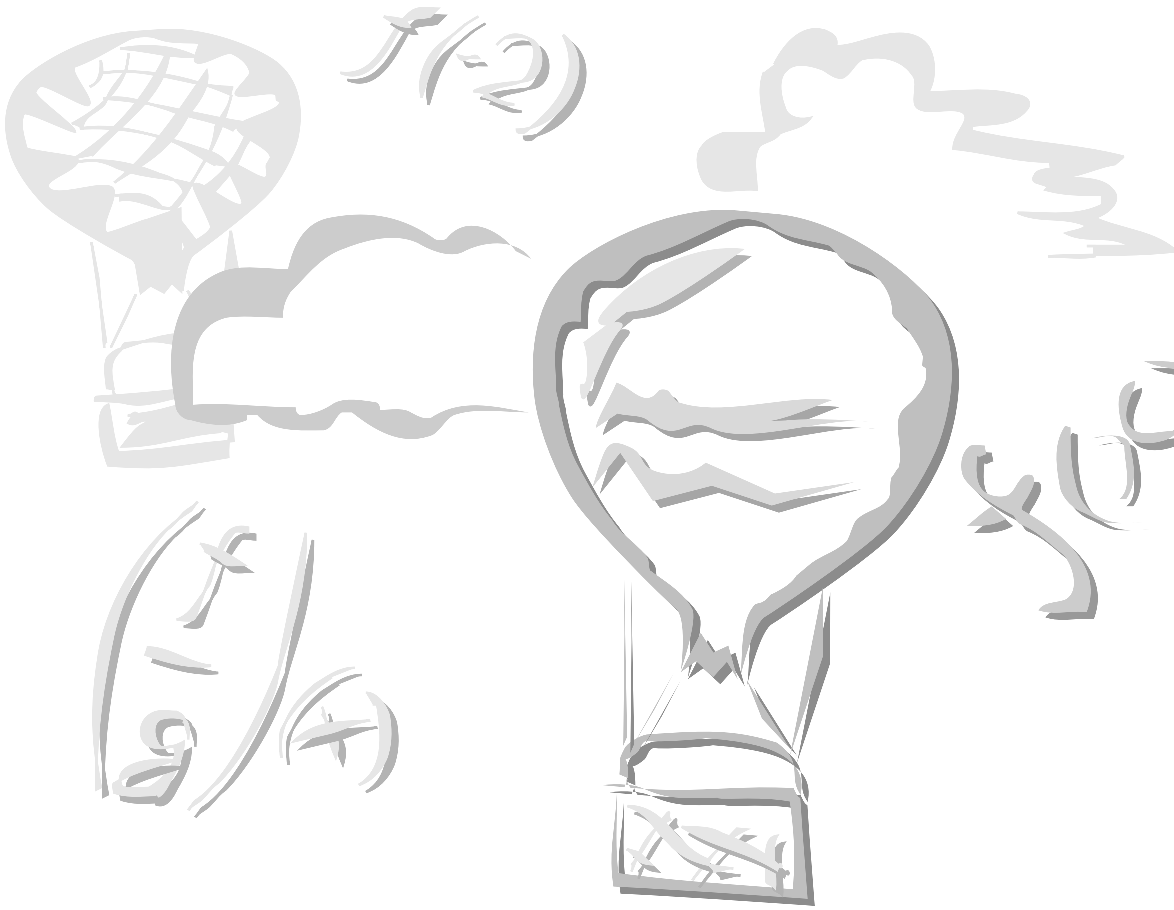
LESSON 11.2 – THE ALGEBRA OF FUNCTIONS



OVERVIEW

Here's what you'll learn in this lesson:

The Algebra of Functions

a. The sum and difference of functions

b. The product and quotient of functions

c. The composition of functions

Inverse Functions

a. Finding the inverse

b. One-to-one: checking whether a function has an inverse

c. Graphing inverse functions

Suppose you are a car salesman with a great idea for drawing people to your sales lot — you'll hire a balloonist to give free "tethered" balloon flights to anyone who test drives a new car!

In order to sell this idea to your boss, you've determined the cost of running the balloon promotion as a function of the number of days the promotion will be run. In addition, you've determined the cost of the liability insurance you'd have to pay. Again, this is a function of the number of days. In order to figure out the total cost of running the promotion, (so your boss can see what a good idea it really is), you add these two functions.

In this lesson, you will learn how to add functions. You will also learn how to subtract, multiply, and divide functions, and how to find the composition of two functions. In addition, you'll learn how to find and graph the inverse of a function.

EXPLAIN

THE ALGEBRA OF FUNCTIONS

Summary

The Sum and Difference of Functions

In algebra, if you are given two functions f and g with the same domain, you can produce new functions called the sum $(f + g)$ or the difference $(f - g)$ of the two functions. The sum $(f + g)$ is defined as follows: $(f + g)(x) = f(x) + g(x)$. In other words, to get $(f + g)(x)$ you add the value $f(x)$ and the value $g(x)$. Similarly, the difference $(f - g)$ is defined as follows: $(f - g)(x) = f(x) - g(x)$. So to get $(f - g)(x)$ you subtract the value $g(x)$ from the value $f(x)$.

In general, to find the sum $(f + g)(x)$ of two functions $f(x)$ and $g(x)$ with the same domain:

1. Add the value $f(x)$ to the value $g(x)$.
2. Combine like terms, if any.

For example, if $f(x) = 2x^2 - 3x$ and $g(x) = x^2 + 2x$, to find $(f + g)(x)$:

1. Add the value $f(x)$ to the value $g(x)$. $\quad (f + g)(x) = f(x) + g(x)$
   $= (2x^2 - 3x) + (x^2 + 2x)$
2. Combine like terms. $\quad = 3x^2 - x$

So $(f + g)(x) = 3x^2 - x$.

The graphs of $y = f(x) = 2x^2 - 3x$, $y = g(x) = x^2 + 2x$, and $y = (f + g)(x) = 3x^2 - x$ are shown in Figure 11.2.1.

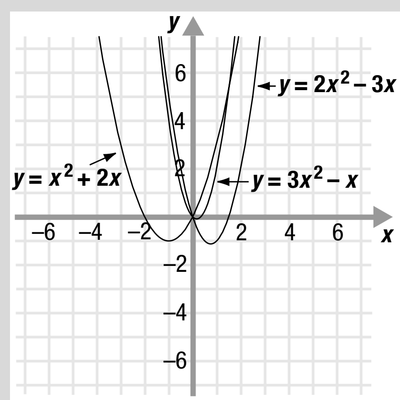


Figure 11.2.1

In general, to find the difference $(f - g)(x)$ of two functions $f(x)$ and $g(x)$ with the same domain:

1. Subtract the value $g(x)$ from the value $f(x)$.
2. Combine like terms, if any.

For example, if $f(x) = 2x^2 - 3x$ and $g(x) = x^2 + 2x$, to find $(f - g)(x)$:

1. Subtract the value $g(x)$ from the value $f(x)$. $\quad (f - g)(x) = f(x) - g(x)$
   $= (2x^2 - 3x) - (x^2 + 2x)$
   $= 2x^2 - 3x - x^2 - 2x$
2. Combine like terms. $\quad = x^2 - 5x$

So $(f - g)(x) = x^2 - 5x$.

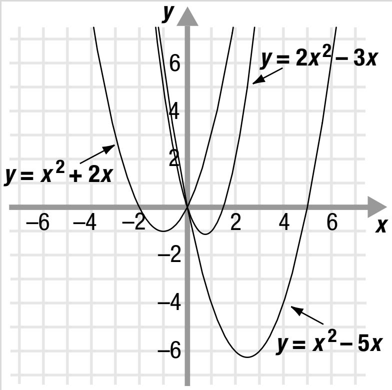


Figure 11.2.2

The graphs of $y = f(x) = 2x^2 - 3x$, $y = g(x) = x^2 + 2x$, and $y = (f - g)(x) = x^2 - 5x$ are shown in Figure 11.2.2.

Given two functions f and g and given a number $x = a$ in the domain of both f and g, you can evaluate the sum $(f + g)$ or the difference $(f - g)$ at $x = a$ using either one of two methods.

Method 1

To find the value $(f + g)(x)$ or $(f - g)(x)$ at $x = a$ using the first method:

1. Find $(f + g)(x)$ or $(f - g)(x)$.
2. Substitute a for x in $(f + g)(x)$ or $(f - g)(x)$.
3. Simplify.

For example, if $f(x) = 2x^2 - 3x$ and $g(x) = x^2 + 2x$, to find the value $(f + g)(x)$ at $x = 4$:

You found $(f + g)(x)$ for these functions in an example above:

$$(f + g)(x) = f(x) + g(x)$$
$$= (2x^2 - 3x) + (x^2 + 2x)$$
$$= 3x^2 - x$$

1.	Find $(f + g)(x)$.	$(f + g)(x) = 3x^2 - x$
2.	Substitute 4 for x in $(f + g)(x)$.	$(f + g)(4) = 3 \cdot 4^2 - 4$
3.	Simplify.	$= 48 - 4$
		$= 44$

So the value $(f + g)(x) = 3x^2 - x$ at $x = 4$ is 44.

As another example, if $f(x) = 2x^2 - 3x$ and $g(x) = x^2 + 2x$, to find the value $(f - g)(x)$ at $x = 4$:

You found $(f - g)(x)$ for these functions in an example above:

$$(f - g)(x) = f(x) - g(x)$$
$$= (2x^2 - 3x) - (x^2 + 2x)$$
$$= 2x^2 - 3x - x^2 - 2x$$
$$= x^2 - 5x$$

1.	Find $(f - g)(x)$.	$(f - g)(x) = x^2 - 5x$
2.	Substitute 4 for x in $(f - g)(x)$.	$(f - g)(4) = 4^2 - 5 \cdot 4$
3.	Simplify.	$= 16 - 20$
		$= -4$

So the value $(f - g)(x) = x^2 - 5x$ at $x = 4$ is -4.

Method 2

To find the value $(f + g)(x)$ or $(f - g)(x)$ at $x = a$ using the second method:

1. Substitute a for x in $f(x)$ and a for x in $g(x)$.
2. Add or subtract the resulting values of the two functions.

For example, if $f(x) = 2x^2 - 3x$ and $g(x) = x^2 + 2x$, to find the value $(f + g)(x)$ at $x = 4$:

1. Substitute 4 for x in both $f(x)$ and $g(x)$.

$$f(x) = 2x^2 - 3x \qquad g(x) = x^2 + 2x$$
$$f(4) = 2 \cdot 4^2 - 3 \cdot 4 \qquad g(4) = 4^2 + 2 \cdot 4$$
$$= 32 - 12 \qquad = 16 + 8$$
$$= 20 \qquad = 24$$

2. Add the resulting values.

$$f(4) + g(4) = 20 + 24$$
$$= 44$$

Again, the value $(f + g)(x) = f(x) + g(x)$ at $x = 4$ is 44.

