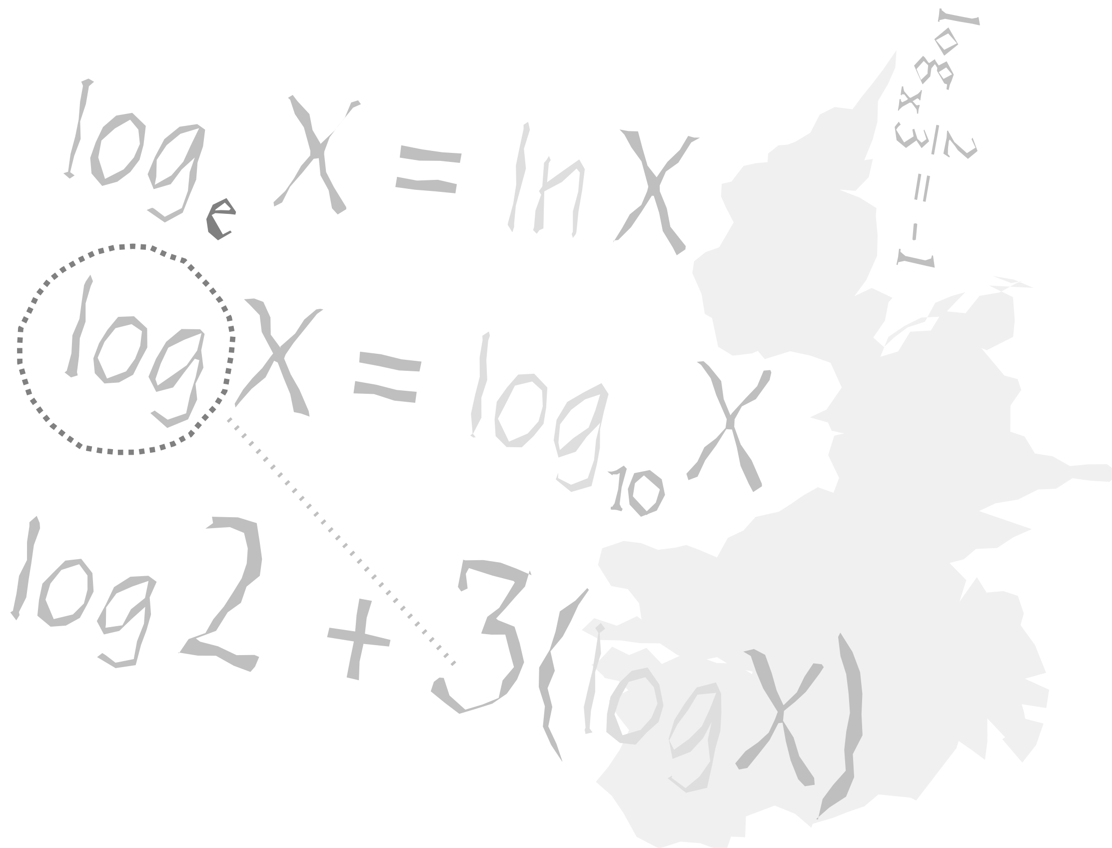
LESSON 12.3 – APPLICATIONS OF LOGS



OVERVIEW

Here's what you'll learn in this lesson:

Natural and Common Logs

a. Base e and natural logarithms

b. Finding logs in base 10 and powers of 10 using a calculator

c. Finding logs in base e and powers of e using a calculator

d. Change of base formula

Solving Equations

a. Solving logarithmic equations

b. Solving exponential equations

Have you ever wondered how to measure the loudness of a noise like an ambulance siren? Or pondered the difference between earthquakes of magnitudes 5 and 6 on the Richter scale? Or thought about how medical examiners can determine the time of someone's death? Well in this lesson, you'll learn more about logarithms, which can be used to investigate all of these things.

In particular, you'll learn about two kinds of logarithms that are useful in many applications: common logarithms and natural logarithms. In addition, you'll see how to use what you've already learned about logarithms to solve equations containing logarithms and to solve exponential equations.

EXPLAIN

NATURAL AND COMMON LOGS

Summary

Logarithms were invented by the Scottish mathematician John Napier (1550 – 1617) in the sixteenth century to make multiplication and division of large numbers easier. Using properties of logarithms, a multiplication or division problem can be replaced with an easier addition or subtraction problem. Today you can often use a calculator or computer to solve problems with large numbers. But you can also use logarithms with base 10 or base e to help you solve problems in the fields of chemistry, earth science, physics, economics, and environmental studies.

Notation for Common Logarithms

You have learned about logarithms such as $\log_{10}100$. Here, the base is 10. Logarithms with base 10 are called common logarithms and they were commonly used for computation in the base 10 number system.

Remember, $\log_{10} 100 = 2$ because $10^2 = 100$.

Here are some examples of common logarithms:

$\log_{10} 22$ $\quad$ $\log_{10} 311$ $\quad$ $\log_{10} 12$ $\quad$ $\log_{10} 1667$

Common logarithms are often written without the base. For example, the logarithms above can be written respectively as:

In general, $\log x = \log_{10} x$. (Here, $x > 0$.)

$\log 22$ $\quad$ $\log 311$ $\quad$ $\log 12$ $\quad$ $\log 1667$

Sometimes, you can find the value of common logarithms without the use of a calculator by switching from logarithmic to exponential notation.

To find the value of a common logarithm $\log b$ when b is a power of 10:
1. Set the logarithm equal to x.
2. Rewrite the log in exponential form.
3. Rewrite the result of (2) in the form $10^x = 10^n$, then solve for x.

For example, find the value of $\log_{10}1000$:

1. Set the logarithm equal to x.	$\log_{10} 1000 = x$
2. Rewrite in exponential form.	$10^x = 1000$
3. Rewrite in the form $10^x = 10^n$,	$10^x = 10^3$
then solve for x.	$x = 3$

So $\log_{10}1000 = 3$.

Finding Logs in Base 10 and Powers of 10 Using a Calculator

You can find the value of a common logarithm when you can't rewrite the exponential form of the logarithm in the form $10^x = 10^n$ by using a calculator.

