LESSON EIII.C – EQUATIONS AND INEQUALITIES

OVERVIEW

Here's what you'll learn in this lesson:

Linear and Absolute Value Equations and Inequalities

a. Solving linear equations

b. Interval notation

c. Solving linear inequalities

d. Solving absolute value equations

e. Solving absolute value inequalities

Quadratic Equations

a. Solving quadratic equations:
by factoring;
by the square root method;
by completing the square;
by the quadratic formula

b. Complex numbers

c. The discriminant

d. Solving other polynomial equations

Linear Equations

a. Cartesian coordinate system

b. Distance formula

c. Midpoint of a line segment

d. Finding the equation of a line:
point-slope form, standard form, slope-intercept form, horizontal line, vertical line, line parallel or perpendicular to a line

In this lesson, you will review how to solve three very important types of equations: linear, absolute value, and quadratic. You will also solve linear and absolute value inequalities.

Then you will work with equations in two variables. You will graph lines, find the slope of a line, and find the equation of a line.

EXPLAIN

LINEAR AND ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Summary

Solving Linear Equations

Here are some examples of linear equations:

$2x + 3 = 7$ $\qquad$ $3(5x - 4) - 4(3 - 11x) = 8x + 7$

$\frac{4}{5}x - 3 = \frac{2}{3}(3x - 1)$ $\qquad$ $\frac{3}{x-4} - 5 = \frac{x+2}{x-4}$

Here are some steps that you can use to solve a linear equation:

1. If the equation contains fractions, clear those fractions by multiplying both sides of the equation by the least common denominator of the fractions.
2. Remove any parentheses using the distributive property.
3. Combine like terms on each side of the equation.
4. Do the following, as necessary, to isolate the variable:
   - Add the same quantity to both sides of the equation.
   - Subtract the same quantity from both sides of the equation.
   - Multiply or divide both sides of the equation by the same non zero quantity.
5. Check the solution.

For example, to solve the equation $3(2x - 5) = x - 4(1 - 6x)$:

1. There are no fractions in the equation.
2. Remove parentheses using the distributive property. $\qquad 6x - 15 = x - 4 + 24x$
3. Combine like terms on each side of the equation. $\qquad 6x - 15 = 25x - 4$
4. To isolate x on the left side:
   - Add 15 to both sides. $\qquad 6x = 25x + 11$
   - Subtract $25x$ from both sides. $\qquad -19x = 11$
   - Divide both sides by –19. $\qquad x = -\frac{11}{19}$

A linear equation is an equation that can be put in the form $ax + b = 0$, where $a \neq 0$.

You don't have to clear the fractions to solve a linear equation. But it may make solving the equation easier.

The distributive property states that for all real numbers a, b, and c, $a(b + c) = a \cdot b + a \cdot c$.

If you multiply both sides of an equation by a variable, you may introduce extraneous, or false, solutions. Check for this by substituting your solution back into the original equation to see if you get a true statement.

Remember, it's possible to get an equation that has no solution.

5. Check the solution, $x = -\frac{11}{19}$. Is $3[2\left(-\frac{11}{19}\right) - 5] = -\frac{11}{19} - 4\left[1 - 6\left(-\frac{11}{19}\right)\right]$?

Is $3\left(-\frac{22}{19} - \frac{95}{19}\right) = -\frac{11}{19} - 4\left(\frac{19}{19} + \frac{66}{19}\right)$?

Is $3\left(-\frac{117}{19}\right) = -\frac{11}{19} - 4\left(\frac{85}{19}\right)$?

Is $-\frac{351}{19} = -\frac{11}{19} - \frac{340}{19}$?

Is $-\frac{351}{19} = -\frac{351}{19}$? Yes.

So, the solution of the equation $3(2x - 5) = x - 4(1 - 6x)$ is $x = -\frac{11}{19}$.

Here's another example. To solve the equation $\frac{4}{3x} + 1 = \frac{2x - 5}{3x}$:

1. Multiply both sides of the equation by the least common denominator, $3x$, to clear the fractions. $3x\left(\frac{4}{3x} + 1\right) = 3x\left(\frac{2x - 5}{3x}\right)$

2. Remove parentheses using the distributive property. $4 + 3x = 2x - 5$

3. There are no like terms to combine.

4. To isolate x on the left side:
   - Subtract 4 from both sides. $3x = 2x - 9$
   - Subtract $2x$ from both sides. $x = -9$

5. Check the solution, $x = -9$. Is $\frac{4}{3(-9)} + 1 = \frac{2(-9) - 5}{3(-9)}$?

   Is $\frac{4}{-27} + \frac{27}{27} = \frac{-18 - 5}{-27}$?

   Is $\frac{23}{27} = \frac{23}{27}$? Yes.

So, the solution of the equation $\frac{4}{3x} + 1 = \frac{2x - 5}{3x}$ is $x = -9$.

Solving Formulas

Equations that contain more than one letter are called formulas, or literal equations. Here are some examples of formulas:

$A = L \cdot W$ $\qquad$ $V = \frac{K \cdot T}{P}$

$\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$ $\qquad$ $2xy + 3yz = 5 - 4x - 3y$

You can solve for one of the letters in a formula in terms of the other letters by using techniques similar to those you just learned for solving linear equations.

For example, here is a formula used for combining resistors in electronics: $R = \frac{r_1 r_2}{r_1 + r_2}$.

To solve the formula for r_1:

Here r_1 and r_2 stand for the two resistors you are combining, and R stands for the single resistor that can replace them in a circuit.

1. Multiply both sides of the formula by the least common denominator, $(r_1 + r_2)$, to clear the fractions. $\quad (r_1 + r_2)R = (r_1 + r_2)\frac{r_1 r_2}{r_1 + r_2}$

2. Remove parentheses using the distributive property. $\quad r_1 R + r_2 R = r_1\, r_2$

3. To isolate r_1, move the terms containing r_1 to the left side.
   - Subtract $r_2 R$ from both sides. $\quad r_1 R = r_1 r_2 - r_2 R$
   - Subtract $r_1 r_2$ from both sides. $\quad r_1 R - r_1 r_2 = -r_2 R$
   - Factor the left side of the equation. $\quad r_1(R - r_2) = -r_2 R$
   - Divide both sides by $R - r_2$. $\quad r_1 = -\frac{r_2 R}{R - r_2}$

$$= \frac{r_2 R}{r_2 - R}$$

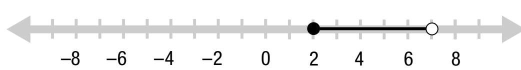
So solving the formula $R = \frac{r_1 r_2}{r_1 + r_2}$ for r_1 gives $\frac{r_2 R}{r_2 - R}$.

Inequalities and Intervals

Next you will solve linear inequalities. First, though, here is a review of three different ways to represent an inequality.

For example, all real numbers between 2 and 7, including 2, can be expressed

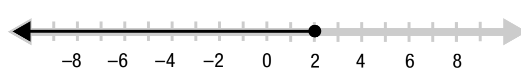
- Using inequality signs: $2 \le x < 7$
- Using interval notation: $[2, 7)$
- As an interval on the number line:



You use a square bracket, or a closed dot on the graph, to indicate that the end point is included in the interval. You use parentheses, or an open dot on the graph, to indicate that the end point is not included in the interval.

Here's another example. All real numbers less than 2, including 2, can be expressed

- Using inequality signs: $x \le 2$
- Using interval notation: $(-\infty, 2]$
- As an interval on the number line:



Solving Inequalities

Here are some examples of linear inequalities:

$$5(2-3x) \geq 4-7x \qquad 17 < 3x-4 \leq 25 \qquad \frac{3}{5} + 2x < \frac{3x-1}{4}$$

To solve a linear inequality you can use the same steps to isolate the variable that you used to solve a linear equation or formula. However, there is one important extra step:

- When you multiply or divide an inequality by a negative number, you must reverse the direction of the inequality sign.

For example, to solve the linear inequality $-3x < 7$:

*Remember, when you multiply or divide an inequality by a positive number, the inequality sign does **not** change. Also, when you add or subtract any number (positive or negative) to or from an inequality, the inequality sign does **not** change.*

1. Isolate x on the left side by:
   - Dividing both sides by –3. $\quad x > -\frac{7}{3}$
   - Since you are dividing by a negative number, reverse the direction of the inequality sign.

So the solution of the inequality $-3x < 7$ is $x > -\frac{7}{3}$.

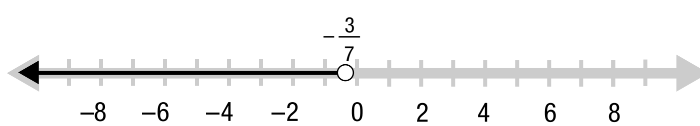
Here's another example of solving an inequality.

