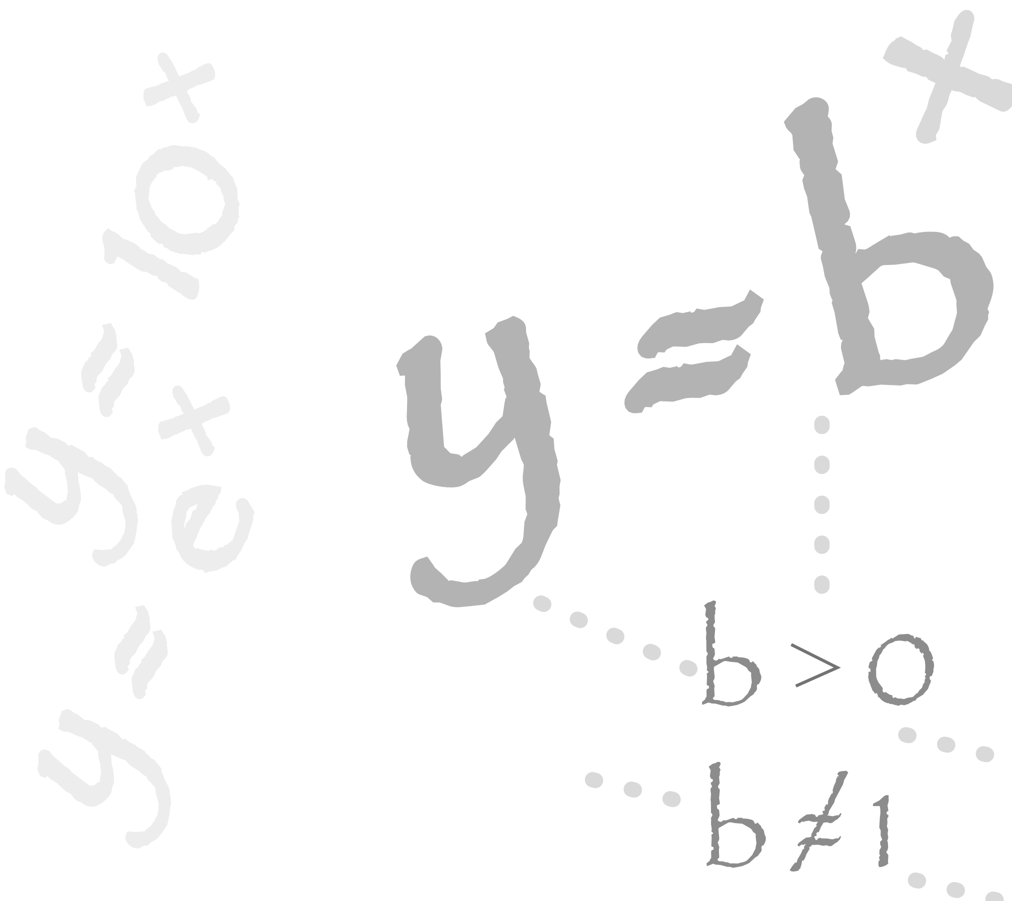
# LESSON 12.1 – EXPONENTIAL FUNCTIONS



## OVERVIEW

***Here's what you'll learn in this lesson:***

***The Exponential Function***

*a. Recognizing and graphing an exponential function*

*b. Applications of the exponential function*

*c. Solving exponential equations*

Suppose you have one amoeba that splits into two. Then suppose that these two amoebas each split, producing four. If this process continues, you will have eight amoebas, then sixteen, then thirty-two, then sixty-four, and so on. The growth of your amoeba population can be described using a function called an exponential function. In this lesson, you'll learn about exponential functions.

# EXPLAIN

## THE EXPONENTIAL FUNCTION

### Summary

#### Recognizing and Graphing an Exponential Function

You have studied linear functions of the form $y = ax + b$, and quadratic functions of the form $y = ax^2 + bx + c$. Now you will learn about exponential functions.

An exponential function is a function of the form $y = b^x$. Here the number $b$ is the base and $x$ is the exponent. Also, $b > 0$ and $b \neq 1$. Examples of exponential functions are $y = 2^x$, $y = 3^x$, and $y = \left(\frac{1}{3}\right)^x$.

In general, to graph an exponential function $f(x) = b^x$, where $b > 0$ and $b \neq 1$:

1. Make a table of ordered pairs for $y = f(x) = b^x$ by substituting several values for $x$ into $y = b^x$, and solving for $y$.
2. Plot these ordered pairs on a grid, and join them with a smooth curve.

