Test 3
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Math 125 – Winter 2009

No work = no credit

No symbolic calculators

Name: key.

I know not what I appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell, whilest the great ocean of truth lay all undiscovered before me.

Isaac Newton (1643 - 1727)
English mathematician

Warm-ups (1 pt each) $-\pi^0 =$ -1 $\frac{\pi}{0} =$ und. $\cos(\pi) =$ -1

1.) (1 pt) According to the quote (see above), what portion of truth did Newton discover? Answer using complete sentences.

Newton just found the pebbles on the shore of the ocean of truth.

2.) (10 pts) Use a Comparison Theorem to determine whether the integral $\int_0^8 \frac{|\sin(x)|}{\sqrt[3]{x}}dx$ converges or diverges. Remember to state your conclusion.

$0 \le \frac{|\sin x|}{\sqrt[3]{x}} \le \frac{1}{\sqrt[3]{x}}$ on $(0, 8]$

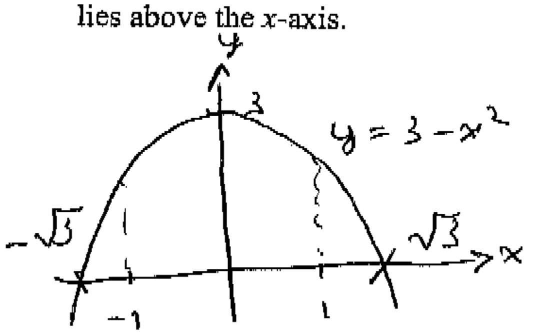
since $\int_0^8 \frac{dx}{\sqrt[3]{x}}$ converges by the p-test, $\int_0^8 \frac{|\sin x|}{\sqrt[3]{x}}dx$ also converges by comparison.

Solution: ______________________

3.) (10 pts) Use Part 1 of the Fundamental Theorem of Calculus to find $\frac{d}{dx}\int_{1}^{\sin(x)} \arcsin(u)\,du$.

Solution: $\arcsin(\sin x)) \cdot \cos x = x \cos x.$

4.) (10 pts) Find the centroid of the region bounded by $y = 3 - x^2$ on the interval $-1 \le x \le 1$ that lies above the x-axis.



$\bar{x} = 0$ by symmetry

$A = \int_{-1}^{1} 3 - x^2\, dx$

$= 2\left(3x - \frac{1}{3}x^3\right)\Big|_0^1$

$= 2 \cdot \frac{8}{3} = \frac{16}{3}.$

$\bar{y} = \frac{3}{16} \cdot \frac{1}{2} \int_{-1}^{1} (3 - x^2)^2\, dx$

$= \frac{3}{32} \cdot 2 \int_0^1 (9 - 6x^2 + x^4)\, dx$

$= \frac{3}{16}\left(9x - 2x^3 + \frac{1}{5}x^5\right)\Big|_0^1$

$= \frac{3}{4\cancel{16}} \cdot \frac{\cancel{36}\,9}{5}$

$= \frac{27}{20}$

Solution: centroid @ $(0, \frac{27}{20})$.

$\overline{20}$

5.) (10 pts) Derive the circumference of a circle with radius r by finding the arclength of the appropriate function and using symmetry (where necessary).

$$y = \sqrt{r^2 - x^2}$$

$$y' = \frac{-2x}{2\sqrt{r^2 - x^2}}$$

$$C = 4\int_0^r \sqrt{1 + \frac{x^2}{r^2 - x^2}}\,dx$$

$$= 4\int_0^r \frac{r}{\sqrt{r^2 - x^2}}\,dx$$

$x = r\sin\theta$

$dx = r\cos\theta\,d\theta$

$$= 4\int_0^{\pi/2} \frac{r^2\cancel{\cos\theta}}{r\cancel{\cos\theta}}\,d\theta$$

$$= 4r\left[\theta\right]_0^{\pi/2}$$

$$= 4r \cdot \frac{\pi}{2}$$

Solution: $C = 2\pi r$

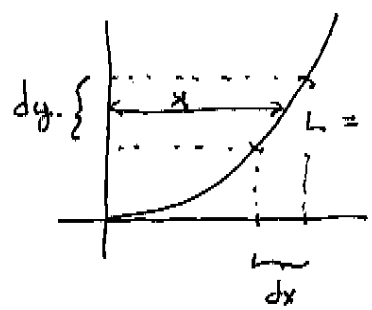
Set-up ~~the~~ an integral to find.

6.) (10 pts) ~~Find~~ the area of the surface obtained by rotating the curve $y = x^2$ on $0 \le x \le 3$ about the y-axis.

$$y = x^2$$

$$y' = 2x$$

$$SA = \int_0^3 2\pi x\sqrt{1 + 4x^2}\,dx$$



$$u = 1 + 4x^2$$

$$\frac{du}{8} = \frac{8x}{8}\,dx$$

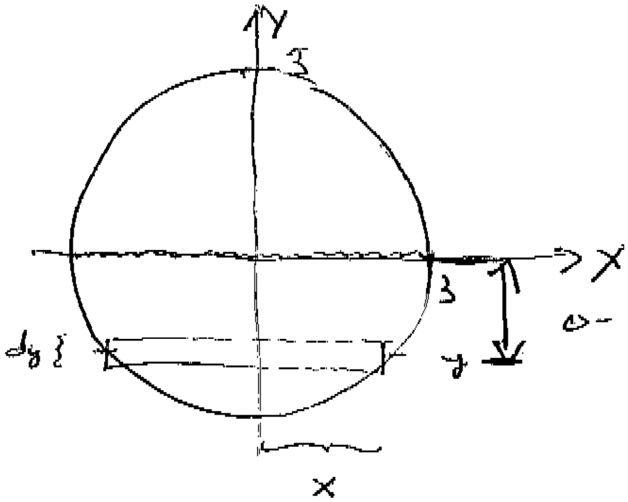
$$\text{so}\quad SA = \int_1^{37} \frac{2\pi}{8} u^{1/2}\,du$$

$$= \frac{\pi}{4}\left[\frac{2}{3}u^{3/2}\right]_1^{37}$$

Solution: $\frac{\pi}{\cancel{4}\,2}\left(\frac{2}{3}\right)\left(37^{3/2} - 1\right) \approx 117.3$