To solve $2(3-4x) > 9-x$:

1. Remove parentheses using the distributive property. $\quad 6-8x > 9-x$
2. To isolate x on the left side:
   - Subtract 6 from both sides. $\quad -8x > 3-x$
   - Add x to both sides. $\quad -7x > 3$
   - Divide both sides by –7. $\quad x < -\frac{3}{7}$
     Reverse the direction of the inequality sign.

So the solution of the inequality $2(3-4x) > 9-x$ is $x < -\frac{3}{7}$.

You can also write this solution as the interval $(-\infty, -\frac{3}{7})$ or you can graph it on the number line as shown.



Solving Compound Inequalities

Inequalities that contain two inequality signs are called compound inequalities. You can solve compound inequalities by using the same steps you've just learned and applying the steps to all three parts of the inequality.

Here's an example.

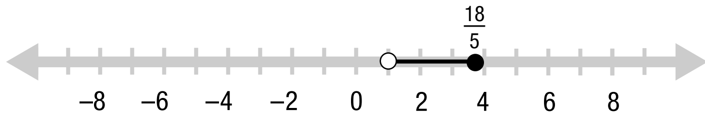
To solve $-3 < 5x-8 \leq 10$:

1. Add 8 to all three parts. $\quad 5 < 5x \leq 18$

2. Divide all three parts by 5. $1 < x \leq \frac{18}{5}$

So the solution of the inequality $-3 < 5x - 8 \leq 10$ is $1 < x \leq \frac{18}{5}$.

You can also write this solution as the interval $\left(1, \frac{18}{5}\right]$ or you can graph it on the number line as shown.



Solving Absolute Value Equations

Recall that the absolute value of x, written $|x|$, is the distance from x to zero on the number line. This distance is never negative.

For example, $|7| = 7$ and $|-9| = 9$.

Here are some examples of absolute value equations.

$|3x| = 9 \qquad 2 - |x - 3| = 7 \qquad |4x + 5| = 7$

Here's how to solve an equation of the form $|z| = a$:

- If $a > 0$, there are two solutions, $z = a$ and $z = -a$.
- If $a < 0$, there are no solutions.
- If $a = 0$, there is one solution, $z = 0$.

For example, the equation $|x| = 8$ has two solutions, $x = 8$ and $x = -8$.

To solve an absolute value equation of the form $|ax + b| = c$, where $c \geq 0$:

1. Substitute z for $ax + b$.
2. Solve for z.
3. Replace z with $ax + b$ to get $ax + b = c$ or $ax + b = -c$.
4. Solve for x.

Notice that if $c < 0$, the equation $|ax + b| = c$ has no solutions.

If $c = 0$, the equation $|ax + b| = c$ has exactly one solution, $x = -\frac{b}{a}$.

For example, to solve the absolute value equation $|3x - 4| = 5$:

1. Substitute z for $3x - 4$.	$\lvert z\rvert = 5$		
2. Solve for z.	$z = 5$	or	$z = -5$
3. Replace z with $3x - 4$.	$3x - 4 = 5$	or	$3x - 4 = -5$
4. Solve for x.	$3x = 9$	or	$3x = -1$
	$x = 3$	or	$x = -\frac{1}{3}$

So the solutions of the equation $|3x - 4| = 5$ are $x = 3$ or $x = -\frac{1}{3}$.

Before you can use these steps to solve some equations you need to make sure the equations are in the form $|ax + b| = c$.

For example, to solve $3 - 4|2x| = 17$:

1. Write the equation in the form $|ax + b| = c$ by isolating the absolute value.
   - Subtract 3 from both sides. $\quad -4|2x| = 14$
   - Divide both sides by –4. $\quad |2x| = \frac{14}{-4}$

     $|2x| = -\frac{7}{2}$
2. Since $-\frac{7}{2} < 0$, the equation has no solutions.

Equations with Two Absolute Value Terms

Some equations have two terms that are absolute values. Here are some examples:

$|2x - 3| = |3x + 5| \qquad 2 + |x| = 4 - |3 - 2x|$

To solve such equations you can use the simpler equation $|z| = |w|$.

If you think in terms of distance, this equation says that the distance from z to 0 is the same as the distance from w to 0. This can happen when $z = w$ or $z = -w$.

Here's how to use substitution to solve the more general equation $|ax + b| = |cx + d|$:

1. Substitute z for $ax + b$ and w for $cx + d$.
2. Solve the equation $|z| = |w|$ to get $z = w$ or $z = -w$.
3. Replace z with $ax + b$ and w with $cx + d$ to get $ax + b = cx + d$ or $ax + b = -(cx + d)$.
4. Solve the two equations for x.

Here's an example.

To solve the equation $|3x - 4| = |x + 6|$:

1. Substitute z for $3x - 4$ and w for $x + 6$. $\quad |z| = |w|$
2. Solve the absolute value equation. $\quad z = w \quad$ or $\quad z = -w$
3. Replace z with $3x - 4$ and w with $x + 6$. $\quad 3x - 4 = x + 6 \quad$ or $\quad 3x - 4 = -(x + 6)$
4. Solve the two equations for x.

$$\begin{aligned} 3x &= x + 10 & 3x &= -x - 2 \\ 2x &= 10 & 4x &= -2 \\ x &= 5 \quad \text{or} & x &= -\tfrac{1}{2} \end{aligned}$$

So the solutions of the equation $|3x - 4| = |x + 6|$ are $x = 5$ or $x = -\frac{1}{2}$.

Solving Absolute Value Inequalities

Here are some examples of inequalities that contain an absolute value.

$|4x - 3| < 5 \qquad |2x + 5| \geq 12 \qquad 3 - |x + 8| < 5 - x$

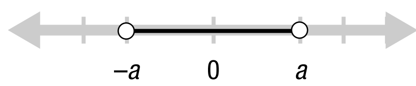
Before you learn to solve these inequalities, start with a simpler case. Here's how to solve an inequality of the form $|z| < a$ or $|z| > a$, where $a > 0$.

To solve $|z| < a$, look for numbers whose distance from 0 is less than a.

The solution is $-a < z < a$.

In interval notation, you can write this solution as $(-a, a)$.

Here's how to picture it on the number line:

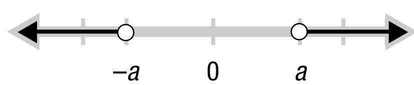


To solve $|z| > a$, look for numbers whose distance from 0 is greater than a.

The solution is $z < -a$ or $z > a$.

In interval notation, you can write this solution as $(-\infty, -a)$ or (a, ∞).

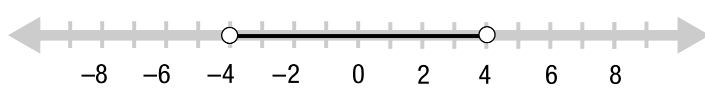
Here's how to picture it on the number line:



For example, to solve $|x| < 4$, look for numbers whose distance from 0 is less than 4.

The solution is $-4 < x < 4$ or the interval $(-4, 4)$.

Here's what the solution looks like on the number line.

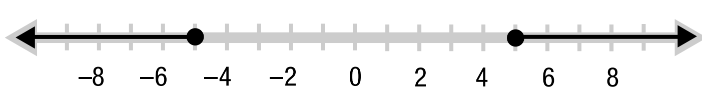


To solve $|x| \geq 5$, look for numbers whose distance from 0 is greater than or equal to 5.

The solution is $x \leq -5$ or $x \geq 5$.

You can write this as the two intervals $(-\infty, -5]$ or $[5, \infty)$.

Here's what the solution looks like on the number line.



Now you can solve an absolute value inequality of the form $|ax + b| < c$ or $|ax + b| > c$. Here are the steps:

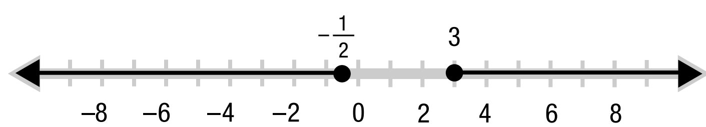
1. Substitute z for $ax + b$ to get $|z| < c$ or $|z| > c$.
2. Solve for z.
3. Replace z with $ax + b$.
4. Solve for x.

Here's an example.