For example, to graph $f(x) = 3^x$:

1. Substitute values for $x$ into $y = f(x) = 3^x$, and make a table of ordered pairs.

Let $x = 2$, then:

$$\begin{aligned} y &= 3^2 \\ &= 9 \end{aligned}$$

Let $x = \frac{3}{2}$, then:

$$\begin{aligned} y &= 3^{\frac{3}{2}} \\ &= \left(\sqrt{3}\right)^3 \\ &= 3\sqrt{3} \\ &\approx 5.196 \end{aligned}$$

Let $x = 1$, then:

$$\begin{aligned} y &= 3^1 \\ &= 3 \end{aligned}$$

Let $x = 0$, then:

$$\begin{aligned} y &= 3^0 \\ &= 1 \end{aligned}$$

Let $x = -1$, then:

$$\begin{aligned} y &= 3^{-1} \\ &= \frac{1}{3} \end{aligned}$$

*The independent variable, x, doesn't have to be an integer. You can pick values for x that aren't integers, then approximate the y-value by using a calculator. You can use the symbol "≈" to mean "is approximately equal to."*

Let $x = -\frac{3}{2}$, then:

$$y = 3^{-\frac{3}{2}}$$
$$= \frac{1}{3^{\frac{3}{2}}}$$
$$= \frac{1}{3\sqrt{3}}$$
$$= \frac{\sqrt{3}}{9}$$
$$\approx .1925$$

Let $x = -2$, then:

$$y = 3^{-2}$$
$$= \frac{1}{3^2}$$
$$= \frac{1}{9}$$

$y = f(x) = 3^x$

| $x$ | $y$ |
|---|---|
| 2 | 9 |
| $\frac{3}{2}$ | 5.196 |
| 1 | 3 |
| 0 | 1 |
| –1 | $\frac{1}{3}$ |
| $-\frac{3}{2}$ | .1925 |
| –2 | $\frac{1}{9}$ |

2. Plot the ordered pairs, and join them with a smooth curve. — See the grid in Figure 12.1.1.

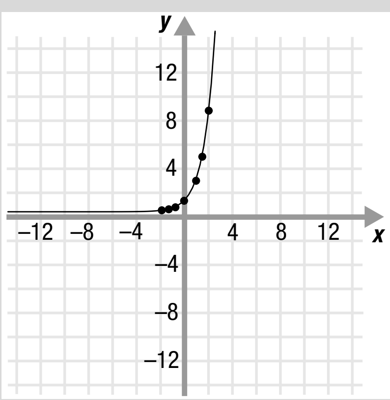



*Figure 12.1.1*

*The domain of $f(x) = 3^x$ is all real numbers, since the input value, x, can be any number. The range is all positive real numbers, since the output value, y, is never 0 or negative.*

As $x$ becomes large (see Figure 12.1.1), the graph rises rapidly. In other words, as the $x$-values increase (that is, as you move to the right along the $x$-axis), the $y$-values become large. Also, regardless of $x$, $3^x$ is never 0 or negative, so the graph never touches or crosses the $x$-axis. Notice that the $y$-intercept of the graph is the point (0, 1).

Here's another example. To graph $f(x) = \left(\frac{1}{3}\right)^x$:

1. Substitute values for $x$ into $y = f(x) = \left(\frac{1}{3}\right)^x$, and make a table of ordered pairs.

Let $x = 2$, then:

$$y = \left(\frac{1}{3}\right)^2$$
$$= \frac{1}{9}$$

Let $x = 1$, then:

$$y = \left(\frac{1}{3}\right)^1$$

$$= \frac{1}{3}$$

Let $x = 0$, then:

$$y = \left(\frac{1}{3}\right)^0$$
$$= 1$$

Let $x = -1$, then:

$$y = 3^{-(-1)}$$
$$= 3$$

Let $x = -2$, then:

$$y = 3^{-(-2)}$$
$$= 9$$

$$y = f(x) = \left(\frac{1}{3}\right)^x$$

| $x$ | $y$ |
|---|---|
| 2 | $\frac{1}{9}$ |
| 1 | $\frac{1}{3}$ |
| 0 | 1 |
| –1 | 3 |
| –2 | 9 |

2. Plot the ordered pairs, and join them with a smooth curve. — See the grid in Figure 12.1.2.

As $x$ becomes large (see Figure 12.1.2), the graph decreases rapidly. In other words, as the $x$-values increase (as you move to the right along the $x$-axis), the $y$-values become small. Also, regardless of $x$, $\left(\frac{1}{3}\right)^x$ is never 0 or negative, so the graph never touches or crosses the $x$-axis. Notice that the $y$-intercept of this graph is also the point (0, 1).

## Exponential Growth and Decay

You have learned that an exponential function with base $b$ is a function of the form $y = f(x) = b^x$, where $b > 0$ and $b \neq 1$. Exponential functions have many applications in the natural and social sciences.