As another example, if $f(x) = 2x^2 - 3x$ and $g(x) = x^2 + 2x$, to find the value $(f - g)(x)$ at $x = 4$:

1. Substitute 4 for x in both $f(x)$ and $g(x)$.

$$f(x) = 2x^2 - 3x \qquad g(x) = x^2 + 2x$$
$$f(4) = 2 \cdot 4^2 - 3 \cdot 4 \qquad g(4) = 4^2 + 2 \cdot 4$$
$$= 32 - 12 \qquad = 16 + 8$$
$$= 20 \qquad = 24$$

2. Subtract the resulting values.

$$f(4) - g(4) = 20 - 24$$
$$= -4$$

Again, the value $(f - g)(x) = f(x) - g(x)$ at $x = 4$ is -4.

The Product and Quotient of Functions

Given two functions f and g with the same domain, you can also produce new functions called the product $(f \cdot g)$ or quotient $\left(\frac{f}{g}\right)$ of the two functions. The product $(f \cdot g)$ is defined as follows: $(f \cdot g)(x) = f(x) \cdot g(x)$. The quotient $\left(\frac{f}{g}\right)$ is defined as follows: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ (here, $g(x) \neq 0$).

In general, to find the product $(f \cdot g)(x)$ of two functions $f(x)$ and $g(x)$ with the same domain:

1. Multiply the values $f(x)$ and $g(x)$.
2. Combine like terms, if any.

For example, if $f(x) = x^2 - 2$ and $g(x) = x^2 + 1$, to find $(f \cdot g)(x)$:

1. Multiply the values $f(x)$ and $g(x)$.

$$(f \cdot g)(x) = f(x) \cdot g(x)$$
$$= (x^2 - 2) \cdot (x^2 + 1)$$
$$= x^2 \cdot x^2 + x^2 \cdot 1 - 2 \cdot x^2 - 2 \cdot 1$$
$$= x^4 + x^2 - 2x^2 - 2$$

2. Combine like terms.

$$= x^4 - x^2 - 2$$

So, $(f \cdot g)(x) = x^4 - x^2 - 2$.

Remember, you can use this pattern to multiply two binomials:

$(a + b)(c + d) = ac + ad + bc + bd$

In general, to find the quotient $\left(\frac{f}{g}\right)(x)$ of two functions $f(x)$ and $g(x)$ with the same domain, and with $g(x) \neq 0$:

1. Divide the value $f(x)$ by the value $g(x)$.
2. Simplify, if possible.

Here, we assume $g(x) = x + 2 \neq 0$. In other words, $x \neq -2$.

For example, to find $\left(\frac{f}{g}\right)(x)$ if $f(x) = x^2 - 4$ and $g(x) = x + 2$:

1. Divide the value $f(x)$ by the value $g(x)$.
2. Simplify.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$
$$= \frac{x^2 - 4}{x + 2}$$
$$= \frac{(x-2)(x+2)}{x+2}$$
$$= \frac{(x-2)\cancel{(x+2)}^{1}}{\cancel{x+2}_{1}}$$
$$= x - 2$$

So, $\left(\frac{f}{g}\right)(x) = x - 2$.

Now you can evaluate the product $(f \cdot g)$ or quotient $\left(\frac{f}{g}\right)$ of two functions f and g at a given value $x = a$ using either one of two methods, just as you can when evaluating the sum or difference of two functions at $x = a$.

Method 1

To find the value $(f \cdot g)(x)$ or $\left(\frac{f}{g}\right)(x)$ at $x = a$ using the first method:

1. Find $(f \cdot g)(x)$ or $\left(\frac{f}{g}\right)(x)$.
2. Substitute a for x in $(f \cdot g)(x)$ or $\left(\frac{f}{g}\right)(x)$.
3. Simplify.

For example, if $f(x) = x^2 - 2$ and $g(x) = x^2 + 1$, to find the value $(f \cdot g)(x)$ at $x = -2$:

You found $(f \cdot g)(x)$ for these functions in an example above:

$(f \cdot g)(x) = f(x) \cdot g(x)$

$= (x^2 - 2) \cdot (x^2 + 1)$

$= x^2 \cdot x^2 + x^2 \cdot 1 - 2 \cdot x^2 - 2 \cdot 1$

$= x^4 + x^2 - 2x^2 - 2$

$= x^4 - x^2 - 2$

1. Find $(f \cdot g)(x)$. $\quad (f \cdot g)(x) = x^4 - x^2 - 2$
2. Substitute -2 for x in $(f \cdot g)(x)$. $\quad (f \cdot g)(-2) = (-2)^4 - (-2)^2 - 2$
3. Simplify. $\quad = 16 - 4 - 2$

$\quad = 10$

So the value $(f \cdot g)(x) = x^4 - x^2 - 2$ at $x = -2$ is 10.

As another example, if $f(x) = x^2 - 4$ and $g(x) = x + 2$, to find the value $\left(\frac{f}{g}\right)(x)$ at $x = -3$:

1. Find $\left(\frac{f}{g}\right)(x)$. $\qquad \left(\frac{f}{g}\right)(x) = x - 2$
2. Substitute –3 for x in $\left(\frac{f}{g}\right)(x)$. $\qquad \left(\frac{f}{g}\right)(-3) = -3 - 2$
3. Simplify. $\qquad = -5$

So the value $\left(\frac{f}{g}\right)(x) = x - 2$ at $x = -3$ is –5.

You found $\left(\frac{f}{g}\right)(x)$ for these functions in an example above:

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}\\ &= \frac{x^2-4}{x+2}\\ &= \frac{(x-2)(x+2)}{x+2}\\ &= \frac{(x-2)\cancel{(x+2)}^{1}}{\cancel{x+2}_{1}}\\ &= x-2\end{aligned}$$

Here, we assume $x \neq 2$.

Method 2

To find the value $(f \cdot g)(x)$ or $\left(\frac{f}{g}\right)(x)$ at $x = a$ using the second method:

1. Substitute a for x in $f(x)$ and a for x in $g(x)$.
2. Multiply or divide the resulting values of the two functions.

For example, if $f(x) = x^2 - 2$ and $g(x) = x^2 + 1$, to find the value $(f \cdot g)(x)$ at $x = -2$:

1. Substitute –2 for x in both $f(x)$ and $g(x)$.

$$\begin{aligned} f(x) &= x^2 - 2 & g(x) &= x^2 + 1\\ f(-2) &= (-2)^2 - 2 & g(-2) &= (-2)^2 + 1\\ &= 4 - 2 & &= 4 + 1\\ &= 2 & &= 5\end{aligned}$$

2. Multiply the resulting values.

$$\begin{aligned} f(-2) \cdot g(-2) &= 2 \cdot 5\\ &= 10\end{aligned}$$

Again, the value $(f \cdot g)(x) = f(x) \cdot g(x)$ at $x = -2$ is 10.

As another example, if $f(x) = x^2 - 4$ and $g(x) = x + 2$, to find the value $\left(\frac{f}{g}\right)(x)$ at $x = -3$:

1. Substitute –3 for x in both $f(x)$ and $g(x)$.

$$\begin{aligned} f(x) &= x^2 - 4 & g(x) &= x + 2\\ f(-3) &= (-3)^2 - 4 & g(-3) &= -3 + 2\\ &= 9 - 4 & &= -1\\ &= 5 & &\end{aligned}$$

2. Divide the resulting values.

$$\begin{aligned} \frac{f(-3)}{g(-3)} &= \frac{5}{-1}\\ &= -5\end{aligned}$$

So, again, the value $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ at $x = -3$ is –5.

The Composition of Functions

You have seen how to define addition, subtraction, multiplication, and division of two functions f and g to produce new functions $(f + g)$, $(f - g)$, $(f \cdot g)$, and $\left(\frac{f}{g}\right)$, respectively. You can also combine two functions f and g in yet another way. This new way is called the composition of f and g. It is denoted by $(f \circ g)$, and is defined as follows: $(f \circ g)(x) = f[g(x)]$. Observe that here the output of the function g is the input of the function f.

When you compose two functions f and g to form $f \circ g$, the range of g must be contained in the domain of f. In other words, all the possible outputs for g must be inputs for f.

Remember, this is the pattern for a perfect square trinomial:

$(a - b)^2 = a^2 - 2ba + b^2$

In general, to find the composition $(f \circ g)(x)$ of two functions $f(x)$ and $g(x)$:

1. Use the output of the function g as the input of the function f. That is, evaluate the function f at the value $g(x)$.
2. Simplify, if possible.

For example, if $f(x) = x^2 - 2$ and $g(x) = 2x - 1$, to find $(f \circ g)(x)$:

1. Evaluate f at $g(x)$.
$$\begin{aligned}(f \circ g)(x) &= f[g(x)] \\ &= f(2x - 1) \\ &= (2x - 1)^2 - 2\end{aligned}$$
2. Simplify.
$$\begin{aligned} &= (2x)^2 - 2(1)(2x) + (-1)^2 - 2 \\ &= 4x^2 - 4x + 1 - 2 \\ &= 4x^2 - 4x - 1\end{aligned}$$

So, $(f \circ g)(x) = 4x^2 - 4x - 1$.

As another example, if $f(x) = 3x^2 - 4$ and $g(x) = x + 2$, to find $(f \circ g)(x)$:

1. Evaluate f at $g(x)$.
$$\begin{aligned}(f \circ g)(x) &= f[g(x)] \\ &= f(x + 2) \\ &= 3(x + 2)^2 - 4\end{aligned}$$
2. Simplify.
$$\begin{aligned} &= 3[x^2 + 2(2)(x) + 2^2] - 4 \\ &= 3[x^2 + 4x + 4] - 4 \\ &= 3x^2 + 12x + 12 - 4 \\ &= 3x^2 + 12x + 8\end{aligned}$$

So, $(f \circ g)(x) = 3x^2 + 12x + 8$.

You can evaluate the composition $(f \circ g)$ of two functions f and g at a given value $x = a$ using either one of two methods, just as you can when evaluating the sum or difference, or the product or quotient of two functions at $x = a$.