To solve $|4x - 5| \geq 7$:

1. Substitute z for $4x - 5$.	$\lvert z\rvert \geq 7$		
2. Solve for z.	$z \leq -7$	or	$z \geq 7$
3. Replace z with $4x - 5$.	$4x - 5 \leq -7$	or	$4x - 5 \geq 7$
4. Solve for x.	$4x \leq -2$	or	$4x \geq 12$
	$x \leq -\frac{1}{2}$	or	$x \geq 3$

So, the solutions to the inequality $|4x - 5| \geq 7$ are $x \leq -\frac{1}{2}$ or $x \geq 3$. You can write this as the intervals $(-\infty, -\frac{1}{2}]$ or $[3, \infty)$. This is shown on the number line.



Sample Problems

1. Solve $\frac{4}{2x-3} + 3 = \frac{3x-1}{2x-3}$ for x.

   ✔ a. Multiply both sides of the the equation by the least common denominator to clear the fractions. $\quad (2x-3) \cdot \left(\frac{4}{2x-3} + 3\right) = (2x-3) \cdot \frac{3x-1}{2x-3}$

   ☐ b. Remove parentheses using the distributive property. __________ = __________

   ☐ c. Combine like terms on each side of the equation. __________ = __________

   ☐ d. Isolate the variable x on the left side. $x =$ __________

2. Solve this formula for the variable a: $3ax + 2bz = \frac{4}{y} - 5az$

Answers to Sample Problems

b. $4 + 3(2x - 3) = 3x - 1$ *or* $4 + 6x - 9 = 3x - 1$

c. $6x - 5 = 3x - 1$

d. $\frac{4}{3}$

✔ a. Multiply both sides of the formula by the least common denominator, y, to clear the fractions. $3axy + 2bzy = 4 - 5azy$

☐ b. To isolate a:
Subtract $2bzy$ from both sides. ______ = ______

☐ c. Add $5azy$ to both sides. ______ = ______

☐ d. Factor a from the left side of the equation. ______ = ______

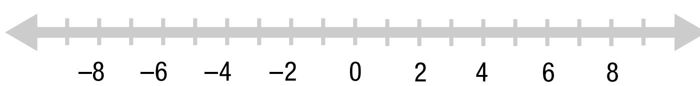
☐ e. Divide both sides of the equation by $3xy + 5zy$. ______ = ______

3. Solve the inequality below and graph the solution on the number line. $9 < 2x - 3 \le 15$

✔ a. Add 3 to all three parts. $12 < 2x \le 18$

☐ b. Divide all three parts by 2. ___ $< x \le$ ___

☐ c. Graph the solution on the number line.



4. Solve: $3 - |2x - 5| = 14$

✔ a. Isolate the absolute value term:
Subtract 3 from both sides. $-|2x - 5| = 11$

☐ b. Multiply both sides by –1. ______ = ______

☐ c. Substitute z for $2x - 5$. and solve. ____________

5. Solve the inequality below and graph the solution on the number line.
$$-2|5x - 1| + 3 \le -5$$

✔ a. Isolate the absolute value term:
Subtract 3 from both sides. $-2|5x - 1| \le -8$

☐ b. Divide both sides by –2. ____________

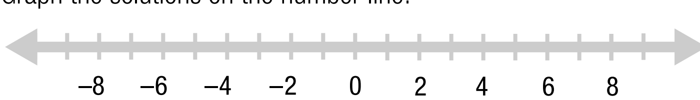
✔ c. Substitute z for $5x - 1$. $|z| \ge 4$

☐ d. Solve $|z| \ge 4$. ____________

☐ e. Replace z with $5x - 1$. ____________

☐ f. Solve for x. ____________

☐ g. Graph the solutions on the number line.



Answers to Sample Problems

b. $3axy = 4 - 5azy - 2bzy$

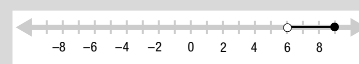
c. $3axy + 5azy = 4 - 2bzy$

d. $a(3xy + 5zy) = 4 - 2bzy$

e. $a = \frac{4 - 2bzy}{3xy + 5zy}$

b. $6 < x \le 9$

c.



b. $|2x - 5| = -11$

c. There are no solutions because the absolute value is never negative.

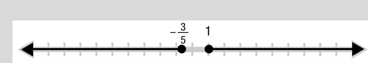
b. $|5x - 1| \ge 4$

d. $z \le -4$ or $z \ge 4$

e. $5x - 1 \le -4$ or $5x - 1 \ge 4$

f. $x \le -\frac{3}{5}$ or $x \ge 1$

g.



QUADRATIC EQUATIONS

Summary

In this concept you will review several ways to solve quadratic equations: by factoring, by using the square root method, by completing the square, and by using the quadratic formula. You will also review complex numbers and the discriminant.

Quadratic Equations

Here are some examples of quadratic equations:

$x^2 = 7$ $\qquad$ $3x^2 - 8x + 11 = 0$ $\qquad$ $(x - 5)(4x + 7) = 2$

In general, a quadratic equation is an equation that can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$. This is called the standard form of a quadratic equation.

Solving Quadratic Equations by Factoring

Some quadratic equations can be solved by factoring and using the zero product property. The zero product property states that if P and Q are polynomials, and $P \cdot Q = 0$, then $P = 0$, or $Q = 0$, or both P and Q are 0.

To solve a quadratic equation by factoring:

1. Write the equation in standard form.
2. Factor the left side.
3. Use the zero product property to set each factor equal to zero.
4. Solve for the variable.

Here's an example.

To solve $x^2 + 3x = 0$:

1. The equation is already in standard form.	$x^2 + 3x = 0$
2. Factor the left side.	$x(x + 3) = 0$
3. Use the zero product property to set each factor equal to zero.	$x = 0$ or $x + 3 = 0$
4. Solve each equation for the variable.	$x = 0$ or $x = -3$

So the solutions to the equation $x^2 + 3x = 0$ are $x = 0$ or $x = -3$.

Here's another example of solving a quadratic equation by the factoring method.

To solve $2x^2 - 7x = 4$:

1. Write the equation in standard form. $2x^2 - 7x - 4 = 0$
2. Factor the left side. $(2x + 1)(x - 4) = 0$
3. Use the zero product property to set each factor equal to zero. $2x + 1 = 0$ or $x - 4 = 0$
4. Solve each equation for the variable. $x = -\frac{1}{2}$ or $x = 4$

So the solutions to the equation $2x^2 - 7x = 4$ are $x = -\frac{1}{2}$ or $x = 4$.

Solving Quadratic Equations by the Square Root Method

Some quadratic equations cannot be solved by factoring, but can instead be solved by using the square root property: if $y^2 = a$, then $y = \sqrt{a}$ or $y = -\sqrt{a}$. Here a is a positive number.

This property can be used to find the two solutions of the quadratic equation $y^2 = a$. The two solutions are often written like this: $y = \pm\sqrt{a}$.

The $\pm$ symbol doesn't mean that you don't know which sign to use. It means that you have two different solutions.

Here's an example of how to solve $5x^2 - 30 = 0$ using the square root method:

1. Write the equation in the form $y^2 = a$. $5x^2 = 30$, $x^2 = 6$
2. Solve for x, using the square root property. $x = \pm\sqrt{6}$

So the equation $5x^2 - 30 = 0$ has two solutions, $x = +\sqrt{6}$ or $x = -\sqrt{6}$.

In general, here's how to solve a quadratic equation of the form $(ax + b)^2 = c$ using the square root method:

1. Substitute y for $ax + b$.
2. Solve the equation $y^2 = c$ to get $y = \sqrt{c}$ or $y = -\sqrt{c}$.
3. Replace y with $ax + b$ and solve each equation for x:
   $ax + b = \sqrt{c}$ or $ax + b = -\sqrt{c}$.

Here's an example.

To solve $(3x - 2)^2 = 7$:

1. Substitute y for $3x - 2$. $y^2 = 7$
2. Solve the equation $y^2 = 7$. $y = \sqrt{7}$ or $y = -\sqrt{7}$
3. Replace y with $3x - 2$ and solve each equation for x.
   $3x - 2 = \sqrt{7}$ or $3x - 2 = -\sqrt{7}$
   $3x = 2 + \sqrt{7}$ or $3x = 2 - \sqrt{7}$
   $x = \frac{2 + \sqrt{7}}{3}$ or $x = \frac{2 - \sqrt{7}}{3}$

So the equation $(3x - 2)^2 = 7$ has two solutions, $x = \frac{2 + \sqrt{7}}{3}$ or $x = \frac{2 - \sqrt{7}}{3}$.

Solving Higher Degree Equations

You can apply the factoring and square root methods to solve some equations that are not quadratic. Here's an example of solving a fourth degree equation.