Look at the graph of the exponential function $y = f(x) = 3^x$ shown on the grid in Figure 12.1.3. Notice that as you move from left to right, the graph rises slowly, and then rises rapidly. This is typical of an exponential function where the base $b$ is greater than 1, and is often called **exponential growth**. Populations often grow exponentially. Money invested where it is earning compounded interest also grows exponentially.

*Remember: $\left(\frac{1}{3}\right)^x = \frac{1^x}{3^x} = \frac{1}{3^x} = 3^{-x}$. When you substitute negative values for $x$ into $y = f(x) = \left(\frac{1}{3}\right)^x$, you might find it easier to substitute these values into $y = f(x) = 3^{-x}$.*

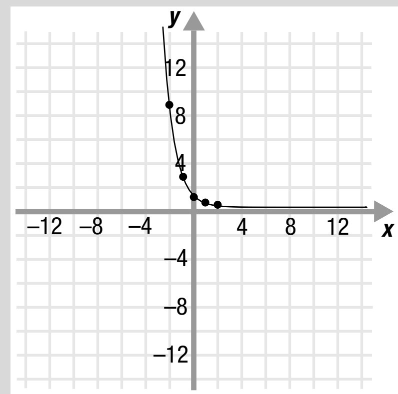



*Figure 12.1.2*

*Once again, the domain of the exponential function $y = f(x) = \left(\frac{1}{3}\right)^x$ is all real numbers, since the input value, x, can be any number. The range of this exponential function is all positive real numbers, since the output value, y, is never 0 or negative.*

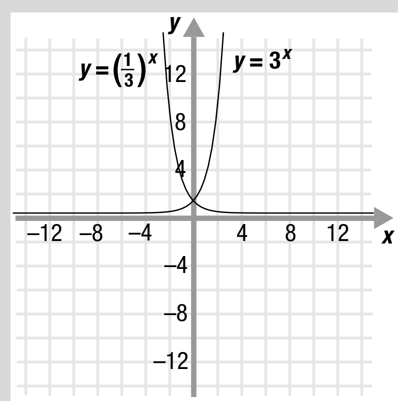



*Figure 12.1.3*

Now look at the graph of the exponential function $y = g(x) = \left(\frac{1}{3}\right)^x$ shown on the grid in Figure 12.1.3. Notice that as you move from left to right, the graph drops sharply and then levels off slowly. This is typical of an exponential function where the base $b$ is between 0 and 1, and is called **exponential decay**. Radioactive substances decay exponentially. The temperature of a round object such as a potato that has been baked then left to cool declines exponentially as well.

## The Exponential Function with Base *e*

There is a particular base that is very important for applications that involve exponential growth and decay. This base is the irrational number $e$, a number that lies between 2 and 3 on the number line. Since $e$ is an irrational number like $\pi$ or $\sqrt{2}$, its decimal representation never ends and never repeats. You will often approximate the value of $e$ to be 2.718.

The exponential function with base $e$ is of the form $y = f(x) = e^x$. This is called the natural exponential function and it arises "naturally" in many applications in the natural and social sciences. You can see the relationship between the graphs of $y = e^x$, $y = 2^x$, and $y = 3^x$ in Figure 12.1.4.

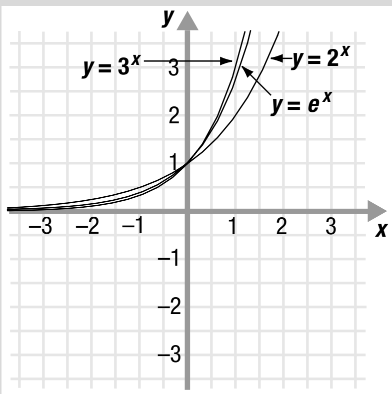



*Figure 12.1.4*

On some calculators there is an "$e^x$" key. So you can find the approximate value of $e$ raised to a power of $x$ by entering the power $x$ first, then pressing the "$e^x$" key.

*If $x$ is a negative value, you may have to use the "±" key to get the negative sign.*

In general, to use a calculator to approximate $e^x$:

1. Enter the power (or exponent), $x$.
2. Press the "$e^x$" key, then round your answer to the desired number of decimal places.

For example, to find $e^{1.8}$ approximated to two decimal places:

*To round 6.0496475 to two decimal places, look at the number in the third decimal place. It is 9, so round the 4 up to 5.*

1. Enter 1.8 in your calculator.
2. Press the "$e^x$" key, then — 6.0496475
   round your answer. — 6.05

So, $e^{1.8} \approx 6.05$.

As another example, to find $e^{-2.2}$ approximated to two decimal places:

1. Enter –2.2 in your calculator.
   Use the "±" key.
2. Press the "$e^x$" key, then — 0.1108031
   round your answer. — 0.11

*To round 0.1108031 to two decimal places, look at the number in the third decimal place. It is 0, so don't round the 1.*

So, $e^{-2.2} \approx 0.11$.