Method 1

To find the value $(f \circ g)(x)$ at $x = a$ using the first method:

1. Find $(f \circ g)(x)$.
2. Substitute a for x in $(f \circ g)(x)$.
3. Simplify.

For example, if $f(x) = x^2 - 2$ and $g(x) = 2x - 1$, to find the value $(f \circ g)(x)$ at $x = -1$:

1. Find $(f \circ g)(x)$. $\quad (f \circ g)(x) = 4x^2 - 4x - 1$
2. Substitute -1 for x in $(f \circ g)(x)$. $\quad (f \circ g)(-1) = 4(-1)^2 - 4(-1) - 1$
3. Simplify. $\quad = 4 + 4 - 1$

   $= 7$

So the value $(f \circ g)(x) = 4x^2 - 4x - 1$ at $x = -1$ is 7.

You found $(f \circ g)(x)$ for these functions in an example above:

$$\begin{aligned}(f \circ g)(x) &= f[g(x)]\\ &= f(2x - 1)\\ &= (2x - 1)^2 - 2\\ &= (2x)^2 - 2(1)(2x) + (-1)^2 - 2\\ &= 4x^2 - 4x + 1 - 2\\ &= 4x^2 - 4x - 1\end{aligned}$$

Method 2

To find the value $(f \circ g)(x)$ at $x = a$ using the second method:

1. Substitute a for x in $g(x)$.
2. Evaluate f at $g(a)$.

For example, if $f(x) = x^2 - 2$ and $g(x) = 2x - 1$, to find the value $(f \circ g)(x)$ at $x = -1$:

1. Substitute -1 for x in $g(x)$.

$$\begin{aligned}g(x) &= 2x - 1\\ g(-1) &= 2(-1) - 1\\ &= -2 - 1\\ &= -3\end{aligned}$$

2. Find $f[g(-1)]$.

$$\begin{aligned}f[g(-1)] &= f(-3)\\ &= (-3)^2 - 2\\ &= 9 - 2\\ &= 7\end{aligned}$$

So, again, the value $(f \circ g)(x) = f[g(x)]$ at $x = -1$ is 7.

Sample Problems

1. Given $f(x) = 3x^2 + 4$ and $g(x) = -6x^2 - 5$, find $(f + g)(x)$.

   ☑ a. Add the value $f(x)$ to the value $g(x)$. $(f + g)(x) = f(x) + g(x)$
   $= (3x^2 + 4) + (-6x^2 - 5)$

   ☐ b. Combine like terms.
   $=$ ____________________

2. Given $f(x) = 2x^3$ and $g(x) = -x^2 + 5x$, find $(f \cdot g)(x)$.

   ☐ a. Multiply the values $f(x)$ and $g(x)$. $(f \cdot g)(x) = f(x) \cdot g(x)$
   $=$ ____________________

   ☐ b. Simplify.
   $=$ ____________________

3. Given $f(x) = 4x^2 - 9$ and $g(x) = 3 + 2x$, evaluate $\left(\frac{f}{g}\right)(x)$ at $x = 1$.

   ☑ a. Divide the value $f(x)$ by the value $g(x)$. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
   $= \frac{4x^2 - 9}{3 + 2x}$

   ☐ b. Simplify. $=$ ____________________

   ☐ c. Find $\left(\frac{f}{g}\right)(1)$. $=$ ____________________

4. Given $f(x) = 4x^2 - 9$ and $g(x) = 3 + 2x$, evaluate $(f \circ g)(x)$ at $x = -2$.

   ☑ a. Substitute -2 for x in $g(x)$.
   $g(x) = 3 + 2x$
   $g(-2) = 3 + 2(-2)$
   $= 3 - 4$
   $= -1$

   ☐ b. Substitute -1 for x in $f(x)$ to find $f[g(-2)]$, and simplify.
   $f(x) = 4x^2 - 9$
   $f[g(-2)] = f(-1) =$ __________

Answers to Sample Problems

b. $-3x^2 - 1$

a. $2x^3 \cdot (-x^2 + 5x)$

b. $-2x^5 + 10x^4$

b. $2x - 3$

c. -1

b. -5

INVERSE FUNCTIONS

Summary

Finding an Inverse Function and the Equation of an Inverse

Given two functions f and g with a common domain, you have seen how to produce new functions by combining these functions in various ways. Now you'll learn about another function, the inverse function.

Suppose you have a function $f(x) = 3x$ and a function $g(x) = \frac{1}{3}x$. The graphs of $y = f(x) = 3x$ and $y = g(x) = \frac{1}{3}x$ are shown in Figure 11.2.3. Notice that f and g are functions since for each function each input value is assigned to exactly one output value.

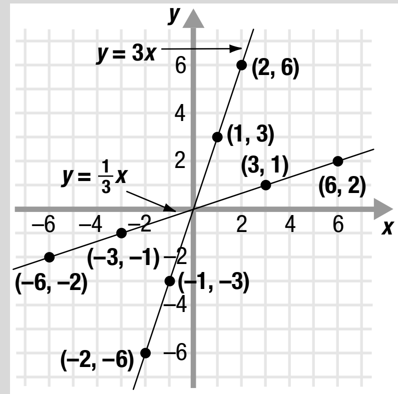


Figure 11.2.3

*Remember, another way to tell whether or not a graph is the graph of a function is to draw vertical lines anywhere on the graph. If any vertical line intersects the graph in more than one place, the graph is **not** the graph of a function.*

Here is a table of some of the ordered pairs that satisfy f:

f

x	y
–2	–6
–1	–3
0	0
1	3
2	6

Here is a table of some of the ordered pairs that satisfy g:

g

x	y
–6	–2
–3	–1
0	0
3	1
6	2

The y-values of f are the x-values of g, and the x-values of f are the y-values of g. Notice that f takes an input, x, and multiplies it by 3 to get the output, y; whereas $g(x)$ takes an input, x, and multiplies it by $\frac{1}{3}$ to get the output, y. So the function g "undoes" what the function f does—that is, the output of g is the original input of f. The function g is called the inverse of the function f. You can write this as $f^{-1}(x) = g(x) = \frac{1}{3}x$.

*Be careful! Here the –1 in $f^{-1}(x)$ is just a notation. It is **not** an exponent, so $f^{-1}(x) \neq \frac{1}{f(x)}$.*

In general, when you find the inverse of a function f, if (x, y) is an ordered pair of f, then (y, x) is an ordered pair of f^{-1}.

To find the equation of the inverse, f^{-1}, of the function f:

1. Replace $f(x)$ with y.
2. In the equation, switch y and x.
3. Solve the equation for y.
4. Replace y with $f^{-1}(x)$.

For example, to find the inverse of $f(x) = 5x - 3$:

1. Replace $f(x)$ with y. $\quad y = 5x - 3$
2. Switch y and x. $\quad x = 5y - 3$
3. Solve for y.
$$x + 3 = 5y$$
$$\frac{x+3}{5} = y$$
$$y = \frac{x+3}{5}$$
4. Replace y with $f^{-1}(x)$. $\quad f^{-1}(x) = \frac{x+3}{5}$

So, if $f(x) = 5x - 3$, then $f^{-1}(x) = \frac{x+3}{5}$.

Here's a slightly different example. If $f(x) = 5x - 3$, you now know that $f^{-1}(x) = \frac{x+3}{5}$. So to find $(f^{-1} \circ f)(x)$:

1. Find f^{-1} at $f(x)$.
$$(f^{-1} \circ f)(x) = f^{-1}[f(x)]$$
$$= f^{-1}[5x - 3]$$
$$= \frac{(5x-3)+3}{5}$$
2. Simplify.
$$= \frac{5x}{5}$$
$$= x$$

So, if $f(x) = 5x - 3$, then $f^{-1}(x) = \frac{x+3}{5}$, and $(f^{-1} \circ f)(x) = x$.

In general, if the inverse f^{-1} of a function f exists, then
$$(f^{-1} \circ f)(x) = f^{-1}[f(x)] = x$$
for every x in the domain of f and
$$(f \circ f^{-1})(x) = f[f^{-1}(x)] = x$$
for every x in the domain of f^{-1}.

That is, the inverse function f^{-1} "undoes" what the original function f does, and f "undoes" what the inverse function f^{-1} does.

Remember: A function is a rule that assigns to each input value exactly one output value.

One-to-One: Checking Whether a Function Has an Inverse

Some functions do not have inverses. As an example, if you look at the table of ordered pairs (x, y) for the function f below and interchange its x-values and y-values, you can get a new table whose ordered pairs do not represent a function.

Here is a table of some ordered pairs that satisfy $f(x) = 2x^2$:

x	y
–2	8
–1	2
0	0
1	2
2	8

Here is a table of these ordered pairs with the x- and y-values interchanged:

x	y
8	–2
2	–1
0	0
2	1
8	2

In the second table, when $x = 2$, $y = -1$ **and** $y = 1$. So the ordered pairs in the second table do not represent a function. If you look back at the table of ordered pairs for f, you see that when $y = 8$, $x = -2$ **and** $x = 2$. It turns out that for a function to have an inverse, each y-value of f must correspond to exactly one x-value. If this is true, the function is said to be one-to-one. The function $f(x) = 2x^2$ is not one-to-one, so it does not have an inverse.

A nice graphical way to tell if a function is one-to-one is to use the horizontal line test. This tests says that if every horizontal line crosses the graph of a function in at most one point, the function is one-to-one. So, for example, the function $y = f(x) = 2x^2$ shown in Figure 11.2.4 is **not** one-to-one since there is at least one horizontal line that crosses the graph in two places.