To solve $x^4 + 6 = 5x^2$:

1. Write the equation in standard form. $x^4 - 5x^2 + 6 = 0$
2. Factor the left side. $(x^2 - 2)(x^2 - 3) = 0$
3. Use the zero product property to set each factor equal to zero. $x^2 - 2 = 0$ or $x^2 - 3 = 0$
4. Solve each equation for x^2. $x^2 = 2$ or $x^2 = 3$
5. Use the square root property to solve for x. $x = \pm\sqrt{2}$ or $x = \pm\sqrt{3}$

So the equation $x^4 + 6 = 5x^2$ has four solutions, $x = \pm\sqrt{2}$ or $x = \pm\sqrt{3}$.

Solving Quadratic Equations by Completing the Square

Some quadratic equations cannot be solved by either the factoring or the square root methods. There are two other methods that can be used to solve any quadratic equation.

The first method that works for any quadratic equation transforms the equation so that one side is a perfect square. This allows you to use the square root property. This method is called completing the square.

To solve any quadratic equation, $ax^2 + bx + c = 0$, by completing the square:

1. Divide both sides of the equation by a, if necessary, so that the coefficient of x^2 is 1.
2. Get the constant term by itself on the right side of the equation.
3. Find the number that completes the square on the left side:
   - Multiply the coefficient of x by $\frac{1}{2}$.
   - Square the result.
4. Add this number to both sides of the equation.
5. Factor the left side of the equation as a perfect square.
6. Solve this equation using the square root property.

Here's an example of completing the square.

To solve $2x^2 + 12x + 14 = 0$:

1. Divide both sides of the equation by 2, so that the coefficient of x^2 is 1. $x^2 + 6x + 7 = 0$
2. Get the constant term by itself on the right side of the equation. $x^2 + 6x = -7$

3. Find the number that completes the square:
   - Multiply the coefficient of x by $\frac{1}{2}$. $\qquad \frac{1}{2} \cdot 6 = 3$
   - Square the result. $\qquad 3^2 = 9$
4. Add this number to both sides of the equation. $\qquad x^2 + 6x + 9 = -7 + 9$
5. Factor the left side of the equation as a perfect square. $\qquad (x + 3)^2 = 2$
6. Solve this equation using the square root property. $\qquad x + 3 = \pm\sqrt{2}$, $\quad x = -3 \pm\sqrt{2}$

So the equation $2x^2 + 12x + 14 = 0$ has two solutions, $x = -3 \pm\sqrt{2}$.

Solving Quadratic Equations by the Quadratic Formula

When you complete the square to solve the general quadratic equation $ax^2 + bx + c = 0$, you get these two solutions:

$$x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a} \text{ (Here } a \neq 0.)$$

This is called the quadratic formula. It is the second method you can use to find the solutions of any quadratic equation.

To solve a quadratic equation using the quadratic formula:

1. Write the equation in standard form.
2. Identify a, b, and c.
3. Substitute these values into the quadratic formula.
4. Simplify.

Here's an example of using the quadratic formula.

To solve the equation $x^2 - 5x - 6 = 0$:

1. The equation is already written in standard form. $\qquad x^2 - 5x - 6 = 0$
2. Identify a, b, and c. $\qquad a = 1,\ b = -5,\ c = -6$
3. Substitute these values into the quadratic formula. $\qquad x = \frac{-(-5) \pm\sqrt{(-5)^2 - 4 \cdot 1 \cdot (-6)}}{2 \cdot 1}$
4. Simplify.

$$x = \frac{5 \pm\sqrt{25 + 24}}{}$$
$$= 5 \pm\sqrt{49}$$
$$= \frac{5 \pm 7}{2}$$
$$= \frac{12}{2} \text{ or } -\frac{2}{2}$$
$$= 6 \quad \text{or } -1$$

Notice that you could have obtained the same solutions by factoring the equation: $x^2 - 5x - 6 = (x - 6)(x + 1) = 0$. If the equation factors, that is usually the easiest way to find the solutions.

So the equation $x^2 - 5x - 6 = 0$ has two solutions, $x = 6$ or $x = -1$.

Notice that when you solve a quadratic equation using factoring or using the quadratic formula, you want 0 on the right side. When you solve using the square root property or completing the square, you want the constant term on the right side.

Here's another example of using the quadratic formula.

To solve $5x^2 - 4x = 6$:

1. Write the equation in standard form. $5x^2 - 4x - 6 = 0$
2. Identify a, b, and c. $a = 5,\ b = -4,\ c = -6$
3. Substitute these values into the quadratic formula. $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 5 \cdot (-6)}}{2 \cdot 5}$
4. Simplify.

$$x = \frac{4 \pm \sqrt{16 + 120}}{10}$$
$$= \frac{4 \pm \sqrt{136}}{10}$$
$$= \frac{4 \pm 2\sqrt{34}}{10}$$
$$= \frac{2 \pm \sqrt{34}}{5}$$

So the equation $5x^2 - 4x = 6$ has two solutions, $x = \frac{2 \pm \sqrt{34}}{5}$.

Imaginary Numbers

In order to solve the quadratic equation $x^2 = -1$, you have to define a new type of number that is not a real number. This number is the imaginary number i which is defined as: $i = \sqrt{-1}$.

Since $i = \sqrt{-1}$, $i^2 = -1$. Also, $(-i)^2 = i^2 = -1$.

So the equation $x^2 = -1$ has two solutions, $x = i$ and $x = -i$.

Using i, you can get other imaginary numbers.

For example, $\sqrt{-9} = i\sqrt{9} = 3i$ and $\sqrt{-7} = i\sqrt{7}$.

In general, if k is a positive number, $\sqrt{-k} = i\sqrt{k}$.

You must be careful when working with imaginary numbers. Not all of the properties of real numbers hold for imaginary numbers.

Here's an example. To find $\sqrt{-25} \cdot \sqrt{-4}$ you cannot use the multiplication property of radicals because that property is true only when the numbers under the square roots are positive. Instead you should write the radicals in terms of i before you multiply:

$$\sqrt{-25} \cdot \sqrt{-4} = 5i \cdot 2i = 10i^2 = -10$$

Powers of i

Here's a table that shows the first few powers of i:

Power of $\boldsymbol{i}$	Value
i^1	i
i^2	-1
i^3	$i^2 \cdot i = -i$
i^4	$i^2 \cdot i^2 = 1$
i^5	$i^4 \cdot i = i$

This pattern, i, –1, $-i$, 1, continues for higher powers of i. However, to find higher powers of i it is often useful to rewrite the number using i^4 as many times as possible (since $i^4 = 1$).

For example, here's a way to find i^{19}:

$$\begin{aligned} i^{19} &= i^{16} \cdot i^3 \\ &= i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^3 \\ &= 1 \cdot 1 \cdot 1 \cdot 1 \cdot (-i) \\ &= -i \end{aligned}$$

Similarly, here's a way to find $= i^{30}$:

$$\begin{aligned} i^{30} &= (i^4)^7 \cdot i^2 \\ &= 1^7 \cdot i^2 \\ &= 1 \cdot (-1) \\ &= -1 \end{aligned}$$

Complex Numbers

A complex number has the form $a + bi$, where a is a real number and b is a real number. Here are some examples:

In a complex number, $a + bi$, the real part, a, and the imaginary part, b, are each real numbers.

$2 + 3i$ $\quad$ $7 - 5i$ $\quad$ $\frac{2}{3} + \sqrt{2} \cdot i$ $\quad$ $0 + \pi i$

In the complex number $a + bi$, the number $\boldsymbol{a}$ is called the real part of the complex number and the number b is called the imaginary part of the complex number.

For example, 2 is the real part of the complex number $2 + 3i$ and 3 is the imaginary part of the complex number $2 + 3i$.

Every real number is a complex number.
For example, the real number 2 can be written as: $2 = 2 + 0i$

Similarly, the real number $\sqrt{5}$ can be written as: $\sqrt{5} = \sqrt{5} + 0i$

In general, any real number, say a, can be written as: $a = a + 0i$
This is of the form $a + bi$, where $b = 0$.

Also, every imaginary number is a complex number.
For example, the imaginary number $3i$ can be written as: $3i = 0 + 3i$
Similarly, the imaginary number πi can be written as: $\pi i = 0 + \pi i$

In general, any imaginary number, say bi, can be written as: $bi = 0 + bi$
This is of the form $a + bi$, where $a = 0$.

Adding and Subtracting Complex Numbers

You can combine complex numbers in many of the same ways that you can combine real numbers.