To find the approximate value of the common logarithm log *a* using a calculator:
1. Enter the number *a* in the calculator.
2. Press the "log" key.
3. Round the result of (2) to the desired number of decimal places.

For example, find the value of $\log_{10}311$ rounded to three decimal places:

1. Enter 311 in the calculator.	
2. Press the "log" key.	2.4927604
3. Round to three decimal places.	2.493

So $\log_{10} 311 \approx 2.493$.

Remember you use the symbol "≈" to mean "is approximately equal to."

Sometimes, you are given the common logarithm of a number, and you want to find the number itself. You can work backwards to find this number using a calculator when necessary. In general, to find the number whose common logarithm is *k*:
1. Let *x* be the number you are trying to find, and let $\log_{10} x = k$.
2. Rewrite the equation in exponential form, $10^k = x$, and solve for *x* using a calculator if necessary.
3. If necessary, round your answer.

For example, suppose you want to find the number whose common logarithm is 4.

1. Substitute 4 for *k* in the equation $\log_{10} x = k$.	$\log_{10} x = 4$
2. Rewrite the equation in exponential and solve for *x*.	$10^4 = x$ $x = 10{,}000$

So 10,000 is the number whose common logarithm is 4.

Here's how to use your calculator to find $10^{-2.23}$ to two decimal places.

1. Enter 2.23, then press the "±" key.

2. Press the "10^x" key.

3. Round to two decimal places. So, $x \approx 0.01$.

Here's an example where you will need to use your calculator. To find an approximation of the number (to two decimal places) whose common logarithm is –2.23:

1. Substitute –2.23 for *k* in the equation $\log_{10} x = k$.	$\log_{10} x = -2.23$
2. Rewrite the equation in exponential form and solve for *x* using your calculator.	$10^{-2.23} = x$ $x \approx 0.0058884$
3. Round to two decimal places.	$x \approx 0.01$

So the number whose common logarithm is –2.23 is approximately 0.01.

Properties of Common Logarithms

You are already familiar with some properties of logarithms. For example, $\log_b b = 1$ since $b^1 = b$. (Here, the base b is a positive number and $b \neq 1$.) When the base b is 10, this property states that $\log_{10} 10 = 1$ or $\log 10 = 1$ because $10^1 = 10$.

Here are some other properties of logarithms written for the case when the base b is 10. Here $x > 0$, $u > 0$, and $v > 0$.

Name of Property	**Property**
Log of a Product	$\log uv = \log u + \log v$
Log of a Quotient	$\log \frac{u}{v} = \log u - \log v$ When $u = 1$, you have this special case: $\log \frac{1}{v} = -\log v$
Log of a Power	$\log u^n = n \cdot \log u$ When $u = 10$, you have this special case: $\log 10^n = n$
Other Properties	$\log 10 = 1$ $\log 1 = 0$ $10^{\log x} = x$

Here's an example of how you can use these properties to write $\log 5 + 3[\log (x - 1)]$ as a single logarithm:

$$\log 5 + 3[\log (x-1)]$$

1. Use the log of a power property. $\quad = \log 5 + \log (x-1)^3$
2. Use the log of a product property. $\quad = \log\left[5(x-1)^3\right]$

So $\log 5 + 3[\log (x - 1)]$ written as a single logarithm is $\log\left[5(x-1)^3\right]$.

Notation for Natural Logarithms

You have learned about common logarithms (logarithms with base $b = 10$). Now you will learn about logarithms with base $b = e$. These logarithms are called natural logarithms. Natural logs have many applications such as describing the growth of populations and the growth of money invested at compound interest.

Here are some examples of natural logarithms:

$\log_e 22 \qquad \log_e 311 \qquad \log_e 12 \qquad \log_e 1667$

Natural logarithms are often written without the base, and abbreviated "ln." For example, the logarithms above can be written respectively as:

$\ln 22 \qquad \ln 311 \qquad \ln 12 \qquad \ln 1667$

Recall that e is an irrational number that lies between 2 and 3 and is approximately equal to 2.718.

***Natural** logarithms get their name from the exponential function with base e, $y = e^x$, which is called the **natural** exponential function. This function arises **naturally** in many applications in the biological and social sciences.*

In general, $\ln x = \log_e x$. (Here, $x > 0$.) Sometimes it may help to rewrite $\ln x$ as $\log_e x$ to remind yourself that you're working with base e, especially when switching to exponential form without the use of a calculator.

Sometimes you can find the value of a natural logarithm without the use of a calculator by switching from logarithmic to exponential notation, just as you did with common logs.

To find the value of a natural logarithm ln b when b is a power of e:

1. Set the logarithm equal to x.

2. Rewrite the logarithm in exponential form $e^x = e^n$.

3. Solve for x.

For example, find the value of $\log_e e^3$ (which is the same as ln e^3):

1. Set the logarithm equal to x.	$\log_e e^3 = x$
2. Rewrite in exponential form, $e^x = e^n$.	$e^3 = e^x$ $e^x = e^3$
3. Solve for x.	$x = 3$

This is an application of $\log_b b^x = x$

So $\log_e e^3 = \ln e^3 = 3$.

Finding Logs in Base *e* and Powers of *e* Using a Calculator

What if you want to find the value of a natural logarithm and you can't rewrite the exponential form of the logarithm in the form $e^x = e^n$? For example, what if you want to find ln 35? If you set ln 35 = x and switch to exponential form, you end up with $e^x = 35$. But 35 isn't a power of e, so it can't be written easily in the form e^n. So how do you find x? You can't find the exact value for x, but you can find the approximate value using a calculator.

To find the approximate value of the natural logarithm $x = \log_e a$ (which is the same as ln a) using a calculator:

1. Enter the number a in the calculator.

2. Press the "ln" key.

3. Round the result of (2) to the desired number of decimal places.

For example, to find the value of ln 35 (which is the same as $\log_e 35$) rounded to two decimal places:

1. Enter 35 in the calculator.	
2. Press the "ln" key.	3.5553481
3. Round to two decimal places.	3.56

So ln 35 ≈ 3.56.