## Applications of the Exponential Function $y = e^x$

There are many situations in the natural and social sciences that can be described using functions that are based on the natural exponential function $y = e^x$.

### A Learning Curve

For example, you can use a function of the form $y = A + Be^{g(x)}$ to predict the number of items that a factory worker can produce on an assembly line. The function $g(x)$ depends upon the particular assembly line. As you might expect, a worker who is just learning how to use the machinery produces very few items. As the worker becomes more experienced with the operation of the machinery, she produces more and more items. But eventually, the number of items that the worker produces will reach a maximum number and level off.

Here's a function that describes this situation for a worker who has $x$ days of experience on the assembly line and produces $y$ items per day: $y = 45 - 30e^{-0.02x}$. Look at the value of this function for different input values of $x$.

- A new worker has 0 days of experience.

Here, $x = 0$, so
$$\begin{aligned} y &= 45 - 30e^{-0.02(0)} \\ &= 45 - 30e^{0} \\ &= 45 - 30(1) \\ &= 45 - 30 \\ &= 15 \end{aligned}$$

So a new worker can expect to produce 15 items per day.

Look at the number of items a worker can produce for various levels of experience:

- A worker who has 3 days of experience.

Here, $x = 3$, so
$$\begin{aligned} y &= 45 - 30e^{-0.02(3)} \\ &= 45 - 30e^{-0.06} \\ &\approx 45 - 30(.942) \\ &= 45 - 28.26 \\ &= 16.74 \end{aligned}$$

*The accuracy of these calculations may differ, depending on which step we decide to round.*

So after 3 days, a worker can expect to produce almost 17 items per day.

- A worker who has 2 months (60 days) of experience.

Here, $x = 60$, so
$$\begin{aligned} y &= 45 - 30e^{-0.02(60)} \\ &= 45 - 30e^{-1.2} \\ &\approx 45 - 30(.301) \\ &= 45 - 9.03 \\ &= 35.97 \end{aligned}$$

So after 2 months, a worker can expect to produce almost 36 items per day, or almost twice as much as she was producing after 3 days.

- A worker who has 6 months (180 days) of experience.

Here, $x = 180$, so $y = 45 - 30e^{-0.02(180)}$
$= 45 - 30e^{-3.6}$
$\approx 45 - 30(.027)$
$= 45 - 0.81$
$= 44.19$

So after 6 months a worker can expect to produce about 44 items per day.

- A worker who has 1 year (365 days) of experience.

Here, $x = 365$, so $y = 45 - 30e^{-0.02(365)}$
$= 45 - 30e^{-7.3}$
$\approx 45 - 30(.001)$
$= 45 - 0.03$
$= 44.97$

So after 1 year, a worker can expect to produce almost 45 items per day, which is not much more than she could produce after 6 months on the job.

Now to graph the function $y = 45 - 30e^{-0.02x}$:

*You already found several ordered pairs for the table, but you will be able to graph the curve more easily if you find a few more.*

1. Substitute values for $x$ into $y = f(x) = 45 - 30e^{-0.02x}$, and make a table of ordered pairs.

Let $x = 20$, then:
$y = 45 - 30e^{-0.02(20)}$
$= 45 - 30e^{-0.4}$
$\approx 45 - 30(.670)$
$= 45 - 20.1$
$= 24.9$

Let $x = 100$, then:
$y = 45 - 30e^{-0.02(100)}$
$= 45 - 30e^{-2}$
$\approx 45 - 30(.135)$
$= 45 - 4.05$
$= 40.95$

Let $x = 275$, then:
$y = 45 - 30e^{-0.02(275)}$
$= 45 - 30e^{-5.5}$
$\approx 45 - 30(.004)$
$= 45 - 0.12$
$= 44.88$

$y = f(x) = 45 - 30e^{-0.02x}$

| $x$ | $y$ |
|---|---|
| 0 | 15 |
| 3 | 16.74 |
| 20 | 24.9 |
| 60 | 35.97 |
| 100 | 40.95 |
| 180 | 44.19 |
| 275 | 44.88 |
| 365 | 44.97 |

2. Plot the ordered pairs, and join them with a smooth curve. See the grid in Figure 12.1.5.

Notice that the graph levels off at approximately $y = 45$. So you would expect that an experienced worker could produce about 45 items per day.