To add or subtract complex numbers:

1. Add or subtract the real parts.
2. Add or subtract the imaginary parts and multiply by i.
3. Write the answer.

For example to find $(11 - 8i) + (3 + 5i)$

1.	Add the real parts.	$11 + 3 = 14$
2.	Add the imaginary parts and multiply by i.	$-8 + 5 = -3$ $-3i$
3.	Write the answer.	$14 + (-3i)$

So $(11 - 8i) + (3 + 5i) = 14 + (-3i)$.

In general, you can write: $(a + bi) + (c + di) = (a + c) + (b + d)i$ and
$(a + bi) - (c + di) = (a - c) + (b - d)i$.

Multiplying Complex Numbers

To multiply a complex number by a real number, you simply multiply the real part by the real number and multiply the imaginary part by the real number and i.

For example, $7 \cdot (3 - 4i) = 7 \cdot 3 - 7 \cdot 4i$

$$= 21 - 28i$$

In general, $k(a + bi) = ka + kbi$. Here, a, b, and k are real numbers.

To multiply a complex number by another complex number, you use steps that are similar to multiplying two binomials.

$$\begin{aligned}(a + bi)(c + di) &= ac + adi + bci + bdi^2 \\ &= ac + bd(-1) + adi + bci \\ &= (ac - bd) + (ad + bc)i\end{aligned}$$

For example, here's how to find $(5 - 3i)(7 + 4i)$:

$$\begin{aligned}(5 - 3i)(7 + 4i) &= 35 + 20i - 21i - 12i^2 \\ &= 35 - 12(-1) + 20i - 21i \\ &= 47 + (-1)i \\ &= 47 - i\end{aligned}$$

Dividing Complex Numbers

To divide one complex number by another complex number you work with the complex conjugate. For example, $2 - 9i$ is the complex conjugate of $2 + 9i$.

In general: $a + bi$ and $a - bi$ are complex conjugates.

To divide complex numbers:

1. Write the division problem as a fraction.
2. Multiply the numerator and denominator by the complex conjugate of the denominator.
3. Simplify.

For example, to find $(3 - 7i) \div (2 + i)$:

1. Write the division problem as a fraction. $\quad \dfrac{3 - 7i}{2 + i}$
2. Multiply the numerator and denominator by the complex conjugate of the denominator. $\quad = \dfrac{(3 - 7i)}{(2 + i)} \cdot \dfrac{(2 - i)}{(2 - i)}$
3. Simplify. $\quad = \dfrac{6 - 3i - 14i + 7i^2}{4 - 2i + 2i - i^2}$

$$= \frac{-1 - 17i}{5}$$

Quadratic Equations and Complex Numbers

When you solve quadratic equations using the methods above, you will sometimes get solutions that are nonreal complex numbers.

For example, to solve $3x^2 + 15 = 0$:

1. Write the equation in the form $x^2 = a$. $\quad 3x^2 = -15$

   $x^2 = -5$
2. Use the square root property. $\quad x = \pm\sqrt{-5}$

   $= \pm i\sqrt{5}$

So the quadratic equation $3x^2 + 15 = 0$ has two nonreal complex number solutions, $x = \pm i\sqrt{5}$.

Notice that you used the standard steps for solving a quadratic equation with the square root property. But here the answer was a nonreal complex number.

Discriminant

In the quadratic formula, the expression that appears under the square root, namely $b^2 - 4ac$, is called the discriminant. It can be used to predict the nature of the solutions of a quadratic equation, without actually solving the equation.

If the discriminant is a perfect square, you can solve the equation by factoring.

For the quadratic equation $ax^2 + bx + c = 0$:

- If $b^2 - 4ac > 0$, then the equation has two unequal, real solutions.
  If $b^2 - 4ac$ is a perfect square, then the solutions are rational numbers.
  If $b^2 - 4ac$ is not a perfect square, then the solutions are irrational numbers.
- If $b^2 - 4ac = 0$, then the equation has one distinct, real solution of multiplicity two.
- If $b^2 - 4ac < 0$, then the equation has two nonreal complex solutions that are complex conjugates of each other.

For example, to determine the type of solutions of the quadratic equation $2x^2 + 5x - 11 = 0$:

Find the discriminant, $b^2 - 4ac$.

$$\begin{aligned} &5^2 - 4 \cdot 2 \cdot (-11) \\ &= 25 + 88 \\ &= 113 \end{aligned}$$

Since $b^2 - 4ac > 0$, there are two unequal real solutions.

Since $b^2 - 4ac$ is not a perfect square, the solutions are irrational numbers.

So the equation $2x^2 + 5x - 11 = 0$ has two unequal solutions that are irrational numbers.

Answers to Sample Problems

b. $(3x - 4)(5x + 2)$

c. $3x - 4 = 0$ or $5x + 2 = 0$

d. $\frac{4}{3}, -\frac{2}{5}$

b. $x^2 + 4x = 7$

c. 2

4

Sample Problems

1. Solve by factoring: $15x^2 - 14x = 8$

   ☑ a. Write the equation in standard form. $15x^2 - 14x - 8 = 0$

   ☐ b. Factor the left side. ________ $= 0$

   ☐ c. Use the zero product property to set each factor equal to zero. ________ $= 0$ or ________ $= 0$

   ☐ d. Solve for x. $x =$ ________ or $x =$ ________

2. Solve by completing the square: $3x^2 + 12x - 21 = 0$

   ☑ a. Divide both sides of the equation by 3 so that the coefficient of x^2 is 1. $x^2 + 4x - 7 = 0$

   ☐ b. Get the constant term by itself on the right side of the equation. ________ = ________

   ☐ c. Find the number that completes the square:

   - Multiply the coefficient of x by $\frac{1}{2}$. ________
   - Square the result. ________

Answers to Sample Problems

☐ d. Add this number to both sides of the equation. __________ = __________

d. $x^2 + 4x + 4 = 11$

☐ e. Factor the left side of the equation as a perfect square. __________ = __________

e. $(x + 2)^2 = 11$

☐ f. Solve this equation using the square root property. x = __________

f. $-2 \pm \sqrt{11}$

3. Solve by using the quadratic formula: $5x^2 + 4x + 2 = 0$

☑ a The equation is already written in standard form. $5x^2 + 4x + 2 = 0$

☐ b. Identify a, b, and c.

a = ______

b = ______

c = ______

b. 5

4

2

☐ c. Substitute these values into the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. x = __________

c. $\dfrac{-4 \pm \sqrt{4^2 - 4 \cdot 5 \cdot 2}}{2 \cdot 5}$

☐ d. Simplify.

x = __________

d. $\dfrac{-2 \pm i\sqrt{6}}{5}$

4. Simplify: $(7 - 2i)(3 + 4i) - (8 - 13i)$

☑ a. Find $(7 - 2i)(3 + 4i)$.

$(7 - 2i)(3 + 4i) - (8 - 13i)$

$= 21 + 28i - 6i - 8i^2 - (8 - 13i)$

☐ b. Simplify the multiplication. = ____________________

b. $29 + 22i - (8 - 13i)$

☐ c. Subtract. = ____________________

c. $21 + 35i$

5. Use the discriminant to predict the type of solutions of $12x^2 - x = 20$.

☑ a. Write the equation in standard form. $12x^2 - x - 20 = 0$

☐ b. Identify a, b, and c.

a = ______

b = ______

c = ______

b. 12

–1

–20

☐ c. Calculate the discriminant. = _______________

c. $(-1)^2 - 4(12)(-20) = 961$

☐ d. Describe the solutions.

d. The discriminant, 961, is greater than 0 and is a perfect square, ($961 = 31^2$), so the equation has two unequal rational number solutions.

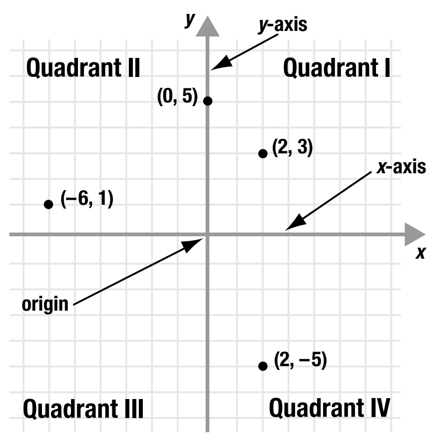
LINEAR EQUATIONS

Summary

In this concept you will review the Cartesian coordinate system. You will then graph linear equations. You will also work with the slope, and the various forms of the equation of a straight line.

The Cartesian Coordinate System

Recall that the Cartesian coordinate system, or *xy*-plane, consists of two number lines placed at right angles to each other. The horizontal line is called the *x*-axis and the vertical line is called the *y*-axis. These lines intersect at the origin, and divide the plane into four quadrants. Individual points can be described using an ordered pair of real numbers (x_1, y_1), where x_1 is the *x*-coordinate and y_1 is the *y*-coordinate. Here are examples of plotted points and their coordinates.



