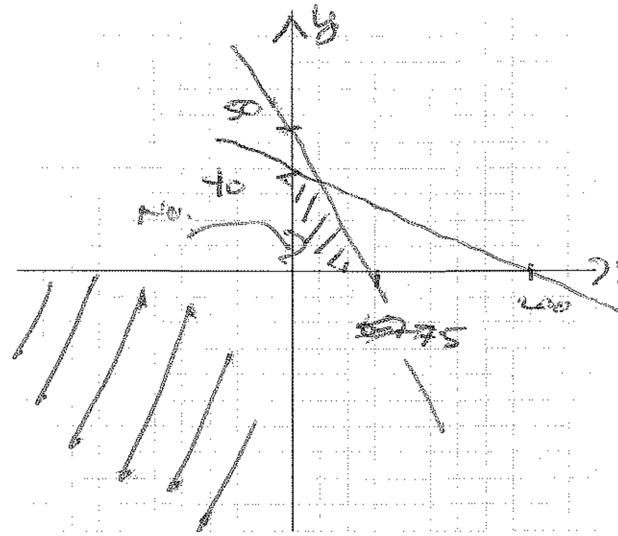
Solution: (6, 9)

$x + y = 15$
$-3x + 4y = 18$

$\Rightarrow x = 15 - y$
$\Rightarrow -3(15 - y) + 4y = 18$
$\Rightarrow -45 + 3y + 4y = 18$
$\Rightarrow 7y = 63$
$\Rightarrow y = 9$
$\Rightarrow x = 6$

4/4

3.) (4 pts) Graph the solution of the system of inequalities: $\begin{cases} x+5y \le 200 \\ 2x+3y \le \cancel{134}\ 150 \\ x \le 0 \\ y \le 0 \end{cases}$



4.) (4 pts) A company manufactures two types of electric hedge trimmers, one of which is cordless. The cord-type trimmer requires 2 hours to make, and the cordless model requires 4 hours. The company has only 800 work hours to use in manufacturing each day, and the packaging department can package only 300 trimmers per day.

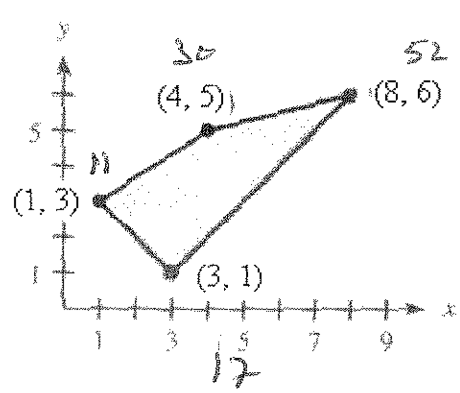
If x = the number of corded trimmers produced and y = the number of cordless trimmers produced, write the system of inequalities that describes the constraints on the number of each type of hedge trimmer produced. There are four constraints (of which, two are trivial).

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ x + y \le 300 \\ 2x + 4y \le 800 \end{cases}$$

5.) (4 pts) Find the maximum and minimum of the objective function $C = 5x + 2y$ over the given feasible region.

a.) Max: 52 at (8,6)

b.) Min: 11 at (1,3)



6.) (4 pts) A candidate wishes to use a combination of radio and television advertisements in her campaign. Research has shown that each 1-minute spot on television reaches 0.09 million people and each 1-minute spot on radio reaches 0.006 million. The candidate feels she must reach at least 2.16 million people, and she must buy a total of at least 80 minutes of advertisement.

Write the inequalities that relate the number of each type of advertising to her needs.