7.) (10 pts) A cylindrical water tank is placed so that the axis of the cylinder is horizontal. Find the fluid force on a circular end of the tank if the tank is half full, assuming that the diameter is 6 meters, the density of water is $1000\,{}^{kg}/_{m^3}$, and the acceleration of gravity is $9.8\,{}^{m}/_{s^2}$.

a.) Draw the picture



$x^2 + y^2 = 9$

$x = \pm\sqrt{9 - y^2}$

$0 - y = -y.$

b.) Set up the integral to determine the force (circle your answer)

$\text{Area} = 2x\,dy.$

$= 2\sqrt{9 - y^2}\,dy$

$\text{Pressure} = 1000(9.8)(3 - y).$

$$\boxed{\text{Force} = \int_{-3}^{0} 1000(9.8)(2)(3 - y)\sqrt{9 - y^2}\,dy.}$$

$= 19600 \int_{-3}^{0} (3 - y)\sqrt{9 - y^2}\,dy$

$= 19600\left(3 \underbrace{\int_{-3}^{0} \sqrt{9 - y^2}\,dy}_{9\pi/4} - \int_{-3}^{0} y\sqrt{9 - y^2}\,dy\right)$ $\quad u = 9 - y^2$

$\frac{du}{-2} = y\,dy$

$= 19600\left(3 \cdot \frac{9\pi}{4} + \frac{1}{2}\int_{0}^{9} u^{1/2}\,du\right)$ $\quad u(-3) = 0;\ u(0) = ($

$\frac{1}{2} \cdot \frac{2}{3} u^{3/2} \Big|_0^9$ $\quad 176{,}400\,N.$

$= 19600\left(\frac{27\pi}{4} + 9\right) = \cancel{592{,}032.71}$

8.) (10 pts) Suppose $f(x)$ is a continuous function on the interval $[1,\infty)$ and

$$\int f(x)\,dx = \frac{\ln x}{\sqrt{x}} + C.$$

Find $\int_4^\infty f(x)\,dx$, if it exists.

$$\int_4^\infty f(x)dx = \lim_{t\to\infty}\int_4^t f(x)dx$$

$$= \lim_{t\to\infty}\left[\frac{\ln x}{\sqrt{x}}\right]_4^t$$

$$= \lim_{t\to\infty}\left(\frac{\ln t}{\sqrt{t}} - \frac{\ln 4}{2}\right)$$

$$\overset{H}{=} \lim_{t\to\infty}\frac{\frac{1}{t}}{\frac{1}{2\sqrt{t}}} - \frac{\ln 4}{2}$$

$$= \lim_{t\to\infty}\frac{2\sqrt{t}}{t} - \frac{\ln 4}{2}$$

Solution: $= -\ln 4/2$ (OR $-\ln(2)$)

9.) (10 pts) Evaluate $\int \arctan x\,dx$ $= x\arctan x - \int \frac{x\,dx}{1+x^2}$ $\quad u = 1+x^2$, $\frac{du}{2} = \frac{2x}{2}dx$

$u = \arctan x$

$du = \frac{dx}{1+x^2}$ $\qquad - \frac{1}{2}\int \frac{du}{u}$

$dv = dx$

$v = x$ $\qquad = x\arctan x - \frac{1}{2}\ln|1+x^2| + C$

Solution: $x\arctan x - \frac{1}{2}\ln|1+x^2| + C$

10.) (10 pts) How large do we have to choose n so that the approximation T_n is an accurate approximation to $\int_1^{\pi} \frac{\cos(x)}{x}\,dx$ within 0.00001?

$$f'(x) = \frac{-\sin x \cdot x - 1 \cdot \cos x}{x^2}$$

$$= -\frac{\sin x}{x} - \frac{\cos x}{x^2}$$

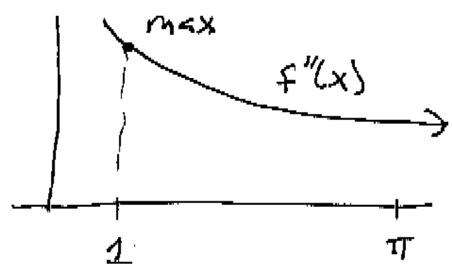
$$f''(x) = \frac{-\cos x \cdot x - 1 \cdot (-\sin x)}{x^2} - \frac{-\sin x \cdot x^2 - 2x \cos x}{x^4}$$

$$= -\frac{\cos x}{x} + \frac{\sin x}{x^2} + \frac{\sin x}{x^2} + \frac{2\cos x}{x^3}$$

$$= -\frac{\cos x}{x} + \frac{2\sin x}{x^2} + \frac{2\cos x}{x^3}$$

Graph $f''(x)$ on $[1, \pi]$

$|f''(x)| \le |f''(1)| \approx 2.23.$



$$|E_T| \le \frac{2.23(\pi-1)^3}{12\, n^2} \le 0.00001$$

$$\Rightarrow \underbrace{\sqrt{\frac{2.23(\pi-1)^3}{12(0.00001)}}}_{\approx 427.24} \le n$$

Solution: $n = 428$