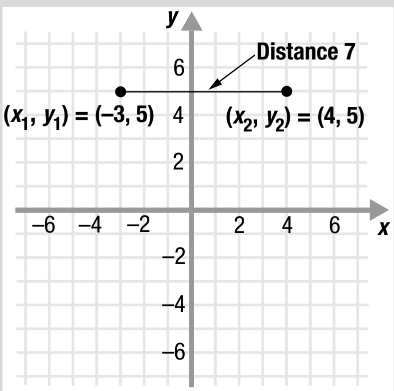


Figure EIII.C.1

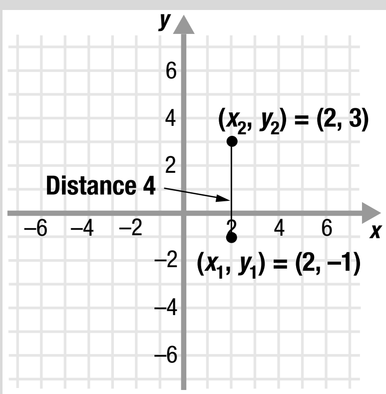


Figure EIII.C.2

The distance formula is derived from the Pythagorean Theorem in geometry, using Cartesian coordinates.

The Distance Formula

You can calculate the distance between any two points (x_1, y_1) and (x_2, y_2) in the *xy*-plane.

If both points lie on a horizontal line, then the distance between them is simply $|x_2 - x_1|$. For example, the distance between (–3, 5) and (4, 5) is $|4 - (-3)| = 7$, as shown in Figure EIII.C.1.

If both points lie on a vertical line, then the distance between them is simply $|y_2 - y_1|$. For example, the distance between (2, –1) and (2, 3) is $|3 - (-1)| = 4$, as shown in Figure EIII.C.2.

Otherwise, the distance between any two points (x_1, y_1) and (x_2, y_2) is given by the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

So, for example, the distance between (–3, 1) and (2, –4) is given by:

$$\begin{aligned} d &= \sqrt{[2-(-3)]^2 + (-4-1)^2} \\ &= \sqrt{25+25} \\ &= \sqrt{50} \\ &= 5\sqrt{2} \end{aligned}$$

You can also calculate the midpoint, M, of the line joining any two points (x_1, y_1) and (x_2, y_2) in the xy-plane by using this formula:

$$M: \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

In the above example, the midpoint, M, of the line joining (–3, 1) and (2, –4) is given by:

$$M: \left(\frac{-3+2}{2}, \frac{1-4}{2}\right) = \left(-\frac{1}{2}, -\frac{3}{2}\right)$$

See Figure EIII.C.3.

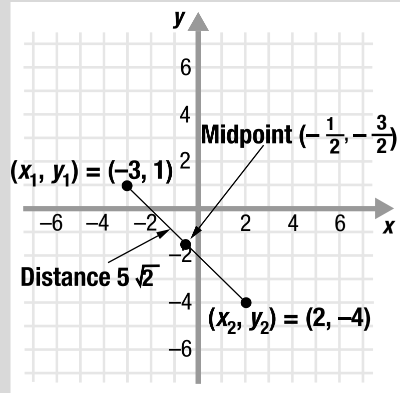


Figure EIII.C.3

Graphing Linear Equations

Here are some examples of linear equations:

$y = 3x - 7$ $\quad 2x + 5y = 12$ $\quad x = 11$ $\quad y = -2$

The graph of a linear equation is a straight line in the xy-plane.

To graph a linear equation:

1. Make a table of ordered pairs that satisfy the equation.
2. Plot these ordered pairs and draw a line through the points.

For example, to graph $3x - 5y = 15$:

1. Make a table of ordered pairs that satisfy the equation.

x	y
0	–3
5	0
10	3

2. Plot these ordered pairs and draw a line through the points. See Figure EIII.C.4.

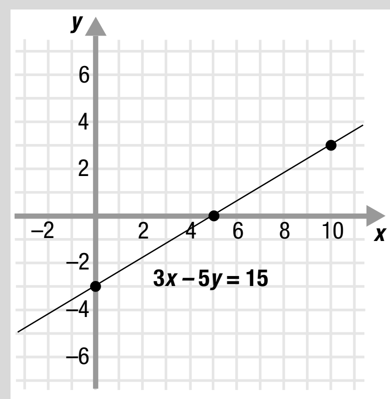


Figure EIII.C.4

Any two points are sufficient to determine a line, although using three points lets you check your calculations. Two points that are often easy to calculate are the x-intercept and the y-intercept.

To find the x-intercept, set $y = 0$ and solve for x.
To find the y-intercept, set $x = 0$ and solve for y.

For example, to find the x-intercept of $3x - 5y = 15$:
Set $y = 0$ and solve for x.

$$3x - 5(0) = 15$$
$$3x = 15$$
$$x = 5$$

So the x-intercept is (5, 0).

To find the y-intercept of $3x - 5y = 15$:
Set $x = 0$ and solve for y.

$$3(0) - 5y = 15$$
$$-5y = 15$$
$$y = -3$$

So the y-intercept is (0, –3).

Horizontal and Vertical Lines

The graph of the linear equation $y = c$ is a horizontal line. All the points on this line have y-coordinate c.

For example, here are some points on the graph of the line $y = 4$:

x	y
–2	4
0	4
3	4

The graph is shown in Figure EIII.C.5.

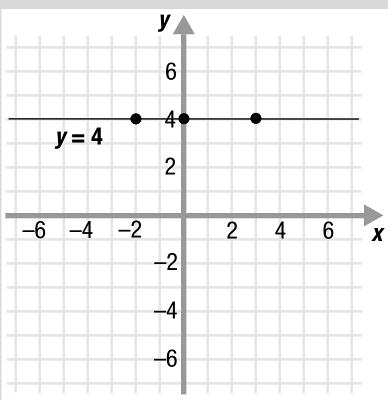


Figure EIII.C.5

Similarly, the graph of the linear equation $x = c$ is a vertical line. All the points on this line have x-coordinate c.

For example, here are some points on the graph of the line $x = -2$:

x	y
–2	–3
–2	0
–2	4

The graph is shown in Figure EIII.C.6.

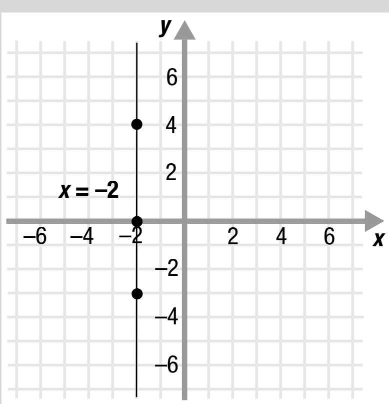


Figure EIII.C.6

Slope

The slope of a line is a number that describes its steepness. It is the ratio of the change in y (rise) to the change in x (run).

The slope, m, of the line joining any two points (x_1, y_1) and (x_2, y_2) in the xy-plane is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

For example, here's how to find the slope of the line joining (–3, –1) and (1, 5):

$$m = \frac{5 - (-1)}{1 - (-3)} = \frac{6}{4} = \frac{3}{2}.$$

This is illustrated in Figure EIII.C.7.

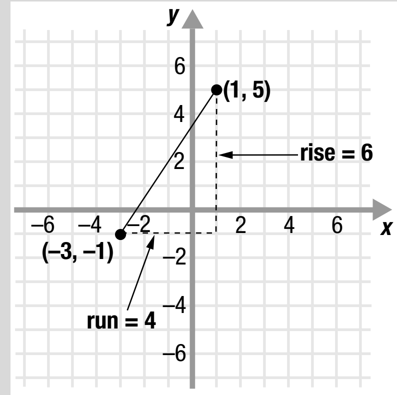


Figure EIII.C.7

The slope describes the steepness of a line:

- If the slope is positive, the line rises from left to right. A larger positive value means that the line rises more steeply.
- If the slope is negative, the line falls from left to right. A more negative value means that the line falls more steeply.
- If the slope is zero, the line is horizontal.
- If the slope is undefined, the line is vertical.

This is illustrated in Figure EIII.C.8.