Sometimes, you are given the natural logarithm of a number, and you want to find the number itself. You can work backwards to find the approximate value of this number using a calculator. In general, to find the number whose natural logarithm is k:

1. Let x be the number you are trying to find, and let $\log_e x = k$.

2. Rewrite the equation in exponential form $e^k = x$, and solve for x using a calculator as follows:
   - Enter k in your calculator.
   - Press the "e^x" key.

3. If necessary, round your answer.

For example, to find the number (rounded to two decimal places) whose natural logarithm is 3.

1. Substitute 3 for k in the equation $\log_e x = k$.	$\log_e x = 3$
2. Rewrite the equation in expontial form and solve for x, using a calculator as follows:	$e^3 = x$
• Enter 3 in your calculator.	
• Press the "e^x" key.	$x \approx 20.085537$
3. Round your answer to two decimal places.	$x \approx 20.09$

So the number whose natural logarithm is 3 is approximately 20.09.

Properties of Natural Logarithms

You are already familiar with some properties of logarithms. For example, $\log_b b = 1$ since $b^1 = b$. (Here, the base b is a positive number and $b \neq 1$.) When the base b is e, this property states that $\log_e e = 1$ or $\ln e = 1$ because $e^1 = e$.

Here are some other properties of logarithms written for the case when the base b is e. Here $x > 0$, $u > 0$, and $v > 0$.

Name of Property	**Property**
Log of a Product	$\ln uv = \ln u + \ln v$
Log of a Quotient	$\ln \frac{u}{v} = \ln u - \ln v$ When $u = 1$, you have this special case: $\ln \frac{1}{v} = -\ln v$
Log of a Power	$\ln u^n = n \cdot \ln u$ When $u = e$, you have this special case: $\ln e^n = n$
Other Properties	$\ln e = 1$ $\ln 1 = 0$ $e^{\ln x} = x$

Here's an example of how you can use these properties to write $2\ln x + \ln(x+1)$ as a single logarithm:

$$2\ln x + \ln(x+1)$$

1. Use the log of a power property. $= \ln x^2 + \ln(x+1)$
2. Use the log of a product property. $= \ln\left[x^2(x+1)\right]$
3. Simplify. $= \ln\left(x^3 + x^2\right)$

So, $2\ln x + \ln(x+1)$ written as a single logarithm is $\ln\left(x^3 + x^2\right)$.

Change of Base Formula

So far you have learned how to find the value of a logarithm with base 10 or base e, and you've seen how to use your calculator to do so. Now you will learn how to find the value of a logarithm with any base b (where $b > 0$ and $b \neq 1$) using a formula called the "change of base formula," which states the following:

$$\log_b x = \frac{\log_c x}{\log_c b}$$

Here, c is any number greater than 0 and not equal to 1.

Because you already know how to find natural logarithms with the "ln" key on your calculator, it is convenient to choose $c = e$. So the formula becomes:

$$\log_b x = \frac{\log_e x}{\log_e b} = \frac{\ln x}{\ln b}$$

In general, to find $\log_b x$ using the change of base formula:

1. Substitute values for b and x in the formula $\log_b x = \frac{\log_e x}{\log_e b} = \frac{\ln x}{\ln b}$.
2. Simplify using your calculator if necessary.
3. Round as required.

Here's an example. To use your calculator to find $\log_5 29$ using the change of base formula (round the answer to two decimal places):

1. Substitute 29 for x and 5 for b in the formula. $\log_b x = \frac{\log_e x}{\log_e b} = \frac{\ln x}{\ln b}$

   $\log_5 29 = \frac{\log_e 29}{\log_e 5} = \frac{\ln 29}{\ln 5}$
2. Simplify using your calculator. $\approx \frac{3.3672958}{1.6094379}$

   ≈ 2.0922185
3. Round the answer to two decimal places. ≈ 2.09

So, $\log_5 29$ is approximately 2.09.

Here is a shortcut for doing this problem with your calculator:

- *Enter 29 in your calculator.*
- *Press the "ln" key.*
- *Press the "÷" key.*
- *Enter 5.*
- *Press the "ln" key.*
- *Press the "=" key*
- *Record the result.*
- *Round the result.*

Sample Problems

Answers to Sample Problems

1. Use your calculator to find $\log_{10} 89$ approximated to four decimal places.

   ☐ a. Enter 89 in your calculator.

   ☐ b. Press the "log" key and write the result. ________ — *b. 1.94939*

   ☐ c. Round the result of (b) to four decimal places. ________ — *c. 1.9494*

2. Use your calculator to find an approximation of the number whose common logarithm is −0.03. Round your answer to three decimal places.

   ☑ a. Substitute –0.03 for k in the equation $\log_{10} x = k$. $\log_{10} x = -0.03$

   ☑ b. Rewrite the equation in exponential form. $10^{-0.03} = x$

   ☐ c. Enter .03 in your calculator, and press the "±" key.

   ☐ d. Press the "10^x" key and write the result. $x \approx$ ________ — *d. 0.9332543*

   ☐ e. Round the result of (d) to three decimal places. $\approx$ ________ — *e. 0.933*

3. Use properties of logarithms to write $2\log x - 3(\log x + \log 3)$ as a single logarithm.

   ☑ a. Use the log of a product property. $2\log x - 3(\log x + \log 3)$
   $= 2\log x - 3(\log 3x)$

   ☐ b. Use the log of a power property. = ________ — *b.* $\log x^2 - \log(3x)^3$

   ☐ c. Use the log of a quotient property. = ________ — *c.* $\log \frac{x^2}{(3x)^3}$

   ☐ d. Simplify. = ________ — *d.* $\log \frac{1}{27x}$

4. Use your calculator to find ln 13 rounded to two decimal places.

   ☐ a. Enter 13 in your calculator.