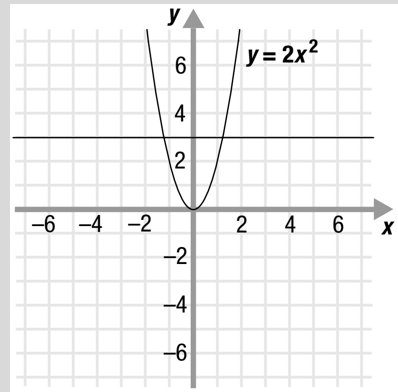


Figure 11.2.4

To determine if a function f is one-to-one, and hence, has an inverse:

1. Graph the function. Use a table of ordered pairs if necessary.
2. Use the horizontal line test.

For example, to determine whether or not the function $f(x) = x^2 - 1$ has an inverse:

1. Graph the function $y = f(x)$. See the grid in Figure 11.2.5.

x	y
-2	3
-1	0
0	-1
1	0
2	3

2. Use the horizontal line test. See the grid in Figure 11.2.5.

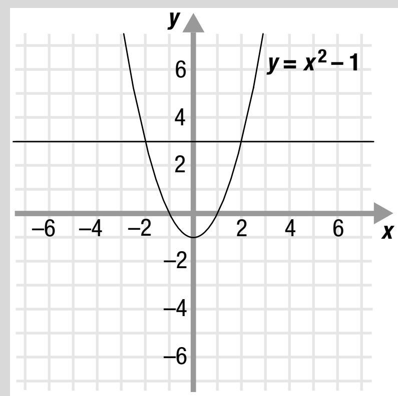


Figure 11.2.5

*If you make a table of values, you can also check the table to see if for every y-value in the table, there is exactly one corresponding x-value. As you can see in this table, when $y = 3$, $x = -2$ **and** $x = 2$, so this function is **not** one-to-one.*

Since you can find a horizontal line that crosses the graph in more than one place, the function $f(x) = x^2 - 1$ is **not** one-to-one, and hence does **not** have an inverse.

Graphing Inverse Functions

If a function f has an inverse f^{-1}, you can find that inverse by interchanging the x- and y-coordinates of f.

To graph the inverse of a function:

1. Graph the original function, $y = f(x)$.
2. Locate some points (a, b) on the graph of f, and then draw the points (b, a).
3. Connect the new points. This is the graph of $y = f^{-1}(x)$.

If the graph of f is a line, then the graph of f^{-1} is also a line and you only need two points to graph f^{-1}. But, if the graph of f is a more general curve, you will need more than two points to help you graph f^{-1}.

For example, to graph the inverse of $y = f(x) = 2x - 5$:

1. Graph the line $y = f(x) = 2x - 5$. — See the grid in Figure 11.2.6.
2. Locate two points (a, b) on the graph of f, and two new points, (b, a). — The points are shown in Figure 11.2.7.
3. Connect the new points. This is the graph of the inverse function, $y = f^{-1}(x) = \frac{x+5}{2}$. — See the grid in Figure 11.2.8.

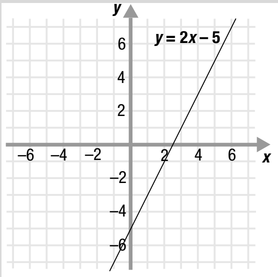


Figure 11.2.6

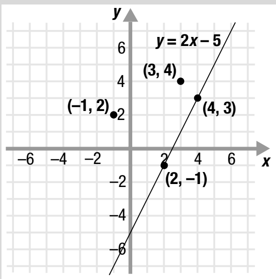


Figure 11.2.7

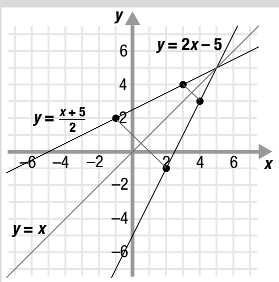


Figure 11.2.8

Notice the line $y = x$ in Figure 11.2.8, and the line segments drawn from points (a, b) to points (b, a). The line $y = x$ is the perpendicular bisector of each segment that joins (a, b) to (b, a), and acts as a mirror for the line $y = f(x) = 2x - 5$. So, another way to graph the inverse f^{-1} of a function f is to reflect the graph of f about the line $y = x$.

For example, to graph the inverse of $y = f(x) = -2x - 2$:

1. Graph $y = f(x) = -2x - 2$. — See the grid in Figure 11.2.9.
2. Reflect the graph of $f(x)$ about the line $y = x$. The result is the graph of $y = f^{-1}(x) = -\frac{x+2}{2}$. — See the grid in Figure 11.2.9.

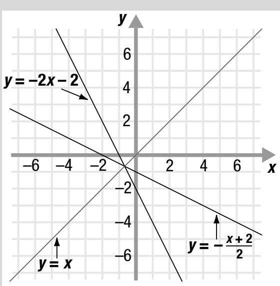


Figure 11.2.9

Sample Problems

1. If $f(x) = 3x + 4$, find $f^{-1}(x)$.

   ☑ a. Replace $f(x)$ with y. $y = 3x + 4$

   ☐ b. Switch y and x. ____________

   ☐ c. Solve for y. ____________

   ☐ d. Replace y with $f^{-1}(x)$. ____________

Answers to Sample Problems

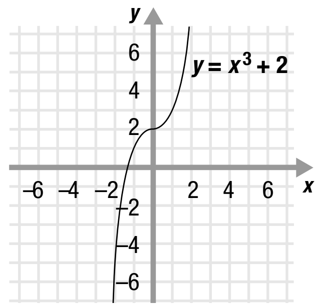
b. $x = 3y + 4$

c. $y = \frac{x-4}{3}$

d. $f^{-1}(x) = \frac{x-4}{3}$

2. Does the function $f(x) = x^3 + 2$ have an inverse?

- ☑ a. Graph the function $y = f(x)$.



- ☐ b. Use the horizontal line test. Does the function have an inverse? (Circle one) — The function has/does not have an inverse.

3. If $f(x) = 6x - 2$, find $f^{-1}(4)$.

- ☐ a. Replace $f(x)$ with y. ____________________
- ☐ b. Switch y and x. ____________________
- ☐ c. Solve for y. ____________________
- ☐ d. Replace y with $f^{-1}(x)$. ____________________
- ☐ e. Find $f^{-1}(4)$. $f^{-1}(4) =$ ________

4. Given $f(x) = 6x - 2$, graph $y = f^{-1}(x)$.

- ☐ a. Graph $y = f(x)$.
- ☐ b. Locate two points (a, b) on the graph, and two new points (b, a).
- ☐ c. Connect the new points. This is the graph of $y = f^{-1}(x) = \frac{x + 2}{6}$.

Answers to Sample Problems

*b. Every horizontal line crosses the graph in at most one point, so the function **has** an inverse.*

a. $y = 6x - 2$

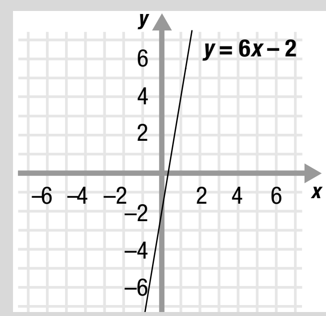
b. $x = 6y - 2$

c. $y = \frac{x + 2}{6}$

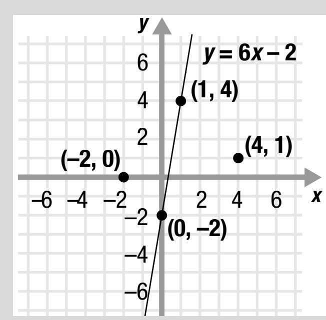
d. $f^{-1}(x) = \frac{x + 2}{6}$

e. 1

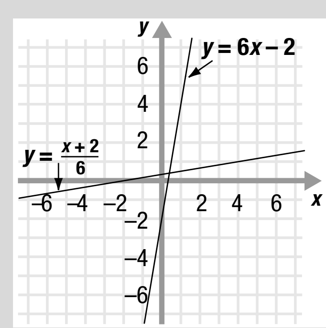
a.



b. Here are some possible points:



c.



EXPLORE

Answers to Sample Problems

Sample Problems

On the computer, you used the Grapher to combine functions by adding, subtracting, multiplying, and dividing them. You also used the Grapher to learn more about the inverse of a function and the composition of two functions. Below are some additional exploration problems.

1. Let $r(x) = 4x + 2$, $c(x) = 2x + 50$, and $p(x) = (r + c)(x)$. Complete the following table.

x	$r(x)$	$c(x)$	$p(x)$
–3			
2		54	
		64	94

☑ a. Find $r(-3)$, $c(-3)$, and $p(-3)$ to complete the first row.

$$r(-3) = 4(-3) + 2$$
$$= -10$$

$$c(-3) = 2(-3) + 50$$
$$= 44$$

$$p(-3) = (r + c)(-3)$$
$$= r(-3) + c(-3)$$
$$= -10 + 44$$
$$= 34$$

b. 10

64

☐ b. Find $r(2)$ and $p(2)$ to complete the second row.

$r(2) =$ ______

$p(2) =$ ______

c. 7

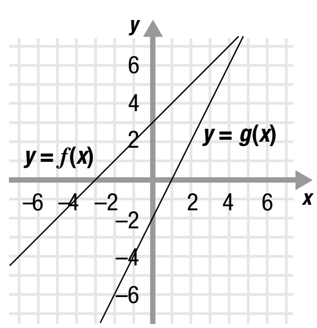
7, 30

☐ c. Find x, then find $r(x)$ to complete the third row.

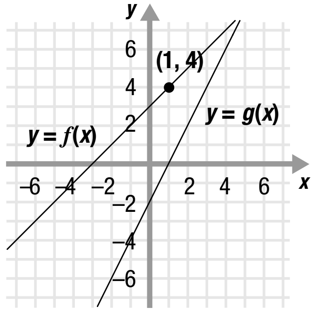
$x =$ ______

$r($____$) =$ ______

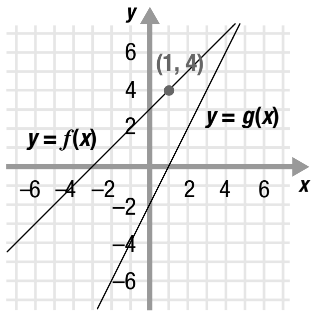
2. The graphs of $y = f(x)$ and $y = g(x)$ are shown on the grid below. Use these graphs to find $(g \circ f)(1)$:



☑ a. Find the y-coordinate of the point on the graph of f where $x = 1$.



☐ b. Use that y-value, 4, as an x-value for g. Find the y-coordinate of the point on the graph of g where $x = 4$.



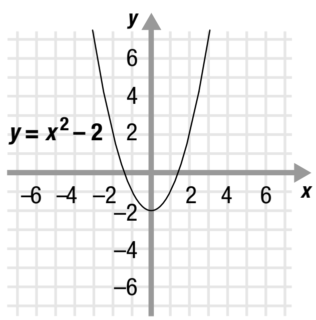
When $x = 4$, $y =$ ________.

☐ c. The value $(g \circ f)(1)$ is the y-value of the point found in (b).

$(g \circ f)(1) =$ ________

3. Graph the function $y = f(x) = x^2 - 2$. If $g(x) = |x|$, sketch the graph of $(g \circ f)(x)$.

☑ a. Graph $y = f(x) = x^2 - 2$.