t = # of min of ~~radio~~ tv

r = # of min of radio

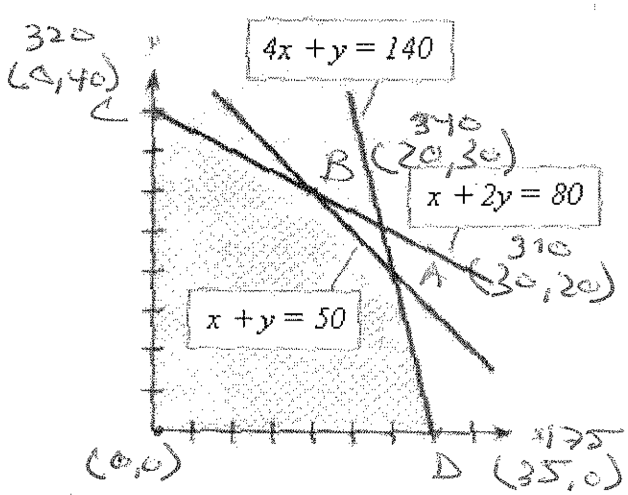
$$\begin{cases} t \geq 0 \; ; \; r \geq 0 \\ t + r \geq 80 \\ 0.09t + 0.006r \geq 2.16 \end{cases}$$

7.) (4 pts) The graph of the feasible region is given. Find the corners of the feasible region, and then find the maximum and minimum of the objective function $f = 5x + 8y$.

Clearly label the corners on the graph.

a.) Max: 340 at (20,30)

b.) Min: 0 at (0,0)



A:

$4x + y = 140$

$x + y = 50$

$3x = 90$

$x = 30$

$y = 20$

B:

$x + y = 50$

$x + 2y = 80$

$y = 30$

$x = 20$

8.) (4 pts) Maximize $f = x + 3y$ subject to $\begin{cases} x + 4y \le 12 \\ x + y \le 9 \\ x \ge 0 \\ y \ge 0 \end{cases}$

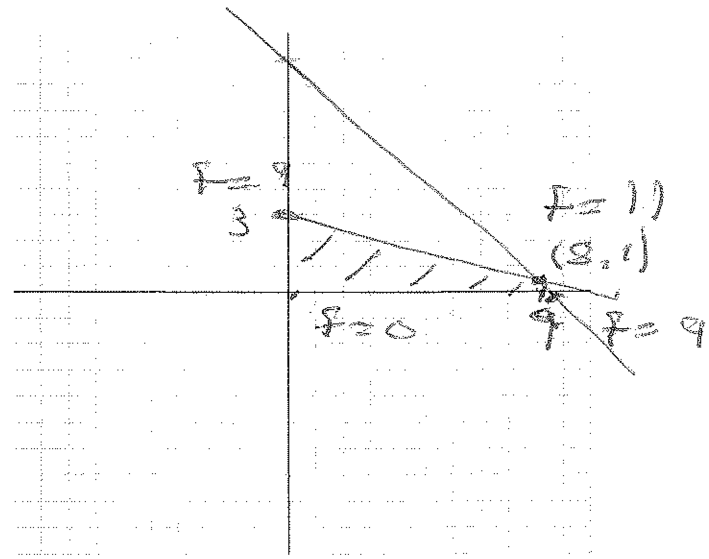
$x + 4y = 12$

$x + y = 9$

$3y = 3$

$y = 1$

$x = 8$



Max: 11 at (8, 1)

9.) (4 pts) If the present value of a $6,000 investment pays a nominal interest rate of 7.5%, compounded continuously, what will be the future value of that investment in 20 years?

$6000e^{.075(20)} = 26890.13$

Future value: $26890.13

10.) (4 pts) If the future value of an investment in 30 years is \$150,000, what was the present value assuming the investment was compounded daily at 5.5% yearly interest.

$$150000 = P\left(1 + \frac{.055}{360}\right)^{360(30)}$$

$$P = \frac{150000}{\left(1 + \frac{.055}{360}\right)^{360(30)}}$$

$$\doteq 28811.12$$

Present value: \$28811.12

11.) (4 pts) Suppose you start with a present value of \$800, how long will it take to double in value if you are compounding the money monthly at a yearly interest rate of 6.5%?

$$1600 = 800\left(1 + \frac{.065}{12}\right)^{12t}$$

$$\Rightarrow \quad 2 = \left(1 + \frac{.065}{12}\right)^{12t}$$

$$\Rightarrow \quad 12t = \frac{\ln 2}{\ln\left(1 + \frac{.065}{12}\right)}$$

$$\Rightarrow \quad t = \frac{\ln 2}{12 \ln\left(1 + \frac{.065}{12}\right)}$$

$$\Rightarrow \quad t = 10.69$$

Number of years: 10.69 yrs.