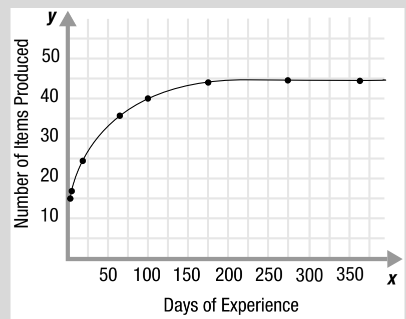


*Figure 12.1.5*

## The Compound Interest Formula

Here's another application of exponential functions. Suppose you open a bank account by investing \$100 for one year at an interest rate of 5%. Also, suppose that interest is compounded 2 times during the year. This means that after 6 months, the bank pays you $\frac{1}{2}$ of the 5% interest, or 2.5%, on the \$100 you invested.

So at the end of 6 months you have: \$100 + \$2.50 = \$102.50

original investment interest

After 12 months, the bank pays you $\frac{1}{2}$ of the 5% interest, or 2.5%, on your original investment **and** on the \$2.50 interest that you have earned.

So at the end of 12 months you have: \$102.50 + .025(102.50) ≈ \$105.06

original investment + \$2.50 interest interest

In general, if you invest $P$ dollars at an interest rate $r$, and interest is compounded $n$ times per year, the amount of money in the account after one year is given by the compound interest formula: $A = P \cdot \left(1 + \frac{r}{n}\right)^n$. For this exponential function, the base is $1 + \frac{r}{n}$ and the exponent is $n$.

The number $e$ arises naturally from a special case of the compound interest formula. Suppose you invest \$1.00 at a rate of 100%, or 1.00 per year. This is a fantastic rate! If you compound this investment $n$ times per year, the interest rate for each individual period is $\frac{1.00}{n}$ or $\frac{1}{n}$.

So after 1 year, your \$1.00 investment will grow to $A = \left(1 + \frac{1}{n}\right)^n$ dollars.

Look at how much the investment will grow in a year for different numbers of compounding periods:

- Interest compounded just 1 time per year.

$$\text{Here, } n = 1, \text{ so } A = \left(1 + \frac{1}{1}\right)^1$$
$$= 2^1$$
$$= \$2.00$$

So you will have \$2.00 at the end of the year if the interest is compounded once a year.

- Interest compounded 2 times per year.

$$\text{Here, } n = 2, \text{ so } A = \left(1 + \frac{1}{2}\right)^2$$
$$= \left(1\frac{1}{2}\right)^2$$
$$= \left(\frac{3}{2}\right)^2$$
$$= \frac{9}{4}$$
$$= \$2.25$$

So you will have \$2.25 at the end of the year if the interest is compounded twice a year.

- Interest compounded 4 times per year (that is, every three months).

$$\text{Here, } n = 4, \text{ so } A = \left(1 + \frac{1}{4}\right)^4$$
$$= \left(1\frac{1}{4}\right)^4$$
$$= \left(\frac{5}{4}\right)^4$$
$$= \frac{625}{256}$$
$$\approx \$2.44$$

So you will have \$2.44 at the end of the year if the interest is compounded four times a year.

From these values, you can see that the more times the bank compounds the interest earned on your investment, the more money you have in your account at the end of the year. So it seems that if the bank compounds your interest many times, your \$1.00 investment will grow very large. To see what really happens, look at the table of increasing values of $n$ and the corresponding values of $\left(1 + \frac{1}{n}\right)^n$ that appears on the facing page.

Notice that as $n$ gets larger, the value of $\left(1 + \frac{1}{n}\right)^n$ increases quickly for a time. Then, as $n$ gets very large, the value of $\left(1 + \frac{1}{n}\right)^n$ levels off to approximately 2.718, which is also the approximation of the number $e$. It turns out that if you deposit $P$ dollars in an account for one year at an interest rate $r$, and the bank compounds the interest as frequently as possible (this is called continuous compounding), your investment will grow to $A = P \cdot e^r$ dollars.

| **Interest Compounded** | $n$ | $\left(1+\frac{1}{n}\right)^n$ **= total amount** |
|---|---|---|
| once a year | 1 | $\left(1+\frac{1}{1}\right)^1 = 2.00$ |
| twice a year | 2 | $\left(1+\frac{1}{2}\right)^2 = 2.25$ |
| four times a year | 4 | $\left(1+\frac{1}{4}\right)^4 \approx 2.4414$ |
| every month | 12 | $\left(1+\frac{1}{12}\right)^{12} \approx 2.6130$ |
| every day | 365 | $\left(1+\frac{1}{365}\right)^{365} \approx 2.7146$ |
| every hour | 8760 | $\left(1+\frac{1}{8760}\right)^{8760} \approx 2.7181$ |
| every minute | 525,600 | $\left(1+\frac{1}{525600}\right)^{525,600} \approx 2.7183$ |

## Solving Exponential Equations

One place you will use what you have learned about exponents is when solving exponential equations. These are equations in which the variable is an exponent.