☐ b. Find $(g \circ f)(x)$.

$(g \circ f)(x) = g[f(x)] =$ ____________

☐ c. Sketch the graph of $(g \circ f)$.

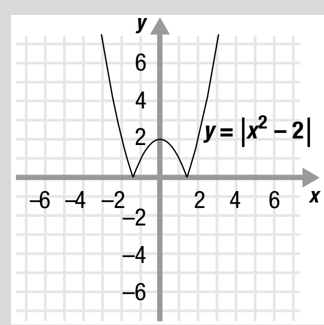
Answers to Sample Problems

b. 6

c. 6

b. $|x^2 - 2|$

c.



Answers to Sample Problems

4. Let $f(x) = 3x + 0.5$, $g(x) = 2x - 8$, and $h(x) = (f \cdot g)(x)$. Complete the following table:

x	$f(x)$	$g(x)$	$h(x)$
–1		–10	
$\frac{1}{2}$			–14
3	9.5		

☐ a. Find $f(-1)$ and $h(-1)$ to complete the first row.

a. –2.5

$f(-1) =$ ________

25

$h(-1) =$ ________

☐ b. Find $f\left(\frac{1}{2}\right)$ and $g\left(\frac{1}{2}\right)$ to complete the second row.

b. 2

$f\left(\frac{1}{2}\right) =$ ______

–7

$g\left(\frac{1}{2}\right) =$ ______

☐ c. Find $g(x)$ and $h(x)$ to complete the third row.

c. –2

$g(x) =$ ______

–19

$h(x) =$ ______

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

The Algebra of Functions

1. Given $f(x) = 5x$ and $g(x) = -6x + 1$, find $(f + g)(x)$.

2. Given $f(x) = 5x$ and $g(x) = -6x + 1$, find $(f - g)(x)$.

3. Given $f(x) = x + 1$ and $h(x) = 3x$, find $(f \circ h)(x)$.

4. Given $f(x) = 5x$ and $g(x) = -6x + 1$, find $(f \cdot g)(x)$

5. Given $f(x) = 4x$ and $g(x) = -3x - 7$, evaluate $(f - g)(x)$ at $x = -3$.

6. Given $f(x) = 2x + 2$ and $h(x) = 3x$, evaluate $(f \circ h)(x)$ at $x = 2$.

7. Given $f(x) = 5x$ and $g(x) = -6x + 1$, evaluate $(f \cdot g)(x)$ at $x = -1$.

8. Given $f(x) = 4x^2$ and $g(x) = 12x$, find $\left(\frac{f}{g}\right)(x)$.

9. The function $h(C) = 331 + \frac{3}{5}C$ represents the speed of sound, h, in meters per second, where C is the temperature in degrees Celsius. The Celsius temperature C can also be expressed as a function of the Fahrenheit temperature F as follows: $C = g(F) = \frac{5}{9}(F - 32)$. Find $h[g(F)]$, the speed of sound expressed in terms of degrees Fahrenheit.

10. Suppose that the function $h(x) = x$ describes the number of days that your store runs a special promotion. And suppose that $g(x) = 4000x + 200$ describes the total cost of running this promotion for x days. Then the average daily cost of running the promotion is given by the quotient function $\left(\frac{g}{h}\right)(x)$, where $x \neq 0$. Find your average daily cost if you were to run the promotion for 5 days.

11. Given $f(x) = 1 - 4x^2$ and $g(x) = 1 + 2x$, evaluate $\left(\frac{f}{g}\right)(x)$ at $x = 2$.

12. Given $f(x) = x^2 - 2$ and $h(x) = \sqrt{x}$, find $(f \circ h)(2)$.

Inverse Functions

13. If $f(x) = 6x - 1$, find $f^{-1}(x)$.

14. Determine whether or not the function graphed in Figure 11.2.10 is one-to-one.

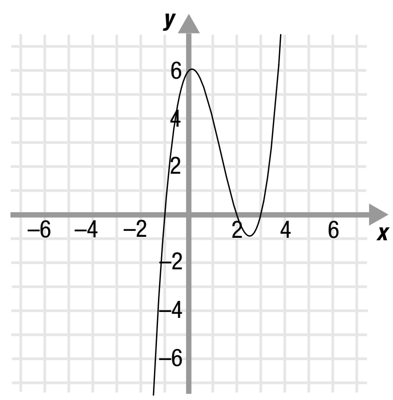


Figure 11.2.10

15. Graph the inverse of the function $y = f(x) = 2 - 3x$. Use the grid in Figure 11.2.11.

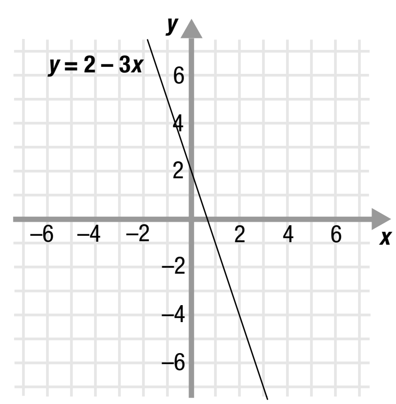


Figure 11.2.11

16. If $f(x) = 6x - 1$, find $(f^{-1} \circ f)(x)$.

17. Is the function $y = f(x) = x^2 - 3$ one-to-one?

18. Graph and find the equation of the inverse of the function $y = f(x) = 4x$. Use the grid in Figure 11.2.12.

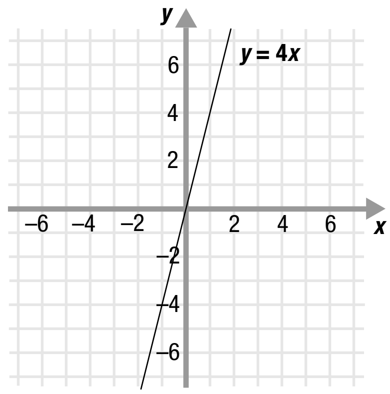


Figure 11.2.12

19. If $g(x) = \frac{2}{x} + 1$, find $g^{-1}(x)$.

20. If $y = f(x) = x^2 - 5$, does f^{-1} exist?

21. Julie parachutes from an airplane at 10,000 feet. She estimates her distance above the ground to be $f(x) = 10{,}000 - 20x$ at a given time x. Find $f^{-1}(x)$.

22. Rolph is in a hot air balloon at 20,000 feet that is descending. He estimates his distance above the ground to be $f(x) = 20{,}000 - 15x$ at a given time x. Find $f^{-1}(50)$.

23. If $y = g(x) = \frac{3}{x} - 1$, determine if g has an inverse, and if it does, find it. Use the graph of $y = g(x) = \frac{3}{x} - 1$ in Figure 11.2.13 to help you.

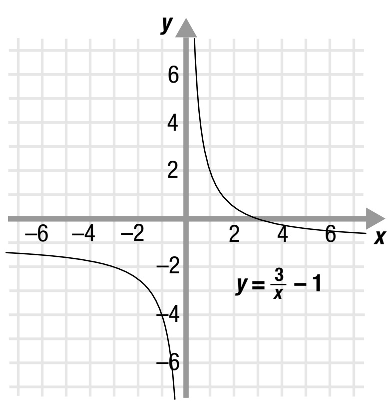


Figure 11.2.13

24. If f is a function that takes an input value x, increases it by 4, then multiplies that quantity by 3, find an equation for f. Then find $f^{-1}(-1)$.

Explore

25. Let $r(x) = 4x$, $c(x) = 20x + 500$, and $p(x) = (r - c)(x)$. Complete the following table.

x	$r(x)$	$c(x)$	$p(x)$
5		600	
11	44		–676
20			

26. The graphs of $y = f(x)$ and $y = g(x)$ are shown on the grid in Figure 11.2.14. Use these graphs to find $(g \circ f)(-1)$.

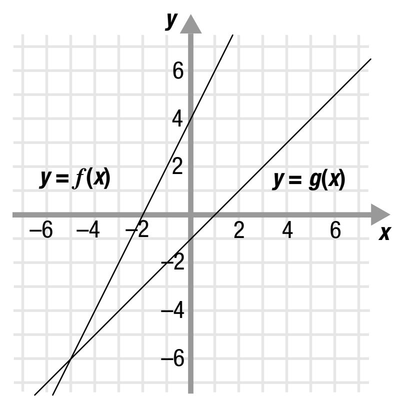


Figure 11.2.14

27. The graph of $y = f(x) = 2x^2 - 2$ is shown on the grid in Figure 11.2.15. If $g(x) = |x|$, which of the following is the graph of $y = (g \circ f)(x)$?

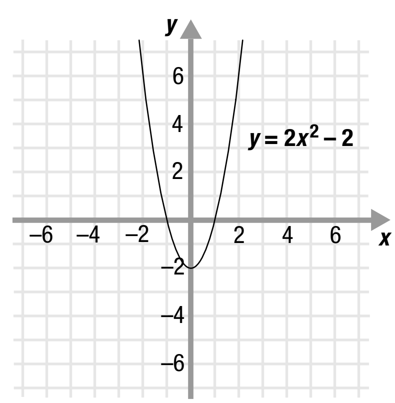
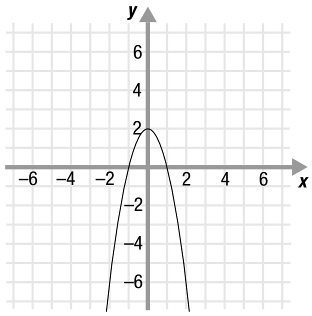
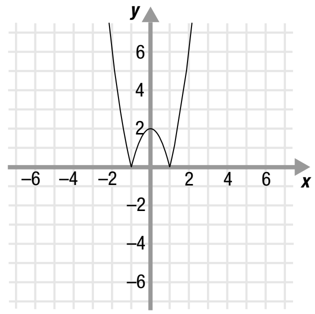
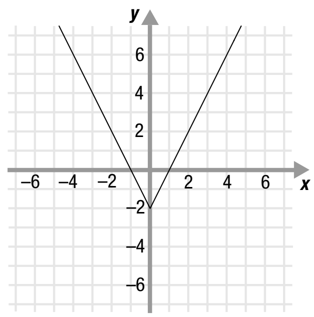


Figure 11.2.15







28. Let $f(x) = 3x$, $g(x) = 3x - 3$, and $h(x) = (f \cdot g)(x) + \left(\frac{f}{g}\right)(x)$. Complete the following table.

x	$f(x)$	$g(x)$	$h(x)$
-1	-3		
$\frac{1}{3}$		-2	
	12		$109\frac{1}{3}$

29. The graphs of $y = f(x) = x^2$ and $y = g(x) = 2x + 1$ are shown on the grid in Figure 11.2.16. Which of the following is the graph of $y = (f + g)(x)$?