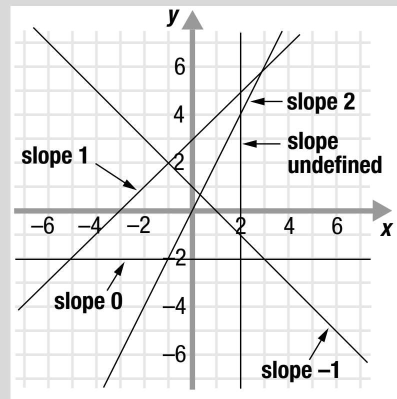


Figure EIII.C.8

The Equation of a Line

There are several different forms for the equation of a line.

Point-Slope Form

The equation of a line with slope m, passing through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$.

For example, the equation of the line with slope 5, passing through the point (2, –8) is $y - (-8) = 5(x - 2)$ or $y + 8 = 5(x - 2)$.

Standard Form

The standard form for the equation of a line is $Ax + By = C$.

Here, A, B, and C are real numbers. A, B, or C can be zero, but A and B cannot both be zero.

For example, here's how to write the equation $y + 8 = 5(x - 2)$ in standard form:

$$y + 8 = 5(x - 2)$$
$$y + 8 = 5x - 10$$
$$-5x + y = -18$$

Slope-Intercept Form

The equation of a line with slope m and y-intercept $(0, b)$ is $y = mx + b$.

For example, here's how to write the equation $-5x + y = -18$ in slope-intercept form:

$$-5x + y = -18$$
$$y = 5x - 18$$

When an equation is in slope-intercept form it's especially easy to identify its slope and y-intercept. For the line $y = 5x - 18$, the slope is 5 and the y-intercept is $(0, -18)$.

Here's another example. To find the slope of the line $5x - 8y = 13$:

1. Write the equation in slope-intercept form. $\quad -8y = -5x + 13$, $\quad y = \frac{5}{8}x - \frac{13}{8}$
2. Identify the slope. $\quad$ The slope is $\frac{5}{8}$.

The other forms for the equation of a line can also be useful.

For example, to find the standard form for the equation of the line passing through the points $(-3, 4)$ and $(5, 7)$:

1. Use the two points to find m. $\quad m = \frac{7-4}{5-(-3)} = \frac{3}{8}$
2. Substitute $m = \frac{3}{8}$ and one of the points, for example, $(x_1, y_1) = (-3, 4)$, into the point-slope form for the equation of a line. Then write the equation in standard form.

$$y - y_1 = m(x - x_1)$$
$$y - 4 = \frac{3}{8}[x - (-3)]$$
$$8(y - 4) = 3(x + 3)$$
$$8y - 32 = 3x + 9$$
$$-3x + 8y = 41$$

In step (2) you used the point (–3, 4). If you used the other point, (5, 7), you would get the same equation for the line.

So the standard form for the equation of the line is $-3x + 8y = 41$.

Parallel and Perpendicular Lines

Parallel lines have the same slope.

For example, the lines $2x + 3y = 7$ and $4x + 6y = 9$ are parallel because they both have the same slope, $-\frac{2}{3}$.

You can check this by writing each equation in slope-intercept form:

$2x + 3y = 7$	$4x + 6y = 9$
$3y = -2x + 7$	$6y = -4x + 9$
$y = -\frac{2}{3}x + \frac{7}{3}$	$y = -\frac{2}{3}x + \frac{3}{2}$

In both cases, the slope is $-\frac{2}{3}$.

Perpendicular lines have slopes that are negative reciprocals of each other. For example, two lines with slopes 3 and $-\frac{1}{3}$ are perpendicular to each other. Two lines with slopes $\frac{1}{14}$ and -14 are also perpendicular to each other.

If two perpendicular lines have slopes m_1 and m_2, then $m_1 \cdot m_2 = -1$.

Here's how to find the slope of a line perpendicular to the line $6x + 9y = 11$:

1. Write the equation of the given line in slope-intercept form. $y = -\frac{2}{3}x + \frac{11}{9}$
2. Find its slope, m. $m = -\frac{2}{3}$
3. The slope of a perpendicular line is the negative reciprocal of $-\frac{2}{3}$. $\frac{3}{2}$

You can use what you know about parallel and perpendicular lines to find the equation of a line that is parallel or perpendicular to a given line.

For example, to find the equation in standard form of the line that passes through the point (1, 5) and is perpendicular to the line $y = 4x - 9$:

1. Find the slope of the new line:
   - The slope of the new line is the negative reciprocal of the slope of the given line.
   - The slope of the given line is 4.
   - So the slope of the new line is $-\frac{1}{4}$.
2. Use the slope, $-\frac{1}{4}$, and the point, (1, 5), to write the equation of the new line. $y - 5 = -\frac{1}{4}(x - 1)$
3. Simplify.
$$4y - 20 = -x + 1$$
$$x + 4y = 21$$

So the equation of the line that passes through the point (1, 5) and is perpendicular to the line $y = 4x - 9$ is $x + 4y = 21$.

Answers to Sample Problems

b. $\sqrt{(-3-2)^2+[4-(-7)]^2}$

c. $\sqrt{146}$

d. $\left(\frac{2+(-3)}{2}, \frac{-7+4}{2}\right)$

e. $\left(-\frac{1}{2}, -\frac{3}{2}\right)$

b. -2

c. Here are some possible ordered pairs.

x	y
0	-2
$\frac{5}{2}$	0
1	$-\frac{6}{5}$
2	$-\frac{2}{5}$

Sample Problems

1. Find the distance between the two points (2, –7) and (–3, 4). Also find the midpoint of the line joining them.

☑ a. Identify x_1, x_2, y_1, y_2

$x_1 = 2$
$x_2 = -3$
$y_1 = -7$
$y_2 = 4$

☐ b. Substitute into the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad d =$ ______________________

☐ c. Simplify. $\qquad d =$ ______________________

☐ d. Substitute into $M: \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $\qquad M$: ______________________

☐ e. Simplify. $\qquad M$: ______________________

2. Graph this equation: $4x - 5y = 10$

☑ a. Find the x-intercept.

Set $y = 0$ and solve for x.

$$4x - 5(0) = 10$$
$$4x = 10$$
$$x = \frac{5}{2}$$

The x-intercept is $\left(\frac{5}{2}, 0\right)$.

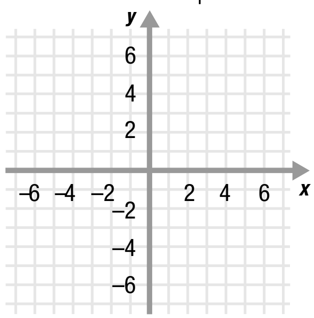
☐ b. Find the y-intercept.

The y-intercept is (0, _____)

☐ c. Make a table of ordered pairs pairs that satisfy the equation.

x	y

☐ d. Plot these ordered pairs and draw a line through the points.



3. Find the slope of each of these lines:
   a. $y = 5x + 6$
   b. $y = -7$
   c. $x = 11$

✔ a. The first equation is written in slope-intercept form, $y = mx + b$. The slope, m, is 5.

☐ b. Find the slope of $y = -7$. __________

☐ c. Find the slope of $x = 11$. __________

4. Find the slope and y-intercept of the line $5x + 7y = 18$.

✔ a. Write the equation in slope-intercept form, $y = mx + b$.

$$7y = -5x + 18$$
$$y = -\frac{5}{7}x + \frac{18}{7}$$

☐ b. Identify the slope. $m =$ __________

☐ c. Identify the y-intercept (0, _____)

5. Find the equation of the line perpendicular to the line $3x + 4y = 12$ that passes through the point (1, 2). Write your answer in standard form.

✔ a. Write the given equation in slope-intercept form. $y = -\frac{3}{4}x + 3$

☐ b. Identify the slope of the given line. ____________________

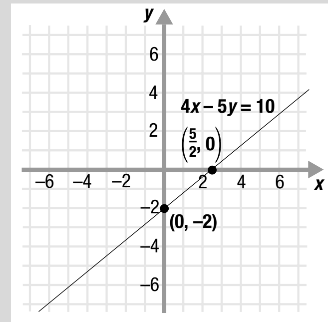
☐ c. Find the slope of a line perpendicular to the given line. ____________________

☐ d. Write the point-slope form of the equation of the new line. ____________________

☐ e. Rewrite the equation in standard form. ____________________

Answers to Sample Problems

d.



b. 0 (The line y = –7 is a horizontal line.)

c. The slope is undefined. (The line x = 11 is a vertical line.)

b. $-\frac{5}{7}$

c. $\frac{18}{7}$

b. $-\frac{3}{4}$

c. $\frac{4}{3}$

d. $y - 2 = \frac{4}{3}(x - 1)$

e. $-4x + 3y = 2$

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Linear and Absolute Value Equations and Inequalities

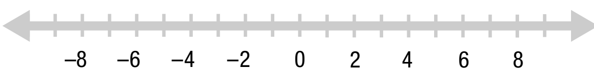
1. Solve this equation for x: $3x + 4 = 5x - 1$

2. Solve this inequality for x: $3x - 1 < 5$. Write your solution using interval notation.

3. Solve this equation for x: $|x + 4| = 9$

4. Solve this equation for x: $3(2x - 7) = 4 - (x - 2)$

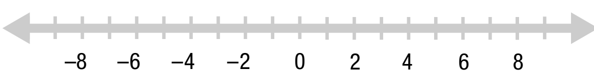
5. Solve this inequality for x: $2(3x - 1) + 5 > 6 - 3(4x - 1)$. Graph your solution on the number line below.



