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Section 9.1

Intro to Differential Equations

Goal:

- Model the spread of an epidemic

Defn A differential Equation (DE) is an equation that contains an ~~unk~~ unknown function and some of its derivatives.

Applications include:
- spread of epidemics
- population growth
- radioactive decay
- free fall
- Newton's Law of Cooling
- Resistors and Inductors (circuits)
- Hooke's Law.

Ex: 1 Population Growth.

Assumption: Under nice conditions, a population grows at a rate proportional to the size of the population.

Population — $P$

rate — derivative

$P' = k \cdot P$ (constant of proportionality) or $\frac{dP}{dt} = kP$ or $\dot{P} = kP$

Ex: 2 A more general population model.

Assumption: For small populations, the population will grow at a rate approximately proportional to the population size.

$P' \approx kP$ for small $P$ (small relative to $K$)

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Assumption 2: If the population gets too big ($P > K$) the population will decrease.

$$\frac{dP}{dt} < \emptyset \quad \text{for} \quad P > K$$

K - carrying capacity.

A differential equation that meets both assumptions is

$$\underset{\text{small } k}{\frac{dP}{dt} = kP}\left(1 - \frac{P}{\underset{\text{big } K.}{K}}\right) \qquad \text{unknown function } P.$$

Assumptions in modeling epidemics

1) Population is stable other than deaths due to epidemic.
2) Number of people that can get sick depends on
   a) Number already sick
   b) Number still healthy.
   c) Constant probability will get sick.

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Section 9.1 (continue)

A) Population Models

1) $P' = kP$ our simplest population model (ex 1 on p1)

2) $P' = kP\left(1 - \frac{P}{K}\right)$ the slightly more complex pop model (ex 2 on p1)

what is the unknown that we must solve for ?

→ $P(t)$.

other apps as DE

B) Hooke's Law (spring)

$F = kx$ $(F = ma)$

$m \cdot a = kx$

$\Rightarrow m \cdot x' = kx$ (might write $x'' = \ddot{x}$)

$\Rightarrow \ddot{x} = \frac{k}{m} x$ ← A specific solution is $x = \emptyset$

If given a potential solution to a DE, how could we test the solution?

Ex: 1 Verify that $y = \tan(x)$ is a solution to to $\dot{y} = 1 + y^2$

$\dot{y} = \sec^2(x)$ : check : $\sec^2(x) = 1 + \tan^2(x)$. ✓

are there other solutions to $\dot{y} = 1 + y^2$

test $y = \cot(x)$

$\dot{y} = -\csc^2(x)$ check: $-\csc^2(x) \neq 1 + \cot^2(x)$

test: $y = \alpha \tan(x)$

$\dot{y} = \alpha \sec^2(x)$ check: $\alpha \sec^2(x) \neq 1 + \alpha^2 \tan^2(x)$

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test: $y = \tan(x+\beta)$

$\dot{y} = \sec^2(x+\beta) =$ check: $\sec^2(x+\beta) = 1 + \tan^2(x+\beta)$. ✓

Method for verification

difference between specific and general solution

↑ all solutions.

Ex:3 a simple example

$\dot{y} = y$

$y = e^x$ is a solution

$y = 0$ is a solution

$y = ce^x \iff \dot{y} = ce^x$ general solution.

Ex:4 verify that $y = \frac{1}{\frac{t^3}{3}+C}$ is a solution to $\frac{dy}{dt} = -t^2y^2$

$\leftarrow\left(\frac{t^3}{3}+C\right)^{-1}$

test: $\frac{dy}{dt} = -\left(\frac{t^3}{3}+C\right)^{-2} \cdot t^2$

$= \frac{-t^2}{\left(\frac{t^3}{3}+C\right)^2}$

$= -t^2 \cdot \left(\frac{1}{\left(\frac{t^3}{3}+C\right)}\right)^2$

$= -t^2y^2$ ✓

What does the constant C do in (Ex:4)?

Ex:4 T+v: Solve $\frac{dy}{dt} = -t^2y^2$

given that $y(0) = 2$ initial condition

General solution: $y = \frac{1}{\frac{t^3}{3}+C}$ $\quad y(0) = 2 = \frac{1}{0+C}$

$\Rightarrow C = \frac{1}{2}$

So, the solution to the initial value problem is $y = \frac{1}{\frac{t^3}{3}+\frac{1}{2}}$.