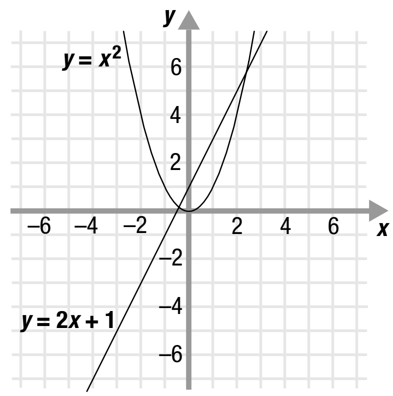
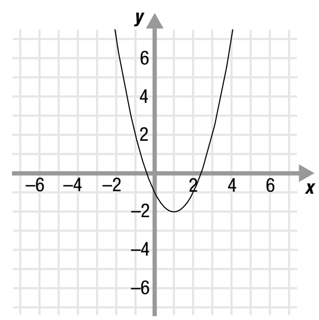
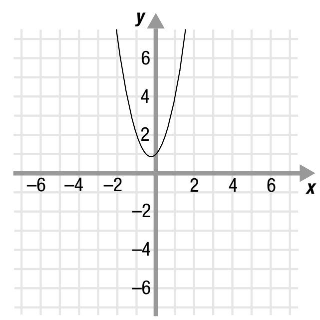
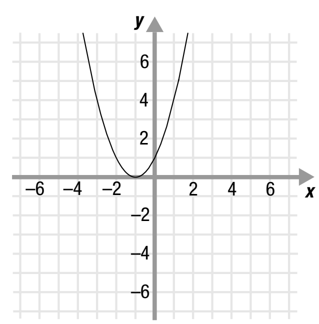


Figure 11.2.16







30. The graph of $y = f(x) = \sqrt{2x + 3}$ is shown on the grid in Figure 11.2.17. Which of the following is the graph of $y = f^{-1}(x)$?

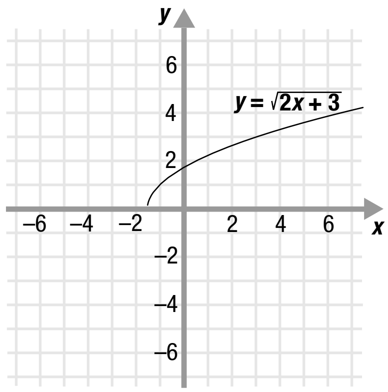
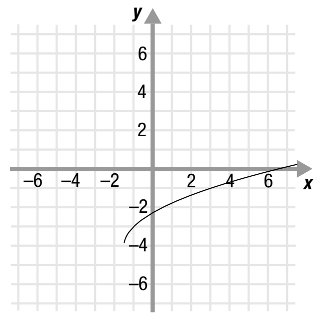
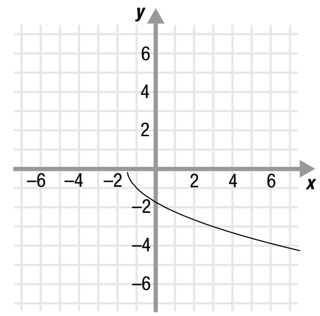
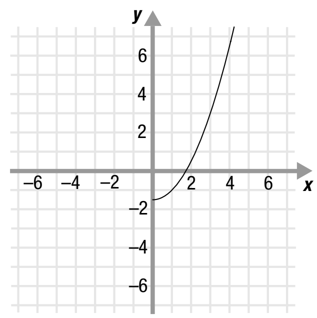


Figure 11.2.17







APPLY

Practice Problems

Here are some additional practice problems for you to try.

The Algebra of Functions

1. Given $g(x) = 3x + 1$ and $h(x) = x^2 - 7x + 4$, find $(g + h)(x)$.
2. Given $f(x) = x^2 - 9$ and $h(x) = x^2 + 3x + 5$, find $(f + h)(x)$.
3. Given $f(x) = 5x$ and $g(x) = 2x^2 - 8x + 7$, find $(f + g)(x)$.
4. Given $f(x) = 3x^2 + 7$ and $g(x) = 2x - 7$, find $(f - g)(x)$.
5. Given $g(x) = x^2 + 3x + 4$ and $h(x) = 3x^2 + 5x - 6$, find $(g - h)(x)$.
6. Given $f(x) = 5x^2 - 2$ and $h(x) = 7x + 2$, find $(f - h)(4)$.
7. Given $f(x) = 10x$ and $g(x) = 4x + 9$, find $(f \cdot g)(x)$.
8. Given $f(x) = x + 1$ and $g(x) = 3x - 5$, find $(f \cdot g)(x)$.
9. Given $g(x) = x^2 - 5$ and $h(x) = 4x + 7$, find $(g \cdot h)(x)$.
10. Given $f(x) = 3x + 5$ and $h(x) = 5x - 8$, find $(f \cdot h)(1)$.
11. Given $g(x) = -x^2 + 2x$ and $h(x) = 3x + 11$, find $(g \cdot h)(0)$.
12. Given $f(x) = -2x^2 - 3x$ and $g(x) = 6x + 2$, find $(f \cdot g)(-2)$.
13. Given $f(x) = 2x$ and $g(x) = x + 4$, find $\left(\frac{f}{g}\right)(x)$.
14. Given $f(x) = x^2 - 1$ and $h(x) = x + 3$, find $\left(\frac{f}{h}\right)(x)$.
15. Given $f(x) = x - 3$ and $g(x) = 4x^2 - 5$, find $\left(\frac{f}{g}\right)(x)$.
16. Given $f(x) = 2x$ and $h(x) = x^2 + 1$, find $(f \circ h)(x)$.
17. Given $g(x) = 3x + 7$ and $h(x) = 2x - 4$, find $(g \circ h)(x)$.
18. Given $f(x) = 4x - 3$ and $g(x) = 9x - 1$, find $(f \circ g)(x)$.
19. Given $f(x) = x^2$ and $g(x) = x + 2$, find $(f \circ g)(x)$.
20. Given $f(x) = 2x^2 + 1$ and $g(x) = x - 3$, find $(f \circ g)(x)$.
21. Given $f(x) = x^2 + x$ and $h(x) = x - 2$, find $(f \circ h)(x)$.
22. Given $f(x) = x^2$ and $g(x) = \sqrt{x + 1}$, find $(f \circ g)(3)$.
23. Given $g(x) = x^2 - 4$ and $h(x) = \sqrt{x - 2}$, find $(g \circ h)(3)$.
24. Given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x - 5}$, find $(f \circ g)(7)$.
25. Given $g(x) = x^2 + 4$ and $h(x) = \sqrt{x + 5}$, find $(g \circ h)(4) - (h \circ g)(4)$.
26. Circle the pairs of functions f and g below for which the composition $(f \circ g)(x) = x$.

$f(x) = x + 7$	$f(x) = 3x$	$f(x) = 2x - 5$
$g(x) = x - 7$	$g(x) = -3x$	$g(x) = \frac{x}{2} + 5$

27. Circle the pairs of functions f and g below for which the composition $(f \circ g)(x) = x$.

$f(x) = 2x$	$f(x) = 3 - 6x$	$f(x) = 4x + 7$
$g(x) = \frac{x}{2}$	$g(x) = 2x - 1$	$g(x) = \frac{x - 7}{4}$

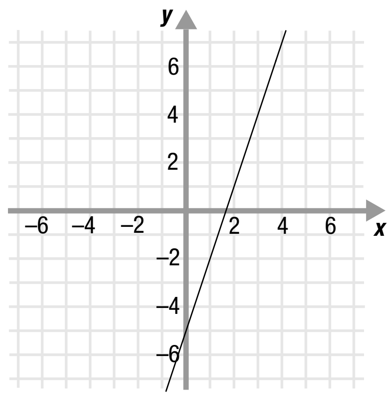
28. Circle the pairs of functions f and g below for which the composition $(f \circ g)(x) = x$.

$f(x) = 4 - x$	$f(x) = 6x$	$f(x) = 3x + 2$
$g(x) = \frac{x}{4} - 1$	$g(x) = \frac{x}{6}$	$g(x) = \frac{x - 2}{3}$

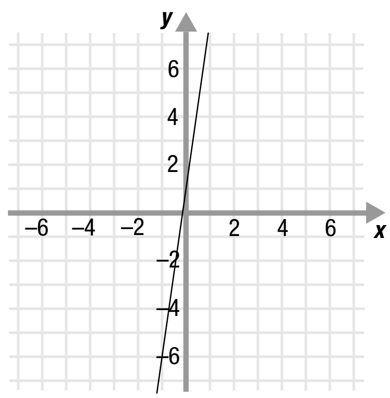
Inverse Functions

29. Given $f(x) = x + 8$, find $f^{-1}(x)$.
30. Given $f(x) = 3x - 2$, find $f^{-1}(x)$.
31. Given $f(x) = 4x - 1$, find $f^{-1}(x)$.
32. Given $g(x) = 7x + 8$, find $g^{-1}(x)$.
33. Given $f(x) = \frac{1}{x} + 2$, find $f^{-1}(x)$.
34. Given $f(x) = \frac{4}{5x} - 3$, find $f^{-1}(x)$.
35. Given $f(x) = \frac{2}{3x} + 1$, find $f^{-1}(x)$.

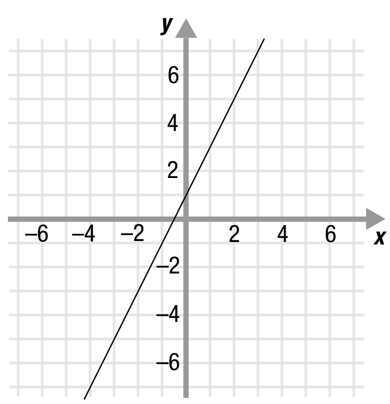
36. Use the graph of the function $y = f(x) = 3x - 5$ to determine if the function is one-to-one.