   ☐ b. Press the "ln" key and write the result. ________ — *b. 2.5649494*

   ☐ c. Round the result of (b) to two decimal places. ________ — *c. 2.56*

5. Use your calculator to find $\log_9 146$ using the change of base formula. Round your answer to two decimal places.

   ☑ a. Substitute 146 for x and 9 for b in the formula. $\log_b x = \frac{\ln x}{\ln b}$

   $\log_9 146 =$ ________ — *a.* $\frac{\ln 146}{\ln 9}$

   ☐ b. Simplify using your calculator. $\approx$ ________ — *b. 2.2681371*

   ☐ c. Round the result of (b) to two decimal places. $\approx$ ________ — *c. 2.27*

SOLVING EQUATIONS

Summary

Solving Logarithmic Equations

Remember, a common logarithm is a log with base 10, like $\log_{10} 7$ (which is the same as log 7). A natural logarithm, like $\log_e 7$ (which is the same as ln 7) is a log with base e.

You have studied logarithms and learned how to switch between logarithmic and exponential notation. You have also studied properties of common and natural logarithms and learned how to approximate the values of common and natural logarithms. Now you will learn how to solve equations that contain logarithms.

To solve some equations that contain logarithms you will need to switch from logarithmic to exponential notation. Recall that the following are equivalent:

logarithmic notation	exponential notation
$\log_b x = L$	$b^L = x$

Here, $b > 0$, $b \neq 1$, and $x > 0$.

For example, here's how to write $\log_5 25$ in logarithmic and exponential notation:

logarithmic notation	exponential notation
$\log_5 25 = 2$	$5^2 = 25$

In general, to solve a logarithmic equation that can be written in the form $\log_b x = L$, where $b > 0$, $b \neq 1$, and $x > 0$:

1. Rewrite the equation in exponential form $b^L = x$.

2. Solve for x.

3. Check your answer.

You need to check your answer when you solve equations like $\log_2 x = 5$ to make sure that the answer makes sense. For the equation $\log_2 x = 5$, x must be a positive number.

For example, to solve $\log_3 x = 4$ for x:

1. Rewrite the equation in exponential form. $\quad 3^4 = x$

2. Solve for x. $\quad 81 = x$
   $x = 81$

3. Check. $\quad$ The answer, 81, is positive, so it is an appropriate solution.

So if $\log_3 x = 4$, then $x = 81$.

Here's an equation where the variable, x, is the base of the logarithm. You will need to check the answer to make sure that $x \neq 1$ and $x > 0$.

To solve $\log_x 25 = 2$ for x:

1. Rewrite the equation in exponential form. $\quad x^2 = 25$

2. Solve for x. $x = 5$ or $\cancel{x = -5}$

3. Check. Since x is the base of a logarithm, you need to check that $x > 0$ and $x \neq 1$. The number 5 satisfies these conditions but –5 does not.

So if $\log_x 25 = 2$, then $x = 5$.

Here's another equation where the variable, x, is the base of the logarithm. Again, you will need to check the answer to make sure that $x \neq 1$ and $x > 0$.

To solve $\log_x \frac{1}{9} = -1$ for x:

1. Rewrite the equation in exponential form. $x^{-1} = \frac{1}{9}$

2. Solve for x:

- Substitute $\frac{1}{x}$ for x^{-1}. $\frac{1}{x} = \frac{1}{9}$
- Multiply both sides by $9x$. $9x \cdot \frac{1}{x} = 9x \cdot \frac{1}{9}$
- Cancel out the common factors. $9\cancel{x}^{1} \cdot \frac{1}{\cancel{x}_{1}} = \cancel{9}^{1}x \cdot \frac{1}{\cancel{9}_{1}}$
- Simplify $9 = x$

  $x = 9$

3. Check. Since x is the base of a logarithm, you need to check that $x > 0$ and $x \neq 1$. Since $9 > 0$ and $9 \neq 1$, it is an appropriate solution.

So if $\log_x \frac{1}{9} = -1$, then $x = 9$.

Using Properties of Logs to Solve Logarithmic Equations

So far, all of the equations you have solved contain only one logarithm. Now you will learn how to solve equations that contain more than one logarithm. You will sometimes find it easier to solve these equations if you use properties of logs to combine the logs into a single logarithm. Below is a chart you can use to review these properties. Here $b > 0$, $b \neq 1$, $u > 0$, and $v > 0$.

Name of Property	**Property**
Log of a Product	$\log_b uv = \log_b u + \log_b v$
Log of a Quotient	$\log_b \frac{u}{v} = \log_b u - \log_b v$
Log of a Power	$\log_b u^n = n \cdot \log_b u$
Other Properties	$\log_b b = 1$ $\log_b 1 = 0$

There is one more property of logs that will help you solve logarithmic equations. This property states that if the logs of two real numbers are equal, the real numbers must also be equal. That is, if $\log_b x = \log_b y$, then $x = y$. You can use this property to solve an equation in which one log is set equal to another log and both have the same base. For example, suppose you want to solve $\log_5 x = \log_5 9$ for x. Since the logs are equal and they have the same base, the quantities are equal. That is, $x = 9$.

Here's an example of how to solve a logarithmic equation using properties of logs. To solve $\ln (3x - 1) = \ln (x + 3)$ for x:

1. Since $\ln a$ is shorthand for $\log_e a$, rewrite the equation. $\quad \log_e (3x - 1) = \log_e (x + 3)$
2. Since the logs are equal and they have the same base, set the quantities to each other and solve for x.
$$3x - 1 = x + 3$$
$$2x = 4$$
$$x = 2$$
3. Check. When $x = 2$, the expressions $3x - 1$ and $x + 3$ must be positive. Since these expressions are equal, you only need to check one of them. When you substitute 2 for x in $3x - 1$, you get $3 \cdot 2 - 1 = 6 - 1 = 5$, which is positive.

So $x = 2$ is an appropriate solution of $\ln (3x - 1) = \ln (x + 3)$.

Now here's an example where you use properties of logs to combine two logs into a single log.

To solve $\log_3 x + \log_3 2 = \log_3 6$ for x:

1. Use the log of a product property to rewrite the left side as a single log. $\quad \log_3 2x = \log_3 6$
2. Since the logs are equal and they have the same base, set the quantities equal to each other and solve for x.
$$2x = 6$$
$$x = 3$$
3. Check. Since 3 is positive, the number $x = 3$ is an appropriate solution.

So if $\log_3 x + \log_3 2 = \log_3 6$, then $x = 3$.

In the next example you use properties of logs to combine two logs into a single log, then you rewrite the equation in exponential form.