*Remember, a function $f$ is one-to-one if each y-value corresponds to one and only one x-value.*

In order to solve some exponential equations, you can use the fact that the exponential function is one-to-one. In other words, if two exponential values are the same, they must have come from the same input value. This can be expressed with the following property:

If $b^m = b^n$, then $m = n$ (here $b > 0$ and $b \neq 1$).

In general, to solve an exponential equation that can be written in the form $b^m = b^n$:

1. Rewrite the equation so that it is in the form $b^m = b^n$.
2. Use the property "If $b^m = b^n$, then $m = n$" to set the exponents equal to each other, then solve.

For example, to solve $2^{4t} = 64$ for $t$:

1. Rewrite the equation. $\quad 2^{4t} = 2^6$
2. Set the exponents equal to each other, then solve for $t$.

$$\begin{aligned} 4t &= 6 \\ t &= \frac{6}{4} \\ &= \frac{3}{2} \end{aligned}$$

So if $2^{4t} = 64$, then $t = \frac{3}{2}$.

Here's another example. To solve $27^{x-1} = 81^{2x}$ for $x$:

1. Rewrite the equation.

$$\begin{aligned} \left(3^3\right)^{x-1} &= \left(3^4\right)^{2x} \\ 3^{3(x-1)} &= 3^{8x} \end{aligned}$$

2. Set the exponents equal to each other, then solve for $x$.

$$3(x-1) = 8x$$
$$3x - 3 = 8x$$
$$-5x = 3$$
$$x = -\frac{3}{5}$$

So if $27^{x-1} = 81^{2x}$, then $x = -\frac{3}{5}$.

In a later lesson you will learn how to solve exponential equations like $2^t = 5$, where you cannot rewrite the equation in the form $b^m = b^n$.

***Answers to Sample Problems***

## Sample Problems

1. Given $y = f(x) = \left(\frac{3}{4}\right)^x$, complete the table of ordered pairs below (to one decimal place), then sketch the graph of the function on the grid.

☑ a. To find the $y$-value for $x = 2$, substitute 2 into $y = \left(\frac{3}{4}\right)^x$, then round to one decimal place.

$$y = \left(\frac{3}{4}\right)^x$$
$$= \left(\frac{3}{4}\right)^2$$
$$= \frac{9}{16}$$
$$= .5625$$
$$\approx .6$$

☐ b. To find the $y$-value for $x = 0$, substitute 0 into $y = \left(\frac{3}{4}\right)^x$, then round to one decimal place. $y =$ _____

*b. 1*

☐ c. To find the $y$-value for $x = -3$, substitute $-3$ into $y = \left(\frac{3}{4}\right)^x$, then round to one decimal place. $y \approx$ _____

*c. 2.4*

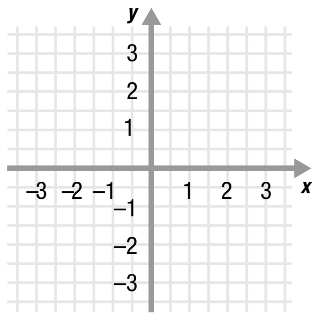
☐ d. Complete the table.

| $x$ | $y$ |
|---|---|
| 3 | .4 |
| 2 | .6 |
| 1 | .8 |
| 0 | ___ |
| –1 | 1.3 |
| –2 | 1.8 |
| –3 | ___ |

*d. 1*

*2.4*

☐ e. Plot the ordered pairs, and join them with a smooth curve.




2. Use your calculator to find $e^{-7.8}$ approximated to five decimal places.

☐ a. Enter 7.8 on your calculator, then press the "±" key.

☐ b. Press the "$e^x$" key and write the result. ________________

☐ c. Round the result of (b) to five decimal places. ________________

3. Solve for $x$: $3^{4-2x} = 9^{8x}$

☐ a. Rewrite the equation in the form $b^m = b^n$, and simplify.

$$3^{4-2x} = 9^{8x}$$
$$3^{4-2x} = 3^{2(8x)}$$

________________

☐ b. Set the exponents equal to each other, then solve for $x$.

________________

$x =$ __________

4. The half-life of radium is approximately 1600 years. This means that approximately half of an original amount of radium remains after 1600 years. Use the formula $A = A_0\left(\frac{1}{2}\right)^{\left(\frac{1}{1600}\right)t}$, where $A$ is the amount of radium left after time $t$ and $A_0$ is the original amount of radium, to find how long it would take for 80mg of radium to disintegrate to 20mg of radium.