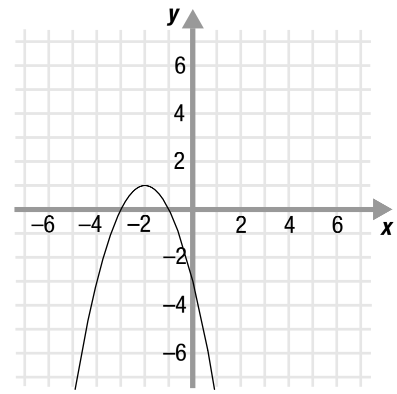
37. Use the graph of the function $y = f(x) = 7x + 1$ to determine if the function is one-to-one.



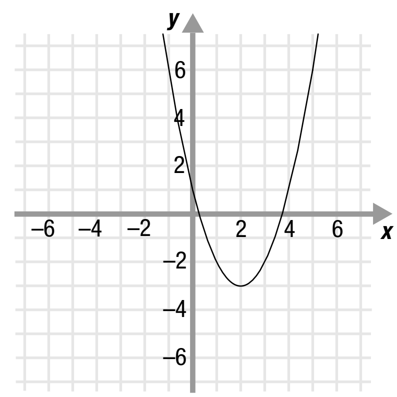
38. Use the graph of the function $y = f(x) = 2x + 1$ to determine if the function is one-to-one.



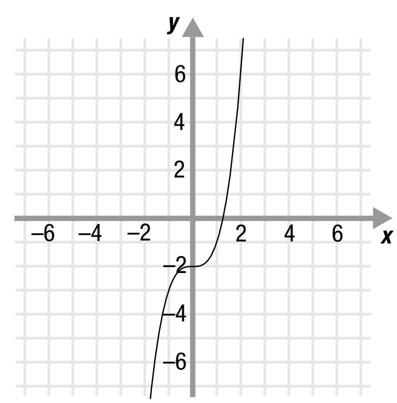
39. Use the graph of the function $y = f(x) = -(x + 2)^2 + 1$ to determine if the function is one-to-one.



40. Use the graph of the function $y = f(x) = (x - 2)^2 - 3$ to determine if the function is one-to-one.



41. Use the graph of the function $y = f(x) = x^3 - 2$ to determine if the function is one-to-one.

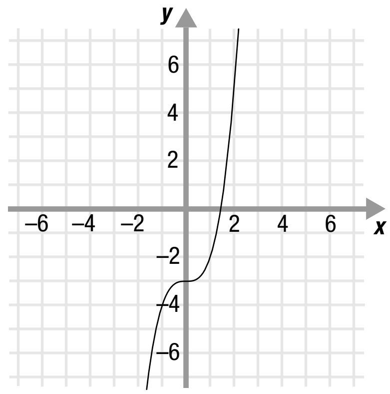


42. Given $f(x) = 8x - 4$, find $(f \circ f^{-1})(x)$.

43. Given $f(x) = 2x + 9$, find $(f^{-1} \circ f)(x)$.

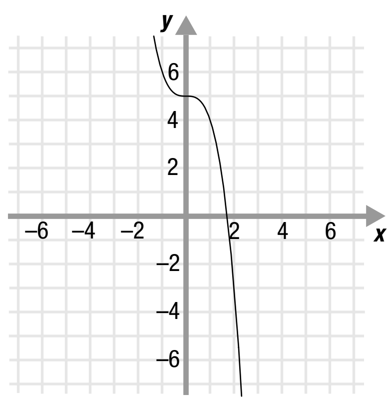
44. Given $f(x) = 5x + 6$, find $(f \circ f^{-1})(x)$.

45. The graph of a function $y = f(x)$ is shown below. Fill in the table below for f^{-1}.



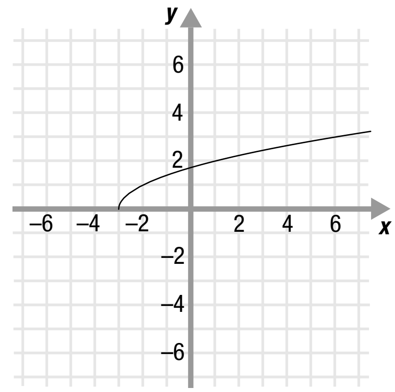
x	$y = f^{-1}(x)$
-4	
-3	
5	

46. The graph of a function $y = f(x)$ is shown below. Fill in the table below for f^{-1}.



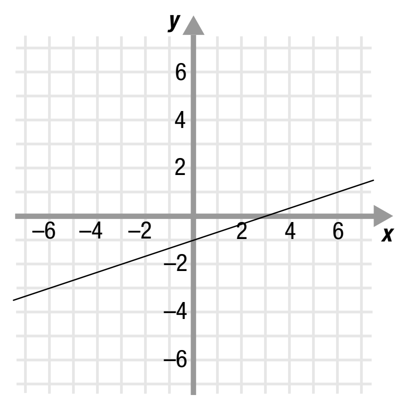
x	$f^{-1}(x)$
-3	
4	
6	

47. The graph of a function $y = f(x)$ is shown below. Fill in the table below for f^{-1}.



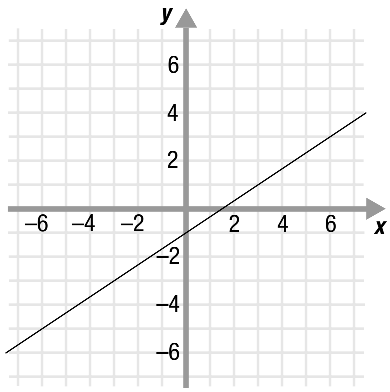
x	$y = f^{-1}(x)$
1	
2	
3	

48. The graph of a function $y = f(x)$ is shown below. Fill in the table below for f^{-1}.



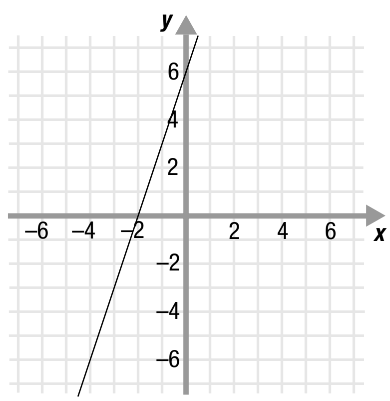
x	$y = f^{-1}(x)$
-3	
-1	
1	

49. The graph of a function $y = f(x)$ is shown below. Fill in the table below for f^{-1}.



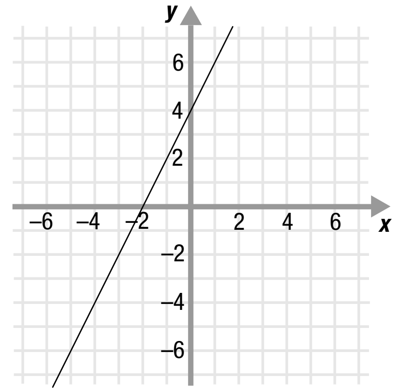
x	$y = f^{-1}(x)$
–3	
–1	
1	

50. The graph of a function $y = f(x)$ is shown below. Fill in the table below for f^{-1}.

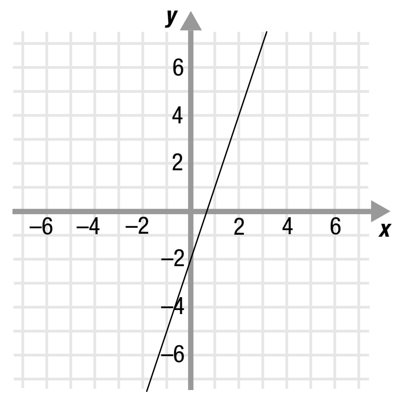


x	$y = f^{-1}(x)$
–3	
0	
6	

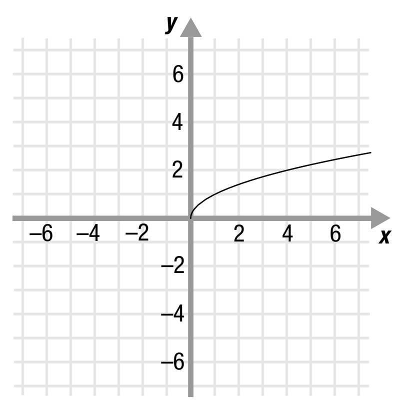
51. The graph of the function $y = f(x) = 2x + 4$ follows. Graph f^{-1}.



52. The graph of the function $y = f(x) = 3x - 2$ is shown below. Graph f^{-1}.



53. The graph of the function $y = f(x) = \sqrt{x}$ is shown below. Graph f^{-1}.



54. Given the function $y = f(x) = x + 5$, find the x-intercept and y-intercept of f^{-1}.

55. Given the function $y = f(x) = 3x + 12$, find the x-intercept and y-intercept of f^{-1}.

56. Given the function $y = f(x) = 2x - 10$, find the x-intercept and the y-intercept of f^{-1}.

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Given $h(x) = -7x$ and $f(x) = 7x$, find $(f - h)(-3)$.

2. Given $f(x) = 6x^4$ and $g(x) = 3x^3$, find $\left(\frac{f}{g}\right)(x) - (f \cdot g)(x)$.

3. Circle the pairs of functions f and g below for which the composition $(f \circ g)(x) = x$.

$f(x) = 2x$	$f(x) = x^2$	$f(x) = 1 - x$	$f(x) = 2x + 1$
$g(x) = \frac{x}{2}$	$g(x) = -x$	$g(x) = x + 1$	$g(x) = \frac{x}{2} - 1$

4. Given $f(x) = -6x + 5$ and $g(x) = -x^2 - 2$, evaluate $(f \circ g)(4)$.

5. The graph of a function, $y = f(x)$, is shown in Figure 11.2.20. Use the points shown on the graph to fill in the table below for f^{-1}:

   f^{-1}

x	y
-5	
-1	0
3	

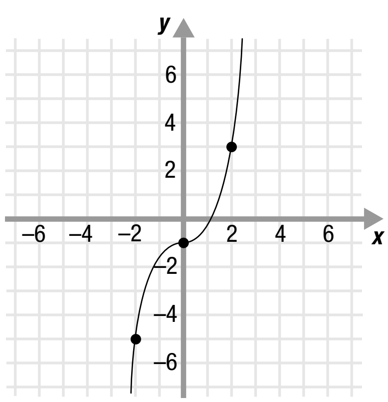


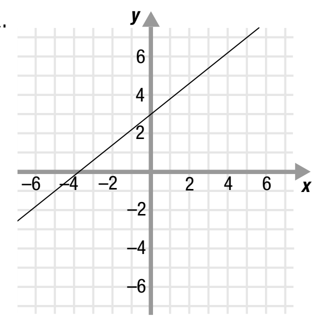
Figure 11.2.20

6. Given $h(x) = \frac{2x - 1}{5} + 2$, find $h^{-1}(x)$.

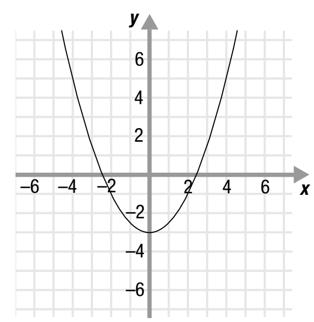
7. If $f(x) = 0.5x - 7$ and $g(x) = f^{-1}(x)$, find $(g \circ f)(x)$.