To solve $\log x + \log (x - 21) = 2$ for x:

1. Use the log of a product property to rewrite the left side as a single log. $\quad \log[x(x - 21)] = 2$

2. Since log a is shorthand for $\log_{10} a$, rewrite the equation. $\log_{10}[x(x-21)] = 2$

3. Rewrite the equation in exponential form. $10^2 = x(x-21)$

4. Finish solving for x.

$$100 = x^2 - 21x$$
$$x^2 - 21x - 100 = 0$$
$$(x - 25)(x + 4) = 0$$
$$x - 25 = 0 \text{ or } x + 4 = 0$$
$$x = 25 \text{ or } \cancel{x = -4}$$

5. Check. For $\log x$ and $\log (x - 21)$ to be defined, x and $x - 21$ must be positive. So $x = 25$ checks, but $x = -4$ does not.

The numbers $x = -4$ and $x - 21 = -4 - 21 = -25$ are both negative, so the logarithm is not defined for $x = -4$.

So the only solution of $\log x + \log (x - 21) = 2$ is $x = 25$.

In the next example, you start with one log term on the left side of the equation, and one log term on the right side of the equation. Since you want to write the equation using a single log term, you need to get both log terms on the left side of the equation.

To solve $2\log_3 x = 2 + \log_3 9$ for x:

1. Subtract $\log_3 9$ from both sides of the equation. $2\log_3 x - \log_3 9 = 2$

2. Use the log of a power property to rewrite $2\log_3 x$. $\log_3 x^2 - \log_3 9 = 2$

3. Use the log of a quotient property to rewrite the left side as a single log. $\log_3\left(\frac{x^2}{9}\right) = 2$

4. Rewrite in exponential form.

$$3^2 = \frac{x^2}{9}$$
$$9 = \frac{x^2}{9}$$

5. Finish solving for x.

$$9 \cdot 9 = x^2$$
$$81 = x^2$$
$$x^2 = 81$$
$$x = 9 \text{ or } \cancel{x = -9}$$

6. Check. For $\log_3 x$ to be defined, x must be positive. So $x = 9$ checks but $x = -9$ does not.

So the only solution of $2\log_3 x = 2 + \log_3 9$ is $x = 9$.

Solving Exponential Equations

You have solved equations that contain logarithms. Now you will use logarithms to solve equations where the variable appears in an exponent. These equations are called exponential equations. Here are some examples:

$2^x = 5$ $\qquad$ $2^x + 9 = 15$ $\qquad$ $4e^{2x-1} = 6$

To solve exponential equations, you will frequently use two properties of logs. The first property states that if two quantities are equal, their logs are equal. That is:

If $x = y$, then $\log_b x = \log_b y$.

The other property is the log of a power property:

$\log_b u^n = n \cdot \log_b u$ (Here $b > 0$, $b \neq 1$, and $u > 0$)

In general, to solve an exponential equation:

1. Isolate the term that contains the exponent.
2. Take the log of both sides of the equation. You may want to use natural logs since you can easily approximate them using a calculator.
3. Use the log of a power property to rewrite the term that contains the exponent. Then the variable will no longer be in the exponent.
4. Finish solving for x.

Remember, a natural log is just a logarithm with base e. So you can let b = e and use the property "If x = y, then ln x = ln y."

Once you find x, you can approximate your answer by using a calculator to compute the natural logs, and then round your answer.

Here's an example. To solve the exponential equation $2^x = 5$ for x:

1. The term that contains the exponent is isolated on the left side. $\qquad 2^x = 5$
2. Take the natural log (ln) of both sides of the equation. $\qquad \ln 2^x = \ln 5$
3. Use the log of a power property to get x out of the exponent. $\qquad x \cdot \ln 2 = \ln 5$
4. Finish solving for x by dividing both sides by ln 2. $\qquad x = \frac{\ln 5}{\ln 2}$

So if $2^x = 5$ then $x = \frac{\ln 5}{\ln 2}$. You can approximate this answer by using a calculator and round your answer. For example, $\frac{\ln 5}{\ln 2}$ rounded to two decimal places is approximately $\frac{1.6094379}{0.6931471} \approx 2.32$.

As another example, to solve $4e^{2x-1} = 5$ for x:

1. Isolate the term e^{2x-1} by dividing both sides by 4.

$$4e^{2x-1} = 5$$
$$e^{2x-1} = \frac{5}{4}$$

2. Take the natural log (ln) of both sides of the equation. $\ln e^{2x-1} = \ln \frac{5}{4}$

3. Use the log of a power property to get $2x - 1$ out of the exponent. $(2x - 1) \cdot \ln e = \ln \frac{5}{4}$

4. Finish solving for x. Recall that $\ln e = 1$.

$$(2x - 1) \cdot 1 = \ln \frac{5}{4}$$
$$2x - 1 = \ln 1.25$$
$$2x = \ln 1.25 + 1$$
$$x = \frac{\ln 1.25 + 1}{2}$$

So if $4e^{2x-1} = 5$ then $x = \frac{\ln 1.25 + 1}{2}$. You can approximate this answer using a calculator and rounding. For example, $\frac{\ln 1.25 + 1}{2}$ rounded to two decimal places is approximately $\frac{0.2231435 + 1}{2} \approx 0.61$.

Sample Problems

1. Solve for x: $\log_{10} x = 3$

- ☑ a. Rewrite the equation in expontential form. $10^3 = x$
- ☐ b. Solve for x. $x =$ __________

2. Solve for x: $\log_x 3 = \frac{1}{2}$

- ☐ a. Rewrite the equation in exponential form. __________________
- ☑ b. Rewrite the equation using a radical sign. $\sqrt{x} = 3$
- ☐ c. Solve for x. $x =$ _________
- ☑ d. Check. The number $x = 9$ is greater than 0 and not equal to 1 so the answer checks.