☑ a. Substitute the values for $A_0$ and $A$ into the formula.

$$A = A_0\left(\frac{1}{2}\right)^{\left(\frac{1}{1600}\right)t}$$

$$20 = 80\left(\frac{1}{2}\right)^{\left(\frac{1}{1600}\right)t}$$

☑ b. Simplify.

$$\frac{20}{80} = \left(\frac{1}{2}\right)^{\left(\frac{1}{1600}\right)t}$$

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{\left(\frac{1}{1600}\right)t}$$

☐ c. Rewrite the equation in the form $b^m = b^n$. ________________

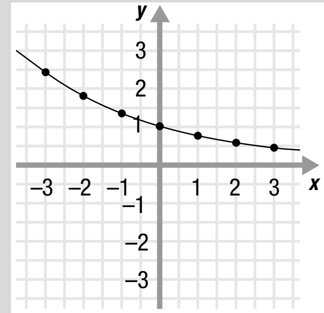
☐ d. Set the exponents equal to each other, then solve for $t$.

$$2 = \frac{1}{1600}t$$

$t =$ __________ years

### *Answers to Sample Problems*

*e.*




*b.* .0004097

*c.* .00041

*a.* $3^{4-2x} = 3^{16x}$

*b.* $4 - 2x = 16x$

$\frac{2}{9}$

*c.* $\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{\left(\frac{1}{1600}\right)t}$

*d.* 3200

# HOMEWORK

## HOMEWORK PROBLEMS

Circle the homework problems assigned to you by the computer, then complete them below.

### Explain
### The Exponential Function

1. Given $y = f(x) = 2^x$, complete this table of ordered pairs.

| $x$ | $y$ |
|---|---|
| 3 | |
| 2 | |
| 1 | 2 |
| 0 | 1 |
| –1 | $\frac{1}{2}$ |
| –2 | |
| –3 | $\frac{1}{8}$ |

2. The graph of the function $y = f(x) = 5^x$ is shown on the grid in Figure 12.1.6. As the value of $x$ increases, does the value of $y$ increase or decrease?

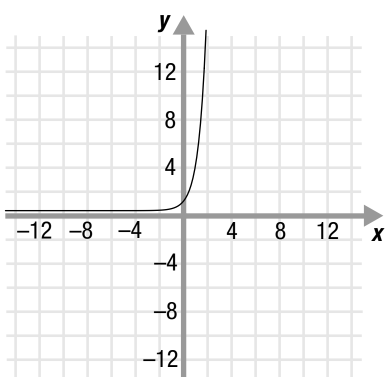


*Figure 12.1.6*

3. Find $e^{-2.6}$ approximated to two decimal places. (Hint: Use your calculator.)

4. If the graph of $f(x) = b^x$ passes through the point (2, 9), find $f(4)$.

5. The graphs of $y = f(x) = b^x$ and $y = g(x) = a^x$ are shown on the grid in Figure 12.1.7. If $b > 1$ and $0 < a < 1$, which graph is the graph of $y = f(x)$, and which graph is the graph of $y = g(x)$?

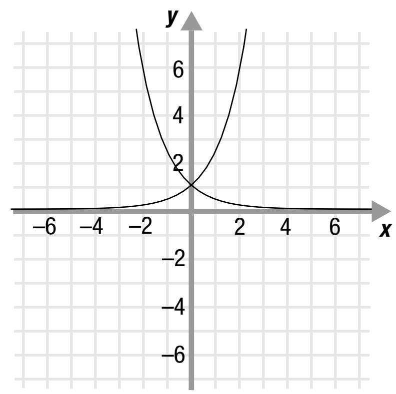


*Figure 12.1.7*

6. Solve for $t$: $4^{2t} = 64$

7. The point (2, 100) lies on the graph of an exponential function $y = b^x$. Find the base $b$ of this exponential function.

8. Graph the function $y = f(x) = \left(\frac{2}{3}\right)^x$. (Hint: Use your calculator and round the $y$-values to the first decimal place.)

9. Pam's biology class is doing an experiment to study the growth of a certain type of bacteria. Suppose that this bacteria grows in such a way that $N = N_0e^{0.04t}$, where $N$ is the number of bacteria present at the end of $t$ hours, and $N_0$ is the number of bacteria present at the start of the experiment. If Pam starts with 12 bacteria in a culture, approximately how many bacteria will be present at the end of 2 days? Round your answer to the nearest whole number. (Hint: 2 days = 48 hours)

10. Use the formula $A = P \cdot e^r$ and a calculator to find the amount $A$ you would have one year after you invested a principal $P = \$1000$ at the rate of $r = 0.03$, compounded continuously.

11. What is the base $b$ of the exponential function $f(x) = b^x$ if $f(-2) = \frac{1}{64}$?

12. Solve for $x$: $e^{3x+1} = \frac{1}{e^{x-2}}$

# APPLY

## Practice Problems

Here are some additional practice problems for you to try.