8. Which of the graphs below are graphs of functions that do **not** have inverses.

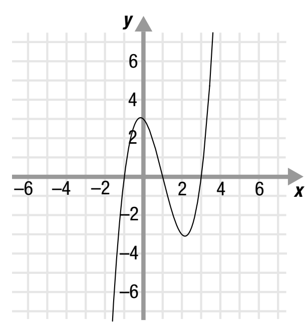
a.



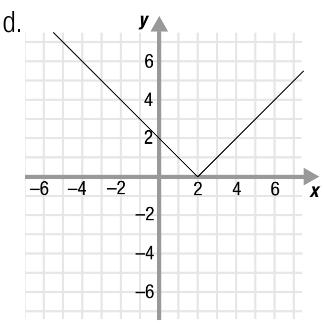
b.



c.



d.



9. If $f(5) = 11$ and $g(5) = 3$, find $(f \cdot g)(5)$.

10. The graphs of two functions, $y = f(x)$ and $y = g(x)$, are shown on the grid in Figure 11.2.21. Use these graphs to find $(f - g)(2)$.

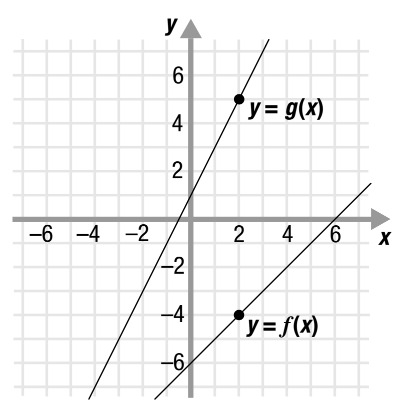


Figure 11.2.21

11. The graphs of $y = f(x)$ and $y = g(x)$ are shown on the grid in Figure 11.2.22. Use these graphs to find $(f \circ g)(2)$.

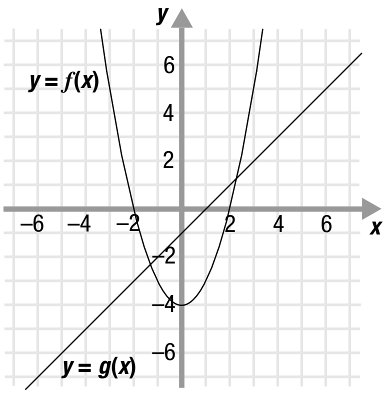


Figure 11.2.22

12. If $f(x) = 2x + 8$, circle the following statements that are true.

$f^{-1}[f(x)] = x$

$f^{-1}(x) = \dfrac{1}{2x + 8}$

$(f \circ f)(x) = 4x + 24$

$(f \cdot f)(x) = (2x + 8)^2$

TOPIC 11 CUMULATIVE ACTIVITIES

CUMULATIVE REVIEW PROBLEMS

These problems combine all of the material you have covered so far in this course. You may want to test your understanding of this material before you move on to the next topic, or you may wish to do these problems to review for a test.

1. Simplify this expression: $\sqrt{\frac{-225}{64}}$

2. Find:
   $(12x^2 - 9xy + 5xy^2 + 2y^2 + 7) - (-4y^2 + 13 + 6x^2y - 11xy)$

3. Solve for x: $3x^2 - 7x + 4 = 0$

4. Find the domain and the range of each of the functions below.

   a. $y = 6x + 13$

   b. $y = x^2 - 3$

   c. $y = \frac{1}{x + 1}$

   d. $y = \sqrt{x^2 - 7}$

   e. $y = \frac{x + 8}{(x - 2)(x + 4)}$

5. If $f(x) = \frac{4}{2x - 4} + 7$, find $f^{-1}(x)$.

6. Find the domain and the range of each of the quadratic functions below.

   a. $y = x^2 + 9$

   b. $y = -x^2 + 3$

   c. $y = 6x2 + 12$

7. Simplify using the properties of exponents: $2a \cdot \frac{(3a)^3}{(3a^2)^2}$

8. Simplify the following expressions:

   a. $\sqrt{-9}$

   b. $\sqrt{-49}$

   c. $\sqrt{-9} \cdot \sqrt{-49}$

   d. $\sqrt{-9} + \sqrt{-49}$

   e. i^{42}

   f. i^{37}

9. Find: $(5\sqrt{3} + 2\sqrt{6})(7\sqrt{3} - \sqrt{6})$

10. Solve for x: $32 + 8x > 13(x + 2)$

11. Given $f(x) = 3x$ and $g(x) = -8x + 6$, find $(f + g)(x)$.

12. Determine whether each of the following parabolas opens up or down.

    a. $y = 7 - 2x^2$

    b. $y = 5x^2 + 8x - 5$

    c. $y = -x^2 + 14x - 11$

13. Solve for a: $\frac{2}{7}(a - 6) = \frac{2}{3}a + \frac{5}{7}$

14. If $g(x) = 7(x + 5)$, find $g^{-1}(-2)$.

15. Which of the following pairs are complex conjugates of each other?

    $4 + 6i$ and $6 - 4i$

    $2 + 9i$ and $2 - 9i$

    $5 + 5i$ and $-5 - 5i$

    $7 + 3i$ and $-3i + 7$

    $8 + 4i$ and $4i - 8$

16. Simplify the expression below. Assume x, y, and z are positive numbers.

    $$\frac{(32)^{\frac{1}{5}} \cdot \sqrt[3]{125x^{27}}}{\sqrt{x^{18} \cdot z^{10} \cdot y^4}}$$

17. Solve for x: $3x^2 - 4x - 7 = 0$

18. Solve for x: $\frac{5x}{3} + 4 = \frac{7}{8}$

19. Find the x- and y-intercepts of this line: $y + 6 = 2(x + 1)$

20. Which of the following functions are quadratic?

    $y = 7 - 2x$

    $y = 4 - 3x^2$

    $y = x^3 + 2x^2 - 6x + 4$

    $y = x + 3$

    $y = 7x^2 - 2x - 7$

21. Given $f(x) = 9x$ and $g(x) = -x + 1$, evaluate $(f \cdot g)(x)$ at $x = -2$.

22. Solve for x: $6 - 4|x + 3| \leq -12$

23. Simplify the expression below. Leave your answer in exponential form.

$$\left(7^{\frac{1}{5}} \cdot 7^{\frac{-2}{3}}\right)^{-5}$$

24. Find the equation of the line through the point $(-4, 2)$ that is perpendicular to the line $y = -3x + 6$. Write your answer in slope-intercept form.

25. If $f(x) = 7x + 4$, find $f^{-1}(x)$.

26. Reduce to lowest terms: $\frac{3ba + 3b - 8a - 8}{ba + b + 2a + 2}$

27. Does the function f, shown in Figure 11.1, have an inverse?

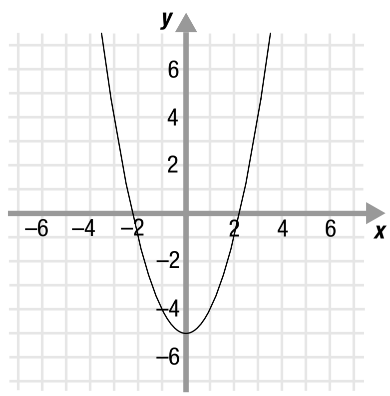


Figure 11.1

28. Which of the following functions are linear?

$y = 3x + 7$ $\qquad$ $y = x^3 + x^2 + 4x + 9$

$y = x^2 - 2x + 2$ $\qquad$ $y = \sqrt{x^3 - 1}$

$y = \frac{2}{3} - 5x$

29. Solve for x: $x^2 + 3x = 9$

30. Combine like terms: $8\sqrt{3y} + 3\sqrt{48y} - 9$

31. Solve for x: $2x^2 - 5x - 20 = -8x - 15$

32. Solve for x: $|x| \geq 7$

33. Find the x- and y-intercepts of each of the functions below.

a. $y = 2x^2 - 3x - 9$

b. $y = 9x^2 - x$

c. $y = 4 - x^2$

34. Factor: $64a^3 + 40a^2 - 24a$

35. Simplify: $\left(a^{\frac{4}{5}} \cdot a^{\frac{-3}{2}} \cdot b^{\frac{3}{5}}\right)^{-5}$

36. Given $f(x) = 4x - 2$ and $g(x) = -2x - 7$, find $(f \circ g)(x)$.

37. If $f(x) = 3x - 8$, find $(f^{-1} \circ f)(x)$.

38. Find the distance d between the points $(3, -4)$ and $(-1, 5)$.

39. Solve for a: $a^2 - 64 = 0$

40. Given $f(x) = 5x^2$ and $g(x) = 15x$, find $\left(\frac{f}{g}\right)(x)$.

41. Find the domain and the range of each of the functions below.

a. $y = 4x - 1$ $\qquad$ b. $y = |x| + 6$

c. $y = |x - 6|$

42. Solve for x: $(x - 4)^2 = 64$

43. Solve for x: $2x^2 - 3x + 5 = 0$

44. Find: $(8 + 9i) - (-3 + 7i)$

45. Given $g(x) = 2x^2 - 4$, calculate:

a. $g(0)$ $\qquad$ b. $g(-1)$

c. $g(2)$ $\qquad$ d. $g(-3)$

e. $g(1)$

46. Given $f(x) = 4x + 1$, calculate:

a. $f(0)$ $\qquad$ b. $f(3)$

c. $f(-1)$ $\qquad$ d. $f\left(\frac{1}{2}\right)$

e. $f(-5)$

47. Solve for x: $|5x + 2| = 9$

48. Find: $(12 + 4i) \div (9 - 6i)$

49. Combine like terms and simplify:

$$4\sqrt{313} + 7\sqrt{47} + \sqrt[3]{64} - 2\sqrt{313} - 5\sqrt[4]{16} - 3\sqrt{47}$$

50. Find: $(4 - 9i)(1 + i)$