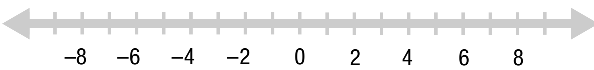
6. Solve this equation for x: $|x + 3| = |2x - 1|$

7. Solve this formula for w: $2xy - 3wz = 12uv$

8. Solve this inequality for x: $-12 \leq 3x - 5 < 8$. Graph your solution on the number line below.

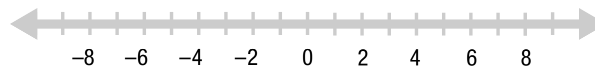


9. Solve this inequality for x: $|3x + 2| \geq 1$. Graph your solution on the number line below.



10. Solve this formula for b: $\frac{1}{a} + \frac{2}{b} = \frac{3}{c}$

11. Solve this inequality for x: $2x - 3(4x - 1) \geq 5 - 4(1 - 6x)$. Write your solution in interval notation, and graph your solution on the number line below.



12. Solve this inequality for x: $17 + 6|13x - 19| < 12$

Quadratic Equations

13. Solve for x: $2x^2 = 6$

14. Solve by completing the square: $x^2 + 8x - 5 = 0$

15. Find: $(2 + 11i) - 3(3 - 4i)$

16. Solve for x using the square root property: $3(2x - 1)^2 = 15$

17. Solve for x completing the square: $4x^2 + 32x + 20 = 0$

18. Find: $(5 - 3i)(7 + 2i)$

19. Solve for x by factoring: $6x^2 + 11x - 35 = 0$

20. Solve for x using the quadratic formula: $x^2 + x - 1 = 0$

21. Simplify: $\frac{3 - i}{5 + 2i}$

22. Solve for x: $2x^4 + 20 = 13x^2$

23. Solve for x: $x^2 + 6x + 12 = 0$

24. Use the discriminant to determine the type of solutions of $5x^2 + 10 = 7x$.

Linear Equations

25. Write the coordinates of the labeled points in Figure EIII.C.9. Also write the quadrant in which each point lies.

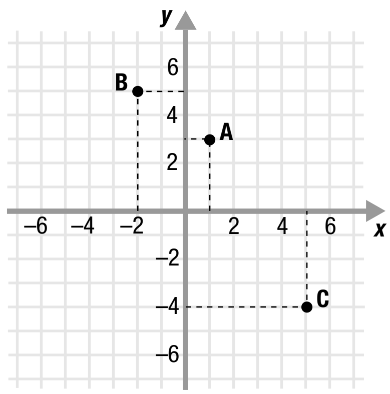


Figure EIII.C.9

26. Graph this equation: $y = 3x - 5$.

27. Find the slope of the line joining the two points (2, 3) and (5, –6).

28. Find the distance between the two points (–7, –3) and (–7, 8).

29. Graph the equation $4x - 7y = 14$.

30. Find the equation of the line with slope 2 that passes through the point (3, 5). Write your answer in standard form.

31. Find the distance between the two points (–4, –3) and (–1, 4).

32. Graph this equation: $x = 5$.

33. Find the equation of the line with slope –4 and y-intercept (0, 11).

34. Find the midpoint of the line joining the two points (–4, –1) and (3, –9).

35. Find the slope and y-intercept of the equation $5x + 2y = 8$.

36. Find the equation of the line perpendicular to $x + 2y = 3$ that passes through the point (–3, 4). Write your answer in slope-intercept form.

APPLY

PRACTICE PROBLEMS

Here are some additional practice problems for you to try.

Linear and Absolute Value Equations and Inequalities

1. Solve for x: $4(2x - 5) = 3x - 8(4 - 5x)$
2. Solve for x: $x - \frac{2}{3} = \frac{1}{8}(5x + 3)$
3. Solve this formula for m: $6mp - 4rs = 3ms + rp$
4. Solve for x: $\frac{2}{5} + 5x < \frac{7}{3}x - 1$

   Write your answer using interval notation.
5. Solve for x: $\frac{x+2}{3} > 2 - \frac{x+2}{6}$

   Graph your solution on a number line.
6. Solve for x: $3 \geq \frac{x+1}{3} \geq 0$

   Graph your solution on a number line.
7. Solve for x: $\frac{|3x+1|}{4} = \frac{2}{5}$
8. Solve for x: $|3x + 7| = |4x - 3|$
9. Solve for x: $|2x - 1| < 4$

   Write your answer using interval notation.
10. Solve for x: $\frac{1}{3}|2x - 1| \geq 4$

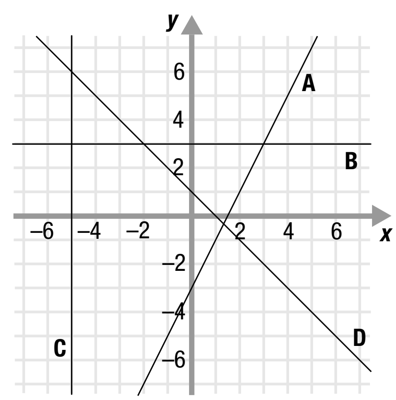
    Graph your solution on a number line.

Quadratic Equations

11. Solve using the square root property: $5x^2 = 40$
12. Solve using the square root property: $(3x + 1)^2 - 5 = 0$
13. Solve by factoring: $8x^2 + 2x = 15$
14. Solve by factoring: $x^3 + 3x^2 - 7x = 21$
15. Solve by completing the square: $3x^2 + 12x + 6 = 0$
16. Simplify: $\frac{2+i}{3-5i}$
17. Solve using the quadratic formula: $3x^2 - 2x + 1 = 0$
18. Simplify: $(4 + 5i) - 3(2 - 9i)$
19. Simplify: $(1 + 8i)(5 - 3i)$

Linear Equations

20. Use the discriminant to predict the nature of the solutions of this equation: $2x^2 + 5x = 7$
21. Identify which line has positive slope, which line has negative slope, which line has zero slope, and which line has no slope.



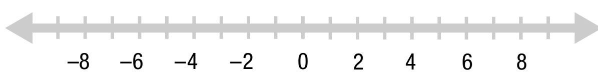
22. Find the distance between the two points (–8, –2) and (–4, –2).
23. Find the distance between the two points (1, 2) and (3, 4).
24. Find the midpoint of the line segment joining the two points (4, –2) and (–10, 15).
25. Graph the equation $4y - 7x = 11$.
26. Graph the equation $4x + 20 = 0$.
27. Find the slope and the y-intercept of the line with equation $5x - 8y = 13$.
28. Find the equation of the line with slope –5 that passes through the point (2, 7). Write your answer in standard form.
29. Find the equation of the line joining the two points (–2, 5) and (–5, –11). Write your answer in standard form.
30. Find the equation of the line perpendicular to $2x + 6y = 27$ that passes through the point (–1, 4). Write your answer in standard form.

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Solve this equation for x: $2(5x - 1) = 4 - \frac{3x}{2}$
2. Solve this formula for h: $A = 2\pi rh + 2\pi r^2$
3. Solve this inequality for x: $\frac{3}{4}x - 5(1 - x) \geq 7 - 4x$. Write your answer using interval notation.
4. Solve for x: $|1 - 3x| < 7$. Graph your solution on the number line below.



5. Solve for x: $(2x - 1)^2 - 30 = 0$
6. Solve by completing the square: $x^2 + 10x + 10 = 0$
7. Simplify:
   a. i^{17}
   b. $(7 - 2i) - (4 - i)$
   c. $(2 - i)(2 + i)$
8. Simplify: $\frac{2 + i}{3 - 4i}$
9. Find the distance between the two points (–3, 6) and (0, –4).
10. Find the equation of the line with slope 6 and y-intercept (0, –13). Write your answer in point-slope form.
11. Find the slope, m, and the y-intercept of the line $5x + 2y = 15$.
12. Find the equation of the line parallel to the line $3x - 4y = 10$ that passes through the point (6, 0).