3. Solve for x: $\ln 2x + \ln (x + \frac{3}{2}) = \ln 5$

- ☐ a. Use the log of a product property to rewrite the left side as a single log. ______________ $= \ln 5$
- ☐ b. Since the two natural logs are equal, set the quantities equal to each other. ____________________
- ☐ c. Solve for x. $x =$ ______ or $x =$ ______

Answers to Sample Problems

1. *b. 1000*

2. *a. $x^{\frac{1}{2}} = 3$*

c. 9

3. *a. $\ln \left[2x(x + \frac{3}{2})\right]$ or $\ln \left(2x^2 + 3x\right)$*

b. $2x\left(x + \frac{3}{2}\right) = 5$ or $2x^2 + 3x = 5$

c. $1, -\frac{5}{2}$

Answers to Sample Problems

☑ d. Check your answer.

The numbers $x = -\frac{5}{2}$,
$2x = 2\left(-\frac{5}{2}\right) = -5$,
and $\left(x + \frac{3}{2}\right) = \left(-\frac{5}{2} + \frac{3}{2}\right) = -1$
are all negative, so the logarithm is not defined for $x = -\frac{5}{2}$.
The only solution is $x = 1$.

4. Solve for x: $2\log_2 x = 2 + \log_2 4$

☑ a. Subtract $\log_2 4$ from both sides. $2\log_2 x - \log_2 4 = 2$

☐ b. Use the log of a power property to rewrite $2\log_2 x$. ________ $- \log_2 4 = 2$

b. $\log_2 x^2$

☐ c. Use the log of a quotient property to rewrite the left side as a single log. _________ $= 2$

c. $\log_2\left(\frac{x^2}{4}\right)$

☐ d. Rewrite in exponential form, and simplify. $2^2 = \frac{x^2}{4}$

____

d. $4 = \left(\frac{x^2}{4}\right)$

☐ e. Finish solving for x.

$x =$ ______ or $x =$ ______

e. 4, –4

☑ f. Check your answer. The number $x = -4$ is negative, so $x = 4$ is the only solution.

5. Solve for x: $2^x + 9 = 15$

☑ a. Subtract 9 from both sides to isolate 2^x. $2^x = 6$

☐ b. Take the natural log of both sides of the equation. ____________________

b. $\ln 2^x = \ln 6$

☐ c. Use the log of a power property to get x out of the exponent. ____________________

c. $x \cdot \ln 2 = \ln 6$

☐ d. Finish solving for x.

$x =$ ______________

d. $\frac{\ln 6}{\ln 2}$

☐ e. Use a calculator to compute the answer to two decimal places. $x \approx$ _________

e. 2.58

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Natural and Common Logs

1. Find: $\log_{10} 10000$
2. Find: $\ln 1 + \ln e$
3. Use properties of logarithms to write $\log 2 + 3(\log x)$ as a single logarithm.
4. Use your calculator to find $\log_{10} 62$ rounded to three decimal places.
5. Use your calculator to find $\ln 31$ rounded to two decimal places.
6. Use properties of logarithms to write $\log x - 2(\log x + \log x^2)$ as a single logarithm.
7. Find an approximation of the number whose common logarithm is –3.56. Round your answer to four decimal places.
8. Use properties of logarithms to write $3\ln x - \ln(x + 3)$ as a single logarithm.
9. Jack is an environmental scientist. A recent experiment of his resulted in some data that he represented with the following logarithmic expressions.

   Expression 1: $2\log(x + 1)$

   Expression 2: $2\log(2 + x)$

   Expression 3: $4 + \log 2 + 3\log x$

   Jack wants to include this data in a report using a single logarithm. Help Jack by writing the sum of the three logarithmic expressions as a single logarithm.

10. Jack, the environmental scientist, now wants to evaluate the sum of the following expressions for $x = 2$:

    Expression 1: $2\log(x + 1)$

    Expression 2: $2\log(2 + x)$

    Expression 3: $4 + \log 2 + 3\log x$

    Evaluate the total of these expressions for $x = 2$. The round your answer to two decimal places.

11. Use your calculator to find $\log_6 801$ using the change of base formula. Round your answer to three decimal places.
12. Use properties of logarithms to write $\frac{1}{2}\log x + 3\log\sqrt{x} - 2\log(x + 1)$ as a single logarithm.

Solving Equations

13. Solve for x: $\log_4 x = 3$
14. Solve for x: $\ln(x + 3) = \ln(2x - 1)$
15. Solve for x: $\log_4 2x - \log_4 2 = \log_4 5$
16. Solve for x: $\log_x 8 = 3$
17. Solve for x: $\ln x + \ln(x + 1) = \ln 2$
18. Solve for x: $2\log_7 x = 2 + \log_7 16$
19. Solve for x: $\log_x 7 = \frac{1}{2}$
20. Solve for x: $4^x + 5 = 12$
21. The number of bacteria, N, at a certain time t is described by the equation $N = N_0e^{4t}$, where N_0 is the number of bacteria present at time $t = 0$. What is t when the number of bacteria has doubled from what it was when $t = 0$? Round your answer to two decimal places. (Hint: You want to know when $N = 2N_0$, so solve the equation $2N_0 = N_0e^{4t}$, which is the same as $2 = e^{4t}$, for t.)

22. On the Richter scale the magnitude M of an earthquake of intensity I is given by the equation $M = \log \frac{I}{I_0}$, where I_0 is a minimum intensity measurement used for comparing earthquakes. If an earthquake has an intensity I that is $10^{6.2}$ times the minimum intensity I_0, what is its magnitude M on the Richter scale?