### The Exponetial Function

1. Circle the ordered pairs below that satisfy the function $y = \left(\frac{1}{4}\right)^x$.

   (0, 1) (1, 0)

   (–1, 4) (–1, –4)

2. Circle the ordered pairs below that satisfy the function $y = \left(\frac{2}{5}\right)^x$.

   (5, 2) (0, 1)

   (–1, 2.5) (–1, –2.5)

3. Circle the ordered pairs below that satisfy the exponential function $y = \left(\frac{2}{3}\right)^x$.

   (0, 1) (1, 0)

   (–1, –1.5) (–1, 1.5)

4. Find $e^{3.4}$ approximated to two decimal places. (Hint: Use your calculator.)

5. Find $e^{1.2}$ approximated to two decimal places. (Hint: Use your calculator.)

6. Find $e^{-2.1}$ approximated to two decimal places. (Hint: Use your calculator.)

7. Graph the function $y = f(x) = 4^x$.

8. Graph the function $y = f(x) = \left(\frac{1}{5}\right)^x$

9. Graph the function $y = f(x) = \left(\frac{1}{3}\right)^x$

10. If the graph of $f(x) = b^x$ passes through the point (2, 64), find $f(-1)$.

11. If the graph of $f(x) = b^x$ passes through the point (–3, 8), find $f(2)$.

12. If the graph of $f(x) = b^x$ passes through the point $\left(3, \frac{125}{8}\right)$, find $f(-1)$.

13. If the graph of $f(x) = b^x$ passes through the point $\left(-2, \frac{1}{16}\right)$, find $f\left(\frac{1}{2}\right)$.

14. The point (3, 27) lies on the graph of the exponential function $y = b^x$. Find the base, $b$, of this exponential function.

15. The point (–2, 9) lies on the graph of an exponential function $y = b^x$. Find the base, $b$, of this exponential function.

16. The point $\left(-2, \frac{9}{16}\right)$ lies on the graph of the exponential function $y = b^x$. Find the base, $b$, of this exponential function.

17. Solve for $x$: $9^x = 27$

18. Solve for $x$: $8^x = 16$

19. Solve for $x$: $125^x = \frac{1}{25}$

20. Solve for $t$: $4^{3t} = 64$

21. Solve for $t$: $9^{4t} = \frac{1}{81}$

22. Solve for $t$: $9^{5t} = \frac{1}{243}$

23. Solve for $x$: $e^{x+1} = e^{3x-1}$

24. Solve for $x$: $e^{2x+3} = \frac{1}{e^{x-9}}$

25. Solve for $x$: $e^{x-1} = \frac{1}{e^{2x-5}}$

26. What is the base, $b$, of the exponential function $f(x) = b^x$ if $f(3) = 64$?

27. What is the base, $b$, of the exponential function $f(x) = b^x$ if $f(2) = \frac{1}{49}$?

28. What is the base, $b$, of the exponential function $f(x) = b^x$ if $f(-2) = \frac{25}{64}$?

# EVALUATE

## Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Circle the ordered pairs below that satisfy the exponential function $y = 2^x$.

   (1, 2) (0, 1)

   (–4, –8) $\left(-2, \frac{1}{4}\right)$

2. Given $f(x) = \left(\frac{1}{2}\right)^x$,
   complete the table of ordered pairs below. Round your answers to four decimal places.

| $x$ | $y$ |
|---|---|
| 2 | $\frac{1}{4}$ |
| 1.5 | |
| 1 | |
| 0 | 1 |
| –1 | 2 |
| –1.5 | |
| –2 | |

3. Circle the statements below that are true for an exponential function of the form $y = f(x) = b^x$.

   The base $b$ can be negative.

   The domain of $f$ is all positive real numbers.

   The range of $f$ includes $y$-values that are negative.

   The function $f$ is one-to-one.

4. Find the approximate value of each of the following to 2 decimal places. (Hint: You may want to use a calculator.)

   a. $e^3$ b. $e^{-1}$

5. The compound interest formula is given by $A = P \cdot \left(1 + \frac{r}{n}\right)^n$, where $A$ is the amount of money in an account after one year, $P$ is the amount invested, $n$ is the number of times the investment is compounded in a year, and $r$ is the rate at which it is compounded. Determine the amount of money in an account at the end of one year if the original investment is \$200 compounded every 4 months at a rate of 3%. (Hint: Since the money is compounded every 4 months, $n = 3$. Also: 3% = .03)

6. A radioactive isotope is said to decay over time. That is, after $t$ years, the original amount of an isotope, $N_0$ grams, decays until the amount is $N$ grams, where $N$ is defined as $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{200}}$. How much of this isotope, in terms of $N_0$, remains at time $t = 600$?

7. Solve this exponential equation for $x$: $3^{2x+1} = 27$

8. Solve this exponential equation for $x$: $e^{x-2} = \frac{1}{e^{3x+4}}$