23. Solve for x: $5^x - 4 = 2$. Round your answer to two decimal places.

24. Solve for x: $2e^{2x+3} - 3 = 5$. Round your answer to three decimal places.

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Natural and Common Logs

1. Find: $\log 400 - \log 40$
2. Find: $\log 30 - \log 3000$
3. Find: $\log 50 - \log 50{,}000$
4. Find: $\ln e^3 + \ln e^7$
5. Find: $\ln e^2 + \ln e^5$
6. Find: $\ln e - \ln e^2$
7. Use properties of logarithms to write $\log x^3 + \log x^5$ as a single logarithm.
8. Use properties of logarithms to write $5\log x^4 + 6\log x^7$ as a single logarithm.
9. Use properties of logarithms to write $5(\log x^2 + \log x^{11})$ as a single logarithm.
10. Use properties of logarithms to write $4\ln x + \ln (x - 7)$ as a single logarithm.
11. Use properties of logarithms to write $7\ln x + \ln (2x + 3)$ as a single logarithm.
12. Use properties of logarithms to write $3\ln x^3 - \ln x^6 - \ln (3x + 2)$ as a single logarithm.
13. Use properties of logarithms to write $2\ln x^2 + \ln x - \ln (4x - 1)$ as a single logarithm.
14. Use your calculator to find log 35 rounded to two decimal places.
15. Use your calculator to find log 24.7 rounded to two decimal places.
16. Use your calculator to find log 28 rounded to two decimal places.
17. Use your calculator to find ln 90 rounded to two decimal places.
18. Use your calculator to find ln 83 rounded to two decimal places.
19. Use your calculator to find ln 54.9 rounded to two decimal places.
20. Use your calculator to find $\log_{11}34$ rounded to two decimal places. (Round your answer at the end of your calculations.)
21. Use your calculator to find $\log_{7}23.4$ rounded to two decimal places. (Round your answer at the end of your calculations.)
22. Use your calculator to find $\log_{12}55$ rounded to two decimal places. (Round your answer at the end of your calculations.)
23. Find an approximation of the number whose common logarithm is 2.125. Round your answer to two decimal places.
24. Find an approximation of the number whose common logarithm is –0.005. Round your answer to two decimal places.
25. Find an approximation of the number whose common logarithm is 1.375. Round your answer to two decimal places.
26. Find an approximation of the number whose natural logarithm is 1.07. Round your answer to two decimal places.
27. Find an approximation of the number whose natural logarithm is –1.24. Round your answer to two decimal places.
28. Find an approximation of the number whose natural logarithm is –2.2. Round your answer to two decimal places.

Solving Equations

29. Solve for x: $\log_2 x = 4$

30. Solve for x: $\log_5 x = 3$

31. Solve for x: $\log_3 x = 5$

32. Solve for x: $\log_x 64 = 3$

33. Solve for x: $\log_x 125 = 3$

34. Solve for x: $\log_x 6 = \frac{1}{2}$

35. Solve for x: $\log_x 4 = \frac{1}{3}$

36. Solve for x: $\log_3 3x + \log_3 9x = 4$

37. Solve for x: $\log_{12} 6x + \log_{12} 4x = 2$

38. Solve for x: $\log 12x^2 - \log 6x = 2$

39. Solve for x: $\log_8 32 = 5\log_8 x$

40. Solve for x: $\log_{13} 125 = 3\log_{13} x$

41. Solve for x: $4\log_{15} x = \log_{15} 81$

42. Solve for x: $\log (x + 1) = 2$

43. Solve for x: $\log_6 (x + 2) - \log_6 5 = \log_6 (x - 2)$

44. Solve for x: $\log_5 (x + 1) - \log_5 7 = \log_5 (x - 1)$

45. Solve for x: $2\ln (x - 3) = \ln (30 - 2x)$

46. Solve for x: $2\ln (x + 2) = \ln (7x + 44)$

47. Solve for x: $\ln (x - 3) + \ln 5 = \ln (x + 17)$

48. Solve for x: $4^x + 7 = 31$

49. Solve for x: $5^x + 2 = 21$

50. Solve for x: $12^x - 8 = 32$

51. Solve for x: $8 + 3^{x-2} = 30$

52. Solve for x: $19 - 4^{x+1} = 7$

53. Solve for x: $20 = 14 + 7^{x-3}$

54. Solve for x: $8e^{3x+2} - 3 = 45$

55. Solve for x: $5e^{4x-1} + 12 = 37$

56. Solve for x: $6e^{5x-1} + 2 = 44$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Use a calculator to approximate the values of x below to two decimal places.

   a. $\log_{10} 113 = x$

   b. $\log_{10} x = 0.34$

2. Use properties of logs to rewrite $\log 5 + \log 2x + 2\log(x - 2)$ as a single logarithm.

3. Circle the statements below that are always true. Assume $x > 0$, $a > 0$, and $b > 0$.

   $\ln 1 = 0$

   $\log a + \log b = \log(a + b)$

   $\log_e 5 = \ln 5$

   $2\log x = \log x + \log x$

4. Use a calculator to approximate $\log_{13} 230$ to two decimal places.

5. Solve for x: $\log_x 36 = 2$

6. Solve the following equations for x:

   a. $\log_3 x = -2$

   b. $\log_x \frac{2}{3} = -1$

7. Solve for x: $\log_3(2x - 2) - 1 = \log_3 8$

8. Solve for x: $6^x - 5 = 17$. Round your answer to two decimal places.

TOPIC 12 CUMULATIVE ACTIVITIES

CUMULATIVE REVIEW PROBLEMS

These problems combine all of the material you have covered so far in this course. You may want to test your understanding of this material before you move on to the next topic, or you may wish to do these problems to review for a test.

1. Solve for t: $3^{t-2} = 27^{4t}$
2. Find: $(-3 + 4i) - (-2 + 9i)$
3. Find: $(4 + 3i) \div (11 - 6i)$
4. Find the domain and range of each of the functions below.
   a. $y = x - 8$
   b. $y = |x| - 8$
   c. $y = |x - 8|$
5. Rewrite each of the following using properties of logarithms:
   a. $\log_7 (4^6)$
   b. $\log_2 \left(\frac{1}{x^3}\right)$
   c. $\log_3 (3^x)$
   d. $\log_5 100 - \log_5 10$
   e. $\log_{10} \sqrt{31}$
6. Given $f(x) = 14x$ and $g(x) = 3x - 5$, evaluate $\left(\frac{f}{g}\right)(x)$ at $x = 2$.
7. The graph of the function $y = -2x^2$ is show on the grid in Figure 12.1. Determine the equations of the other parabolas shown.

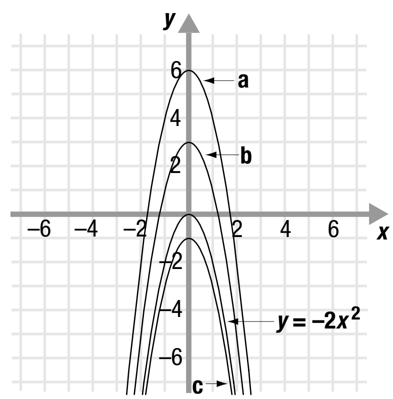


Figure 12.1

8. Find: $(x + 7)(4x - 1)$
9. Evaluate the following:
   a. $\log_8 8$
   b. $\log_{41} 1$
   c. $6^{\log_6 14}$
10. Solve for x: $2\ln x = \ln(x + 6)$
11. Write the equation $4(y + 2) = 3x + 4$ in slope-intercept form.
12. Solve for x: $\ln x + \ln 9 = \ln(2x + 7)$
13. Combine like terms and simplify:

    $5\sqrt{13} + 2\sqrt{41} - \sqrt[3]{-27} - 6\sqrt{41} + \sqrt{13} + 1$

14. Rewrite the following in logarithmic form:
    a. $3^4 = 81$
    b. $10^6 = 1{,}000{,}000$
    c. $2^7 = 128$
    d. $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$
    e. $49^{\frac{1}{2}} = 7$

15. Find the domain and range of each of the functions below.

a. $f(x) = x^2 - 2$ b. $f(x) = 21x + 13$

c. $f(x) = \frac{14x}{(2x-1)(x+7)}$ d. $f(x) = \sqrt{x^2 - 4}$

16. Rewrite each of the following using more than one logarithm. Use properties of logarithms.

a. $\log_4 (8 \cdot 3)$ b. $\log_{17}\left(\frac{m}{n}\right)$

c. $\log_{10} [5(a+1)]$ d. $\log_9 \left(\frac{7}{8}\right)$

e. $\log_{10}\left(\frac{5x}{x^2}\right)$

17. Rewrite using properties of logarithms: $\ln \frac{x^2}{x+2}$

18. Find the reciprocal: $6a \cdot \left(\frac{2b^2 + 9c}{a + c}\right)$

19. Solve for x: $x^2 - 70 = 11$

20. Find the equation of the line through the point (–3, 2) that is parallel to the line $y + 4 = 2(x - 1)$. Write your answer in slope-intercept form.

21. Find the inverse of the function $y = 3^x$ in logarithmic form. Then graph $y = 3^x$ and its inverse.

22. Using your calculator and the change of base formula, find each of the following. Round your answers to two decimal places.

a. $\log_4 72$ b. $\log_8 16$

c. $\log_{21} 11$ d. $\log_5\left(\frac{7}{9}\right)$

23. Does $y = f(x) = x^3$, shown in Figure 12.2, have an inverse? If yes, find and graph the inverse $y = f^{-1}(x)$.

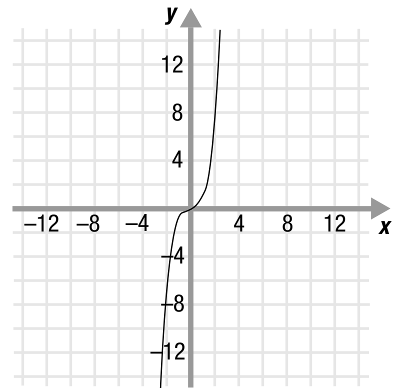


Figure 12.2

24. Find: $(11 + 2i)(2 + 11i)$

25. Solve for y: $5y^2 - 7y + 2 = 0$

26. Solve for y: $-8|y + 2| > 7$

27. Solve for x: $4e^{3x-6} + 2 = 11$

28. Find: $\frac{3z^2}{x^2} + \frac{10x}{xy}$

29. Using the compound interest formula, $A = P\left(1 + \frac{r}{n}\right)^n$, determine how much money, A, you would have after one year if you invested $P = \$500$ in an account that compounded interest quarterly ($n = 4$) at an interest rate of $r = 3\%$. Use a calculator.

30. Find: $\log_5 125$

31. Simplify the following expressions:

a. $\sqrt{-81}$ b. $\sqrt{25}$

c. $\sqrt{-81} \cdot \sqrt{25}$ d. $\sqrt{-81} \cdot \sqrt{-25}$

e. i^{31} f. i^{50}

32. Simplify: $\frac{\sqrt{ab^2c}}{a^{\frac{1}{4}}}$

33. Solve for q: $2q^2 - 11q = 7 - 6q$

34. Find the x- and y-intercepts of each of the functions below.

a. $y = x^2 - 10x - 6$ b. $y = -8x^2$

c. $y = x^2 - 3x - 4$

35. Complete the table of ordered pairs below for $y = 2^{x+1}$, then graph the function. As the value of x becomes smaller and smaller, what value does y approach?

x	y
0	
–1	
1	
–2	
2	

36. Solve for x: $\log_x 81 = 2$

37. Factor: $2x^2 - 6x + xy - 3y$

38. Solve for x: $3x(x - 2) - 2 = 4x^2 - 9x + 3$

39. Solve for d: $\frac{d-7}{3} = \frac{2d}{5} - 1$

40. Using your calculator, find x for each of the following to two decimal places.

a. $\log 365 = x$

b. $\log 727 = x$

c. $\log x = -0.78$

d. $\ln x = 3.14$

e. $\ln 216 = x$

41. If $f(x) = 33x - 18$, find $f^{-1}(x)$.

42. Solve for z: $z^2 + 6z + 7 = 0$

43. Complete the table below for the function $y = x + 4$. Then use the table to graph the function.

x	y
−6	
−4	
–2	
0	
1	
3	

44. If $f(x) = \frac{3}{x+9}$, find $f^{-1}(x)$.

45. Solve for m: $-4(2m - 5) \geq -3m + 27$

46. Which of the following functions are linear?

$y = 4x + 3$

$y = x^2 + 2x - 6$

$y = \sqrt{x}$

$y = 244x$

$y = x^3 - 7x + 2$

47. Find: $\left(6\sqrt{5} - 3\sqrt{6}\right)\left(5\sqrt{5} + 9\sqrt{6}\right)$

48. Solve for p: $11 - 2|3p + 1| = 9$

49. Given $f(x) = x^2 + 2x - 8$ and $g(x) = x + 1$, find $(f \circ g)(x)$.

50. Solve for x: $(2x - 6)^2 = 4$